6.5 Work \div Energy

Work = Force \times Distance.

Note: Force = Mass \times Acceleration.

**English System**

Force = Pounds. (Mass = Slugs)

\[ \text{Work} = F \times D \]  
\text{(Pounds) \times (Feet)}

**Example:**

Textbook weighs 6 pounds. Lift 4 feet.

\[ \text{Work} = 6 \times 4 = 24 \text{ Foot Pounds} \]

**Metric System**

Force = Mass \times Acceleration.

\[ = 3 \text{kg} \times g \text{ m/sec}^2 \quad g \approx 9.8 \text{ m/sec}^2 \]

\[ = 3 \text{kg} \times 9.8 \text{ m/sec}^2 = 29.4 \text{ kg.m/sec}^2 \]

\[ 1 \text{ N} = 1 \text{ kg.m/sec}^2 \]

\[ \text{Newton} \]
Ex 60 pound bag move 20 ft

\[ \text{Work} = 60 \cdot 20 = 1200 \text{ foot-pounds} \]

Ex 25 kg, move 6 m.

\[ \text{Work} = 25 \cdot 9.8 \cdot 6 = 1470 \text{ kg m}^2 \text{sec}^{-2} \text{ m} \]

\[ = 1470 \text{ N m} \]

1 Newton-meter = 1 Joule.

If \( F \) varies, distances may vary.

\[ \text{Work} = 60 \cdot 20 \text{ pounds feet} \]

\[ = 1600 \text{ foot-pounds} \]

Start with 80 lb at base. Weighs 60 lb at top. Look at = \( \Delta x \)

Force \( = F(x) = 80 - x \)
\[ W \approx \sum_{i=1}^{n} F(x) \cdot \Delta x \]

\[ \Delta x \to 0 \]

\[ \text{WORK} = \int_{\text{START}}^{\text{END}} F(x) \, dx = \int_{0}^{2} (80-x) \, dx \]

\[ = 80x - \frac{x^2}{2} \bigg|_{0}^{2} = 80(20) - \frac{(40^2)}{2} \]

\[ = 1600 - 200 = 1400 \text{ ft. pounds} \]

**Springs**

**Hooke's Law**

**Force \propto \text{Displacement}**

\[ F = \frac{k \cdot x}{\text{constant of proportionality}} \]

**Example**

\[ F = 15 \text{N stretches a spring from 20 cm (equilibrium) to 23 cm.} \]

\[ F = 15 \text{N} \]

\[ 15 = k (0.03) \]

\[ 15 \cdot \frac{1}{0.03} = k \]

\[ 500 = k \]
Find work done in stretching spring from 23 cm to 30 cm. \[ F = kx = 500 \cdot x \text{ N} \]

For \( 3 \text{ cm} \leq x \leq 10 \text{ cm} \)

\[ 0.03 \leq x \leq 0.10 \]

\[
W = \int_{0.03}^{0.10} (500x) \, dx = 500 \left( \frac{x^2}{2} \right) \bigg|_{0.03}^{0.10}
\]

\[
= 250 \left( (0.1)^2 - (0.03)^2 \right)
\]

\[
= 250(0.0091) = 2.275 \text{ N m}
\]

EX: Drawing (pulling) a weight up

- Chain weighs 2 pounds/foot
- Lift a 50 pound bucket out of a 30 foot well. Find work required.

\[ 50 \text{ lb} \cdot 30 \text{ ft} = 1500 \text{ ft lb} \]

Required work to separate bucket from chain.
 WEIGHT OF EACH "CHUNK"

\[ \int_{0}^{30} 2 \, dx \times \frac{x}{\text{foot}} \text{ foot} \]

\[ = \frac{x^2}{2} \bigg|_{0}^{30} = (300^2 - 0) = 900 \text{ F.T.-Lb} \]

TOTAL WORK = 1500 + 900 = 2400 F.T.-Lb

TOTAL WEIGHT
AT START = 110 lb. √
AT END = 50 lb. √

WEIGHT = FORCE = 110 - 2y
(AT ANY POINT IN 'LIFT')

WORK = \[ \int_{y=0}^{y=30} (110-2y) \, dy = \int_{0}^{30} (110-2y) \, dy \]

= 10y - y^2 \bigg|_{0}^{30} = (3000 - 900) - 0
Pumping fluid out of a container.

Less work (small distance)

More work (greater distance)

Look at all fluid at a certain depth

Note: Water "weighs" 62.5 lb/ft³

Force = 62.5 (Volume)
METRIC SYSTEM

\[
\frac{1000 \text{ kg}}{(\text{Volume})} \times \frac{9.8 \text{ m/s}^2}{\text{m}^2} = 9800 \text{ Volume} = \text{Force}.
\]