Chapter 12 - Vectors and the Geometry of Space

12.1 Three-Dimensional Coordinate Spaces

Coordinate axes: Three axes, mutually perpendicular.

Right-hand rule for z-axis.

Notation: coordinate planes

Octants, first octant.

Three-dimensional coordinate system:

\[ \mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) | x, y, z \in \mathbb{R}\} \]

Graphs of equations: \( x = 2 \)

\[ x + y = 5 \]

Distance formula: If \( P_1 = (x_1, y_1, z_1) \) and \( P_2 = (x_2, y_2, z_2) \), then

\[ |P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \]

Example: find distance from \((2, -1, 4)\) to \((5, 3, -2)\)

Equation of a sphere with center \( C = (h, k, l) \), radius = \( r \)

\[ (x - h)^2 + (y - k)^2 + (z - l)^2 = r^2 \]

Example: Find the center and radius of the sphere whose equations is

\[ x^2 + y^2 + z^2 - 10x - 6y - 14z + 3 = 0 \]