1. Given: \( \mathbf{u} = \langle 8, -2, 6 \rangle, \quad \mathbf{v} = \langle 3, -4, -1 \rangle, \quad \mathbf{w} = \langle 1, 3, 1 \rangle \)

Find:

a. \( \mathbf{u} \cdot (2\mathbf{v} - 5\mathbf{w}) \)

b. A unit vector in the opposite direction of \( \mathbf{w} \).

c. \( \mathbf{v} \times \mathbf{w} \)

d. The angle (in radians) formed by \( \mathbf{u} \) and \( \mathbf{v} \)

e. \( \text{proj}_\mathbf{w}(\mathbf{u}) \) [the vector projection of \( \mathbf{u} \) onto \( \mathbf{w} \)]

2. a. Find the parametric equations of the line \( \mathbb{L} \) passing through the points \((4, -1, 5)\) and \((6, 2, 1)\).

b. Find the point at which the line \( \mathbb{L} \) passes through the plane whose equation is \( x + 2y + 3z = 5 \).

c. The line \( \mathbb{L} \) intersects the line given by the vector equation \( \mathbf{r} = \langle 5, -4, 3 \rangle + s \langle 1, 2, -2 \rangle \).

Find the point of intersection.

3. Given the points \((5, 0, 1), (0, -1, 2), \) and \((8, -1, 1)\):

a. Find the equation of the plane that passes through these points. Write your answer in the form \( ax + by + cz = d \).

b. Find the area of the triangle with these three vertices.

4. Evaluate: \( \int_0^2 \left( e^{3t} \mathbf{i} + t e^t \mathbf{j} - t e^t \mathbf{k} \right) dt \)

5. Given the vector function \( \mathbf{r}(t) = \langle 2t^3, 3t^2, -3t \rangle \)

a. Find the length of the curve from \( t = 0 \) to \( t = 2 \).

b. Find the unit tangent vector \( \mathbf{T} \) and the unit normal vector \( \mathbf{N} \) at the point \( (2, 3, -3) \).

6. Given \( f(x, y) = x^3 \tan^{-1}y + \ln(4x + y^2) \), find:

a. \( f_x(x, y) \)

b. \( f_{yy}(x, y) \)

c. \( f_{yx}(x, y) \)

d. The equation of the tangent plane to the graph of \( f(x, y) \) at the point \((2, 0, f(2, 0))\).
1. a. $12$ 
   b. $< -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}} >$
   c. $<-1, -4, 13>$ 
   d. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$
   e. $<\frac{8}{11}, \frac{24}{11}, \frac{8}{11}>$

2. a. $x = 4 + 2t$
   b. $(10, 8, -7)$
   c. $(14, 14, -15)$ 
   [can also 'start' with other point]

3. a. $x+3y+8z=13$ 
   b. $\text{Area} = \frac{|AB \times AC|}{2} = \frac{\sqrt{74}}{2}$

4. $<\frac{e^6-1}{3}, e^2+1, \frac{1-e^4}{2}>$

5. a. $\int_0^2 \sqrt{(6t^2)^2 + (6t)^2 + (-3)^2} dt = \int_0^2 \sqrt{36t^4 + 36t^2 + 9} dt$

   $\int_0^2 \sqrt{(6t^2 + 3)^2} dt = \int_0^2 (6t^2 + 3) dt = (2t^3 + 3t)|^2_0 = 22$

   b. $\mathbf{T} = <\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}>$
   $\mathbf{N} = <\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}>$

6. a. $f_x(x, y) = 3x^2\tan^{-1}y + \frac{4}{4x+y^2}$
   
   b. $f_{yy}(x, y) = \frac{-2x^3y}{(1+y^2)^2} + \frac{8x-2y^2}{(4x+y^2)^2}$
   
   c. $f_{yx}(x, y) = \frac{3x^2}{(1+y^2)^2} - \frac{8y}{(4x+y^2)^2}$
   
   d. $x + 16y - 2z = 2 - 2\ln 8$