12.5 EQUATIONS OF LINES / PLANES (WRAP-UP)

PLANE: THROUGH $(x_0, y_0, z_0)$ W/ NORMAL VECTOR $\vec{n} = \langle a, b, c \rangle$ IS

$$a x + b y + c z + d = 0$$

$d$ OBTAINED BY SUBBIN' $(x_0, y_0, z_0)$ INTO EQN:

$$a(x_0) + b(y_0) + c(z_0) + d = 0$$

$$d = -(a x_0 + b y_0 + c z_0)$$

NOTE: TWO PLANES ARE PARALLEL (NEVER INTERSECT) IF NORMAL VECTORS ARE PARALLEL. (ie $\vec{n}_2 = k \vec{n}_1$)

2) ANGLE BETWEEN PLANES IS THE (ACUTE) ANGLE FORMED BY NORMAL VECTORS.
EX 4. FIND ANGLE BETWEEN

\[ p_1: 4x + 4y + 2 = 8 \text{ AND } \]
\[ p_2: 2x - y + 3z = 6 \]

\[ \vec{n}_1 = \langle 4, 4, -1 \rangle \]
\[ \vec{n}_2 = \langle 2, -1, 3 \rangle \]

\[ \cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} \]

\[ \cos \theta = \frac{8 + (-4) + (-3)}{\sqrt{16+16+1} \cdot \sqrt{4+1+9}} = \frac{1}{\sqrt{33} \sqrt{14}} \]

\[ \theta = \cos^{-1} \left( \frac{1}{\sqrt{33} \sqrt{14}} \right) \cong 87.3^\circ \]

\[ 1.52 \text{ RADIANS} \]

3) DISTANCE FROM A POINT \((x_1, y_1, z_1)\) TO A PLANE \(P: ax + by + cz + d = 0\) is.

\[ D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \]

\[ D = \text{COMP}_\vec{n} (\overrightarrow{QP}) = \frac{\vec{n} \cdot (\overrightarrow{QP})}{|\vec{n}|} \]
1. Find distance from $(1,-2,4)$ to plane $3x + 2y + 6z = 5$.

$$D = \frac{|3(1) + 2(-2) + 6(4) - 5|}{\sqrt{9 + 4 + 36}} = \frac{3 - 4 + 24 - 5}{\sqrt{49}}$$

$$D = \frac{18}{7}$$

12.6 Cylinders and Quadratic Surfaces

**A Cylinder is a surface that consists of all lines (rulings) that are parallel to a given line and pass through a given plane curve.**

- Circular Cylinder
- Parabolic Cylinder
ELLiptical CURVE.

Elliptical CYLINDER

Cylinders in \( \mathbb{R}^2 \) (Easiest) are represented by equations w/ a variable missing.

Example in \( \mathbb{R}^3 \): \( y = x^2 \)

Any point on surface of form \((x, x^2, y)\)
\[ \frac{y^2}{4} + z^2 = 4 \]

X-axis goes through center of cylinder \((4, 0, 2)\)

\[ \{ (x, y, z) \mid \text{any real; } y^2 + z^2 = 4 \} \]

**QUADRIC SURFACES:**

**Recall:** Conic section in \(\mathbb{R}^2\)

From equations, \(Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0\)

Equations of form:

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J < 0 \]

"Rotate" surfaces.

A rotation will eliminate these terms.
EQNS: \( A x^2 + B y^2 + C z^2 + Dx + Ey + Fz + G = 0 \)

E \[ H \] ELIMINATE BY
E \[ H \] COMPLETING THE
E \[ H \] SQUARE.

\[ A(x-h)^2 + B(y-k)^2 + C(z-l)^2 + H = 0 \]

OR EQUATIONS W/ ONE LINEAR TERM.

\[ A(x)^2 + B(y)^2 + C(z)^2 = K \]

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

\[ q^2 = \frac{K}{A} \]

TRACE: CURVE FOR
A CONSTANT
COORDINATE.

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} \]

X-TRACES: PARABOLAS
Y-TRACES: "
Z-TRACES: ELLIPSES.
\( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \)

**X-traces:** parabolas

**Y-traces:** parabolas (opposite directions)

**Z-traces:** hyperbolas

**Hyperboloid:**

\( \frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \rightarrow \text{cone} \)

**Hyperboloids:**

**One sheet:**

\( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \)  \( \text{One neg.} \)

**Two sheets:**

\( \frac{x^2}{a^2} \bigg( \frac{y^2}{b^2} + \frac{z^2}{c^2} \bigg) = 1 \)  \( \text{Two new} \)