1. Given the points $A = ( 5, 3, 0)$, $B = ( 6, 1, 1)$, and $C = (1, 5, 2)$, find:

   a. parametric equations for the line that passes through $C$ and is parallel to the line that passes through $A$ and $B$.
   
   b. the angle $\angle BAC$.
   
   c. the equation of the plane (in the form $ax + by + cz = d$) through these three points.

2. Given the equation $z^2 = 36 - 4x + 9y^2$:

   a. Classify this quadric surface.
   
   b. Find the equation of the tangent plane to this surface at the point $(5, 1, -5)$. Express your answer in the form $ax + by + cz = d$.

3. Find $r(t)$ if $r'(t) = (t)i + (e^{2t})j + (te^t)k$ and $r(0) = i + 2j + 4k$.

4. Given the curve defined by $r(t) = \langle -2t + 3, t^2 - 1, \frac{1}{3}t^3 + 2 \rangle$, find:

   a. the vector equation for the line $L(t)$ tangent to this curve when $t = 1$.
   
   b. the length of this curve between the points $\left(1, 0, \frac{7}{3}\right)$ and $\left(-3, 8, 11\right)$.
   
   c. the curvature $\kappa$ at the point when $t = 1$.

5. Given $f(x, y, z) = 6x^3y + 4x^2y^3z - 5x^2z^2$, find:

   a. $f_{xz}(x, y, z)$
   
   b. $\nabla f(1, 2, -1)$
   
   c. the directional derivative of $f(x, y, z)$ at $(1, 2, -1)$ in the direction of the point $(5, 1, 7)$.
6. A cardboard box without a top needs to have a volume of 4000 cubic centimeters. Find the dimensions of the box that minimizes the amount of cardboard used. List the dimensions in the following order: length, width, height.

7. Evaluate by reversing the order of integration: 
\[ \int_0^8 \int_{\sqrt[3]{x}}^{\sqrt[2]{x}} e^{y^4} \, dy \, dx \]

8. Switch to polar coordinates to evaluate: 
\[ \int_0^3 \int_y^{\sqrt{18-y^2}} e^{x^2+y^2} \, dx \, dy \]

9. Find the volume of the solid in the first octant bounded by the cylinder \( y = x^2 \) and the plane \( 2x + 4y + z = 20 \).

10. Use a triple integral to find the volume of the 'spherical cap' inside the sphere \( x^2 + y^2 + z^2 = 16 \) that is above the plane \( z = 2 \).
Math 22 - Calculus of Several Variables  
M. Eastman - Winter 2016

1. a. \( y = 5 - 2t \)
   b. \( \cos^{-1} \left( -\frac{1}{2} \right) = \frac{2\pi}{3} \)
   c. \( x + y + z = 8 \)

2. a. hyperbolic paraboloid
   b. \( 2x - 9y - 5z = 26 \)

3. \( \mathbf{r}(t) = \left( \frac{1}{2} t^2 + 1 \right) \mathbf{i} + \left( \frac{1}{2} e^{2t} + \frac{3}{2} \right) \mathbf{j} + (te^t - e^t + 5) \mathbf{k} \)

4. a. \( \mathbf{L}(t) = < 1 - 2t, 2t, \frac{7}{3} + t > \)
   b. \( \frac{38}{3} \)
   c. \( \frac{2}{9} \)

5. a. \( 8xy^3 - 10z \)
   b. \( < -33, -42, 42 > \)
   c. \( \frac{82}{3} \)

6. Minimize : \( A = lw + 2lh + 2wh \) subject to \( V = lwh = 4000 \)
   Setting \( \nabla A = \lambda \nabla V \) yields \( l = 20 \text{ cm}, w = 20 \text{ cm}, h = 10 \text{ cm} \)

7. \( \int_0^2 \int_0^3 e^y dy dx = \ldots = \frac{1}{4} (e^{16} - 1) \)

8. \( \int_0^\pi \int_0^{3\sqrt{2}} e^{r^2} r dr d\theta = \ldots = \frac{\pi}{8} (e^{18} - 1) \)

9. \( V = \int_0^2 \int_{x^2}^{5 - \frac{1}{2} x} (20 - 2x - 4y) dy dx = \ldots = \frac{244}{5} \)

10. Cylindrical coordinates yield simplest triple integral :
   \( V = \int_0^{2\pi} \int_0^{2\sqrt{3}} \int_0^{\sqrt{16 - r^2}} r dz dr d\theta = \ldots = \frac{40\pi}{3} \)