8.2 Fluid Pressure and Force

**Fluid Force:** the force on an object submerged in a fluid.

**Fluid Pressure exerted by a fluid:** is proportional to the depth \((h)\), and does not act in a specific direction.

Definition: The \( p \) at depth \( h \) in a fluid of mass density \( \rho \) is \( p = \rho gh \), where \( g \) is the acceleration due to gravity \((g = 9.8 \text{ m/sec}^2)\).

Force on a plate submerged at a specific depth in a fluid is given by
\[
F = \text{pressure} \times \text{area} = pA
\]

Note: units for force are Newtons \((N = kg \text{m/sec}^2)\);
units for pressure are Pascals \((Pa = N/m^2 = kg/m \text{sec}^2)\)
units for mass density is mass per unit volume \((\rho = kg/m^3)\)

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Note: pressure at any point in a liquid is the same in all directions at a given depth \(d\).

Example: Find the fluid force on the bases of a truncated inverted pyramid submerged in water, where the larger base has an area of 25 \(m^2\) and is 2 meters below the surface, and the smaller base has an area of 4 \(m^2\) and is 7 meters below the surface. Note the density of water is \(\rho = 1000 kg/m^3\).

Total force on a plate that is at varying depths requires a Riemann sum of strips of the plate at various (constant) depths: \( F \approx \rho g \sum_{j=1}^{N} y_j (f(y_j) \Delta y) \)

Theorem 1: The fluid force \( F \) of a flat side submerged vertically in a fluid is
\[
F = \rho g \int_{a}^{b} y f(y) \, dy, \text{ where } f(y) \text{ is the width of the side at depth } y.
\]

Example (8.2.4): A plate in the shape of an isosceles triangle with base 1m and height 2m is submerged vertically in a tank of water so that its vertex touches the surface of the water. Find the fluid force \(F\) on the surface.

Example: A trapezoidal dam 2 m in depth, 6 m at the bottom, 10 m at top is filled with water. Find the force on the dam.

Example (8.2.21): The Three Gorges Dam has a height 185m, has a dam face that is a trapezoid 3000m across at the top and 2000m across at the bottom, and is inclined at an angle of 55°. Calculate the force on the dam.