14.4 TANGENT PLANES; LINEAR approximations (cont'd)

If \( f(x, y) = z \) at \((x_0, y_0)\) \( f(x, y) \) are curves on surface through \((x_0, y_0, f(x_0, y_0))\) eq'n of tangent plane to surface \( z = f(x, y) \) at \((x_0, y_0, z_0)\) is:

\[
\begin{align*}
\frac{z - z_0}{f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)} & = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
\end{align*}
\]

Ex: Find eq'n of plane tangent to \( z = x^2 - 4y^2 \) at \((3, 1, 5)\).

\[
\begin{align*}
f(x, y) & = x^2 - 4y^2 \\
\frac{\partial f}{\partial x} & = 2x \\
\frac{\partial f}{\partial y} & = -8y
\end{align*}
\]

\[
\begin{align*}
z - 5 & = 6(x - 3) - 8(y - 1) \\
\Rightarrow \quad z & = 6x - 18 - 8y + 8
\end{align*}
\]
$5 = 6x - 8y - 7$

**Recall:** $y = f(x)$

$m_T = f'(a)$

"Close" to $(a, f(a))$

_Y-VALUE ON LINE_ $\approx f(x)$-VALUE ON CURVE

$dy \approx \Delta y$

$\Delta y = f(a + \Delta x) - f(a)$

$e_i y = f'(x) dx$

$\frac{dy}{dx} = f'(x) = \frac{dy}{dx}$

**Back to $R^2/R^3$:**

TANGENT PLANE TO SURFACE $z = f(x, y)$ 'near' $(a, b, f(a, b))$ has $z$-VALUES close to $z$-VALUES ON SURFACE.

The linearization of $f$ at $(a, b)$

$f(x, y) \approx f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b)$
ΔZ = f(x+Δx, y+Δy) - f(x, y)

**DFN** IF Z = f(x, y) THEN f
IS DIFFERENTIABLE AT (x, y)
IF ΔZ CAN BE
EXPRESSED IN FORM

ΔZ = f_x(x, y) Δx + f_y(x, y) Δy + ε_1 Δx + ε_2 Δy

WHERE ε_1, ε_2 → 0 AS Δx, Δy → 0

ex = f(x, y) = 5x^2 + 3xy, NEAR (2, 1)

f(2, 1) = 5(4) + 3(2)(1) = 26

\[ f(x+Δx, y+Δy) = 5(2+Δx)^2 + 3(2+Δx)(1+Δy) \]

\[ = f(2, 1) + \frac{2}{2} \]

ΔZ = 5(4 + 4Δx + Δx^2) + 3(2+2Δy+Δx+Δy) - 26

= 20 + 20Δx + 5Δx^2 + 6 + 6Δy + 3Δx + 3ΔxΔy - 26

= 20Δx + 6Δy + 5Δx^2 + 3Δx + 3ΔxΔy - 26
\[ \Delta z = f_x(2,1) \Delta x + f_y(2,1) \Delta y \]
\[ + (5 \Delta x) \Delta x + (3 \Delta x) \Delta y \]

As \( \Delta x, \Delta y \to 0 \)

**THEOREM:** IF \( f_x(x, y) \) \& \( f_y(x, y) \) EXIST NEAR \((a, b)\) AND ARE CONTINUOUS AT \((a, b)\) THEN \( f(x, y) \) IS DIFFERENTIABLE AT \((a, b)\).

**DEFINITION:** THE (TOTAL) DIFFERENTIAL OF \( z = f(x, y) \) IS DENOTED \( dz \) AND

\[ dz = f_x(x, y) \, dx + f_y(x, y) \, dy = \frac{\partial z}{\partial x} \, dx + \frac{\partial z}{\partial y} \, dy \]

NOTE: NEAR \((a, b)\) \( dz \approx \Delta z \)

\[ f_x(x, y) = 5x^2 + 3xy \]

\( f(2,1) = 26 \)

\[ f(1.9, 1.03) = 5(1.9)^2 + 3(1.9)(1.03) \]

\[ = 23.921 \]

\[ f_x = 10x + 3y \]

\[ f_y = 3x \]

\[ f_x(2,1) = 23 \]

\[ f_y(2,1) = 6 \]

\[ \Delta x = 1.9 - 2 = -0.1 \]

\[ \Delta y = 1.03 - 1 = 0.03 \]
So \( \Delta z = f(1.9, 1.03) - f(2,1) = 23.921 - 26 = -2.079 \)

\[
dz = f_x(x,y)dx + f_y(x,y)dy = \left[ 10x + 3y \right]dx + \left[ 3x^2 \right]dy \bigg|_{(2,1)} = 23(-.1) + (0)(.03)
\]

\[
\begin{align*}
\text{ON FRIDAY WE WILL } & \quad = -2.3 + .18 \\
\text{COVER 14.5 CHAIN RULE(S) } & \quad = -2.12
\end{align*}
\]

\[\text{MIDTERM EXAM: 6 PROBLEMS, (12.1 - 14.3) 15-20 RESPONSES,} \]
\[\text{1 PROBLEM: VECTOR OPERATIONS: } \cdot, \times, \perp, \]
\[\text{ANGLES, PROJECTIONS, COMPONENTS, AREAS.} \]
\[\text{2 PROBLEMS: EQNS OF LINES \& PLANES} \]
\[\text{INTERSECTION 5} \]
\[\text{(2PTS \rightarrow LINE; 3PTS \rightarrow PLANE) AREAS.} \]
2 Problems

Vector Functions \( \mathbf{F}(t) = (x, y, z) \)

- Derivatives,
- Integrals,
- Tangent Vectors/Lines,
- Arc Length,
- \( T, N, B \) (and \( \ddot{\mathbf{B}} \))
- Curvature

1 Prob.

Functions of Several Variables,
Describe Domain Levels, Curves
Partial Derivatives (Higher)