1. a. If \( F(x) = 4x \cdot \tan^{-1}x - 2\ln(1 + x^2) \), show that \( F'(x) = 4\tan^{-1}x \).

b. Use the result of part a. and the Fundamental Theorem of Calculus to evaluate
\[
\int_1^4 (4\tan^{-1}x) \, dx.
\]

2. The traffic flow rate past a certain point on a highway is \( q(t) = 5000 + 1000t - 600t^2 \), where \( t \) is the number of hours after 8 am. How many cars pass by in the time interval between 9 am and 11 am?

3. The area under the curve \( y = \sqrt{x^2 + 2x} \) between \( x = 0 \) and \( x = 3 \) is rotated around the line \( x = -1 \). Find the volume of the solid that is generated.

4. A tank has ends that are trapezoids 8 meters wide at the top and 6 meters wide at the bottom. The tank is 4 meters high and 10 meters long. The tank is filled with water to a depth of 3 meters. Find the work done in pumping the water just over the top of the tank. State the units of your answer. Recall that the density of water is 1000 kg/m³.

5. Evaluate:
\[
\int x^5 \, dx
\]

6. Evaluate:
\[
\int \frac{3x^2 + 19x + 17}{x^3 + 8x^2 + 17x} \, dx
\]

7. Evaluate:
\[
\int \sin(\sqrt{x}) \, dx
\]
1. a. \[ F'(x) = [4 \tan^{-1} x + 4x \frac{1}{1+x^2}] - 2 \frac{1}{1+x^2}(2x) = 4 \tan^{-1} x + \frac{4x}{1+x^2} - \frac{4x}{1+x^2} = 4 \tan^{-1} x \]

b. \[ \int_1^4 (4 \tan^{-1} x) \, dx = 4 \tan^{-1} x - 2 \ln(1 + x^2) \bigg|_1^4 = \ldots = 16 \tan^{-1} 4 + 2 \ln \left( \frac{2}{17} \right) - \pi \]

2. \[ 9\text{am} \rightarrow t = 1 \ ; \ 11\text{am} \rightarrow t = 3 \int_1^3 (5000 + 1000t - 600t^2) \, dt = \ldots = 8,800 \]

3. using shells: \[ V = \int_0^\sqrt{12} 2\pi(x+1)\sqrt{x^2 + 2x} \, dx \quad \text{(substitution: let } u = x^2 + 2x) \ldots = 10\pi \sqrt{15} \]

using washers: \[ V = \int_0^\sqrt{12} \pi \left( (4)^2 - (\sqrt{y^2 + 1})^2 \right) dy = \ldots = 10\pi \sqrt{15} \]

4. vertical \( y \)-values measured from the bottom of the tank: using similar right triangles at the end of the tank yields: \[ \text{width of horizontal strip} = 6 + 2x = 6 + \frac{1}{2}y \quad \text{distance to lift slice} = 4 - y \]

\[ W = \int_0^3 \text{(distance)(force)} = \int_0^3 \text{(distance)(density \cdot volume)} = \int_0^3 (4 - y)(9.8 \cdot 1000 \cdot 10 \left(6 + \frac{1}{2}y\right) dy) = \ldots = 4,851,000 \text{ Joules} \]

5. \[ \int \frac{x^5}{\sqrt{x^2 + 4}} \, dx = \ldots \text{(trig substitution: } \tan \theta = \frac{x}{2}) \ldots = \frac{1}{2} \int \left( \frac{32 \tan^5 \theta}{\sec \theta} \right) 2 \sec^2 \theta \, d\theta = 32 \int \tan^5 \theta \sec \theta \, d\theta = \ldots \text{(substitution: } u = \sec \theta) \ldots = \frac{1}{5} (x^2 + 4)^{5/2} - \frac{8}{3} (x^2 + 4)^{3/2} + 16 (x^2 + 4)^{1/2} + C \]

6. \[ \int \frac{3x^2 + 19x + 17}{x^3 + 8x^2 + 17x} \, dx = \int \left( \frac{1}{x} + \frac{2x + 11}{x^2 + 8x + 17} \right) \, dx \]

\[ \int \left( \frac{1}{x} \right) \, dx = \ln|x| \quad \int \left( \frac{2x + 11}{(x + 4)^2 + 1} \right) \, dx = \ldots \text{(trig substitution: } \tan \theta = x + 4) \]

\[ = \int \left( \frac{2\tan \theta + 3}{\sec^2 \theta} \right) \sec^2 \theta \, d\theta = \int (2 \tan \theta + 3) \, d\theta = 2 \ln|\sec \theta + 3\theta = 2 \ln \sqrt{x^2 + 8x + 17} + 3 \tan^{-1} (x + 4) \]

\[ \int \frac{3x^2 + 19x + 17}{x^3 + 8x^2 + 17x} \, dx = \ln|x| + \ln|x^2 + 8x + 17| + 3 \tan^{-1} (x + 4) + C \]

7. \[ \int \sin(\sqrt{x}) \, dx = \text{(sub: } w = \sqrt{x} \text{ then IBP: } u = w, \ dv = \sin w \, dw) = -2 \sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) + C \]