6.3 Volumes of Revolution

Volume of rotation: A 'strip' of area perpendicular to axis of rotation yields a 'disk' whose volume is $\pi \cdot (\text{radius of disk})^2 \cdot (\text{thickness})$.

Hence volume $\approx \sum \pi(r)^2 \Delta x \rightarrow \int_a^b \pi("r")^2 \, dx$

Volume of Revolution: Disk Method  If $f$ is continuous and $f(x) \geq 0$ on $[a, b]$, then the solid obtained by rotating the region under the graph about the $x$-axis has volume

$$V = \int_a^b \pi("r")^2 \, dx = \pi \int_a^b (f(x))^2 \, dx$$

example: Find the area under $y = x^2$, $y = 0$, $x = 3$ rotated around $x$-axis.

example: Use the method to develop the formula for the volume of a sphere of radius $R$.

Note: If strip does not touch the axis of rotation, a 'washer' is formed, with volume $= \pi(R_{outer}^2 - R_{inner}^2)(\text{thickness})$

Volume of Revolution: Washer Method  If $f$ and $g$ are continuous and $f(x) \geq g(x) \geq 0$ on $[a, b]$, then the solid obtained by rotating the region between the graphs about the $x$-axis has volume

$$V = \pi \int_a^b (R_{outer}^2 - R_{inner}^2) \, dx = \pi \int_a^b \left((f(x))^2 - (g(x))^2\right) \, dx$$

example: Find the volume obtained by rotating the region bounded by $y = x^3$ and $y = x$ around $x$-axis.

Revolving a region about a vertical axis: choose horizontal strips that are perpendicular to the axis of rotation. In this case, the 'thickness' of a strip is $\Delta y$. The volume is calculated with an integral taken with respect to $y$.

example: Area bounded by $x = y - y^2$, $x = 0$ rotated around the $y$-axis.

example: Area bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ rotated around line $y = -1$. 