12.6 (cont’d)

**QUADRIC SURFACES:** Eqs’ns of form 

\[ Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0 \]

\[ \Rightarrow \]

\[ x^2 + y^2 + z^2 = 25 \leftarrow \text{SPHERE} \]

**RECALL:** Conic sections in \( \mathbb{R}^2 \)

**Equations of form** 

\[ Ax^2 + By^2 + Cx + Dy + Ey + F = 0 \]

**Graphs are possible intersections of a plane with a cone.**

**Circle:** 

\[ x^2 + y^2 = r^2 \] (Center at \((0,0)\))

\[ (x-h)^2 + (y-k)^2 = r^2 \] (Center at \((h,k)\))
**PARABOLA**: \( y = x^2 \) (vertex at \((0,0)\))

**ONE SQUARE TERM**.

\( x = y^2 \) (vertex at \((h,k)\))

\((y-k) = (x-h)^2\)

or \((x-h) = (y-k)^2\)

**ELLIPSE**

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Center at \((0,0)\)

Vertices at \((\pm a, 0)\) and \((0, \pm b)\)

Center at \((h, k)\)

\[ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \]

Both squares are in equation, but coeffs not equal (both positive)

\[ 3x^2 + 7y^2 + 6x - 13 = 0 \leftarrow \text{ellipse} \]
**Hyperbola**

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{Center at} \ (0, 0)
\]

\[
y = \pm \frac{b}{a} x
\]

\[
\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1
\]

\[
(x-h)^2 \quad (y-k)^2 = 1 \quad \text{Center at} \ (h, k)
\]

Both square terms in eq'n, but opposite signs. (For hyperbolas which open up/down or left/right.)

\[
\begin{align*}
0 &= 6x^2 - y^2 + 8y = 12 \quad \text{Hyperbola}
\end{align*}
\]

Note:

\[
xy = k
\]

\[
y = \frac{k}{x}
\]
**GRAPHS OF QUADRATIC SURFACES.**

Identify by considering "traces" on surfaces: set one variable = constant.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{< ALL THREE SQUARES, ALL SAME SIGN.>}
\]

Ellipsoid.

\[
\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \quad \text{NO } z^2 \text{ TERM.}
\]

\[x^2, y^2 \text{ SAME SIGN W/Coeff.}\]

X-traces \((x=k)\) are parabolas

Y-traces \((y=k)\) — —

Z-traces \((z=k)\) ellipses.

Elliptic \quad Paraboloid
**Hyperbolic Paraboloid**

\[ \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \]

- No \( z^2 \) terms
- \( x^2, y^2 \) have opposite signs
- \( x, y \)-traces are parabolas (open in opposite directions)
- \( z \)-traces \( \rightarrow \) hyperbolas

**Cones:**

\[ \frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]

- All squares
- Two ones sides
- Once on other

\( x = a: \)

\[ \frac{z^2}{c^2} = 1 + \frac{y^2}{b^2} \]

\[ \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1 \]

\( y = b \)

\( \rightarrow \) Hyperbola.
HYPERBOLIDS

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \]

ONE NEGATIVE SQUARE \( \rightarrow \) OF ONE SHEET

TWO NEGATIVE COEFF'S \( \rightarrow \) OF TWO SHEETS

NOTE "DIFF" ONE IS "AXIS OF SYMMETRY"

ex. \( 25y^2 + z^2 = 100 + 4x^2 \)

\[ -4x^2 + 25y^2 + z^2 = 100 \]

\( 2 \div 100 \)

\( -\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{100} = 1 \) \( \leftarrow \) HYPERBOLOID OF ONE SHEET

\( X \)-DIRECTION.
\[ 9x^2 - 4y^2 - z^2 - 6y + 6z = 0 \]

\[ 9x^2 - 4(y^2 + 2y + 1) - (z^2 - 6z + 9) = 0 + (-4) + (-9) \]

\[ 9x^2 - 4(y+1)^2 - (z-3)^2 = -13 \]

\[ \frac{9x^2}{-13} + \frac{4(y+1)^2}{13} + \frac{(z-3)^2}{13} = 1 \]

**Hyperboloid of one sheet.**