14.7 MAX/MIN VALUES (CONT'D)

EX (CONT'D FROM WED)

Find and classify crit. pt. OP f(x,y)

\[ f_x = 6x^2 + y^2 + 10x \]
\[ f_y = 2xy + 2y \]

Both 0 at (0,0)
\[ \left( -\frac{5}{3}, 0 \right), (-1, \pm 2) \]

Last time

\[ f_{xx} = 12x + 10 \]
\[ f_{yy} = 2x + 2 \]
\[ f_{xy} = 2y = f_{yx} \]

\[ D = (12x+10)(2x+2) - [2y]^2 \]
\[ D(0,0) = (10)(2) - [0]^2 = 20 > 0 \]
\[ D(-\frac{5}{3}, 0) = (-10)(-\frac{4}{3}) - [0]^2 > 0 \]

f(0,0) is a local min = 0

f(-\frac{5}{3}, 0) is a local max

\[ = 2 \left(-\frac{5}{3}\right)^2 + 0 + 5 \left(-\frac{5}{3}\right)^2 + 0 = \frac{-250}{27} + \frac{125}{9} = \frac{-250 + 375}{27} = \frac{125}{27} \]
\[ D(-1,2) = (-2)(0) - [4]^2 = -16 < 0 \]

\( f(-1,2) \) is a SADDLE pt.

\[ D(-1,-2) = (-2)(0) - [-4]^2 = -16 < 0 \]

\( f(-1,-2) \) is a SADDLE pt.

\[ \text{NOTE: } D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - f_{xy}f_{yx} \]

\text{RECALL: EXTREME VALUE THM. IF } y = f(x) \text{ is cont. on } [a, b] \text{ THEN BOTH } f_{\text{max}} \text{ and } f_{\text{min}} \text{ exist: they at CRITICAL NUMBERS or AT ENDPOINTS } \]

\[ D \text{ cont. on closed bounded set in } \mathbb{R}^2 \text{ THEN } f \text{ attains both an abs max } \text{ and an abs min on } D \text{ (EXTREMES OCCUR AT CRITICAL POINTS OR ON BOUNDARY) } \]

1. FIND CRIT. PTS. (AND FUNCTION VALUES)
2. FIND EXTREMES OF \( f(x,y) \) ON BOUNDARY.
3. LARGEST = MAX \text{ SMALLEST = MIN.
Find Extremes of
\[ f(x, y) = 3 + xy - x - 2y \] on D

D is closed triangular region w/ vertices

\[ f_x = y - 1 = 0 \quad y = 1 \]
\[ f_y = x - 2 = 0 \quad x = 2 \]

\[ f(2, 1) = 3 + (2)(1) - (2) - 2(1) = 1 \]

Side A: Note \( y = 0 \) \( 1 \leq x \leq 5 \)

\[ f(x, 0) = 3 + x(0) - x - 2(0) \]
\[ = 3 - x \]
\[ f' = -1 \neq 0 \]
\[ f(1, 0) = 3(1) = 2 \]
\[ f(5, 0) = 3 - (5) = -2 \]

Side B: \( x = 1 \) \( 0 \leq y \leq 4 \)

\[ f(1, y) = 3 + (1)y - (1) - 2y \]
\[ f(1, 0) = 2 \quad = 2 - y \]
\[ f(1, 4) = -2 \]
SIDE C: LINE THROUGH (1,4) AND (5,0)

is \( x + y = 5 \) (or \( y = 5 - x \))

\[ f(x, y) = f(x, 5-x) = 3 + x(5-x) - x - 2(5-x) \]

\[ f = -x^2 + 6x - 7 \]

For \( 1 \leq x \leq 5 \)
\[ f = -2x + 6 = 0 \quad @ \quad x = 3 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>

\( f_{\text{max}} = 2 \quad @ \quad (1,0) ; (3,2) \)

\( f_{\text{min}} = -2 \quad @ \quad (5,0) ; (1,4) \)
14.8 LAGRANGIAN MULTIPLIERS

\[ z = f(x, y) \]: Draw Level Curves to get a Contour Map.

**PATH**: \( g(x, y) = k \)

Normal Vector to Path \( g \) is \( \overline{n} = \nabla g \)

At Local Extremes:

\[ \nabla f(x, y) = \lambda (\nabla g(x, y)) \]

\( \lambda \) (Lambda)

To Find Local Extremes Consider:

\[ \nabla f = \lambda \cdot \nabla g \]

\[ \langle f_x, f_y \rangle = \lambda \cdot \langle g_x, g_y \rangle \]

So \( f_x = \lambda g_x \)

\( f_y = \lambda g_y \)

\( g(x, y) = k \)
Ex (14.8.6) Find Extremes of \( f(x,y) = xe^y \)

subject to \( x^2 + y^2 = 2 \)

\( g(x,y) = x^2 + y^2 \)

\( \nabla f = \langle e^y, xe^y \rangle \)

\( \nabla g = \langle 2x, 2y \rangle \)

\( \nabla f = A \cdot \nabla g \)

\( x(A \cdot 2x) = 2Ax \)

\( x^2 = y \)

\( y = 1 \quad x = \pm 1 \quad \text{Max} \)

\( (+1, 1) \quad f(1, 1) = e \)

\( (-1, 1) \quad f(-1, 1) = -1 \cdot e \)

\( y^2 + y - 2 = 0 \)

\( (y + 2)(y - 1) = 0 \)

\( y = -2 \text{ or } y = 1 \)

\( \checkmark 2 = f(x, y) \)