Chapter 13 - Vector Functions

13.1 Vector Functions and Space Curves

Definition: A vector-valued function is a function with single real input (usually $t$) and a vector output: $\mathbf{r} : \mathbb{R} \to \mathbb{R}^n$

$$\mathbf{r}(t) = < x(t), y(t), z(t) > = < f(t), g(t), h(t) > \quad \text{[component functions]}$$

Domain of the vector function is the largest set common to all functions.

Example: $\mathbf{r}(t) = < \ln t, \sqrt{4-t}, \frac{t}{t^2-9} >$

Definition: limit of a vector function

$$\lim_{t \to a} \mathbf{r}(t) = < \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) > \quad \text{provided all limits exist}$$

Example: Find $\lim_{t \to 0} < t e^t, \frac{\sin t}{t}, \frac{t}{1-e^t} >$

A vector function is \textbf{continuous at} $a$ in case $\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$

If the domain of $\mathbf{r}$ is an interval $I$, then the graph of $\mathbf{r}$ is a \textbf{space curve}.

Example: $\mathbf{r}(t) = < 2-3t, 4-t, 5t >$

- line

Example: $\mathbf{r}(t) = < \cos t, \sin t, t >$

- helix

Example: $\mathbf{r}(t) = t \mathbf{i} + t \mathbf{j} + (\cos t) \mathbf{k}$

Look at problems in text: 13.1 21 - 26 (matching equations w/ graphs)