5.6 Net Change as the Integral of a Rate of Change

Given a quantity that changes at a given rate \( r(t) \) that depends on time, then if \( r(t) = r \) is constant the change in the quantity over an interval is \( r \cdot \Delta t \).

Theorem 1 (Net Change as an Integral of a Rate of Change) The net change in \( s(t) \) over an interval \([t_1, t_2]\) is given by the integral:

\[
\int_{t_1}^{t_2} s'(t) \, dt = s(t_1) - s(t_2)
\]

example (5.6.2): A population of insects increases at a rate of \( 200 + 10t + 0.25t^2 \) insects per day (\( t \) is measured in days). Find the change in the insect population after 3 days assuming that there are 35 insects at \( t = 0 \).

Theorem 2 (The Integral of Velocity) For an object in linear motion with velocity \( v(t) \):

- Displacement during \([t_1, t_2]\) is \( \int_{t_1}^{t_2} v(t) \, dt \)
- Distance traveled during \([t_1, t_2]\) is \( \int_{t_1}^{t_2} |v(t)| \, dt \)

example: If \( v(t) = t^2 - 2t - 8 \), find a) displacement (net distance) and

b) (total) distance traveled during the time \( 1 \leq t \leq 6 \).

Marginal cost: If \( C(x) \) represents the cost of producing \( x \) units of a product, the derivative \( C'(x) \) is call the **marginal cost**. The cost of increasing production from \( a \) to \( b \) units is given by the integral

\[
\int_a^b C'(x) \, dx
\]

example (5.6.17): A small boutique produces wool sweaters at a marginal cost of \( 40 - 5\left[\frac{x}{5}\right] \) for \( 0 \leq x \leq 20 \), where \( [x] \) is the greatest integer function. Find the cost of producing 20 sweaters.