1. Evaluate: for the definite integrals please give the exact answer.

   a. \( \int \sin^3 x \cos^4 x \, dx \)

   b. \( \int_1^9 \frac{\ln x}{\sqrt{x}} \, dx \)

2. Evaluate: \( \int \frac{12x-18}{x^3-9x} \, dx \)

3. Evaluate: show your work and give the exact answer: \( \int_3^{3\sqrt{3}} \frac{1}{x^2 \sqrt{36-x^2}} \, dx \)

4. Find the area of the region bounded by the two curves \( y = \sin x \) and \( y = 2 \sin^2 x \) between \( x = 0 \) and \( x = \frac{\pi}{2} \).

5. Let \( \mathbb{R} \) be the region in the first quadrant under the curve \( f(x) = \frac{1}{x^2+16} \).

   a. Find the area of \( \mathbb{R} \), or explain why such an area cannot be found.

   b. The region \( \mathbb{R} \) is rotated around the \( y \)-axis. Find the volume that is generated, or explain why such a volume cannot be found.
6. The part of the curve given by \( y = \frac{x^3}{2} \) from \((0, 0)\) to \((2, 4)\) is rotated around the \(x\)-axis. Find the area of the surface that is generated.

7. Determine whether the following series converge or diverge. Be specific with your answers when appropriate (absolute vs. conditional convergence). State which test(s) you use.

   a. \[ \sum_{n=0}^{\infty} \frac{n 3^{n-1}}{(n+1)2^{2n+1}} \]

   b. \[ \sum_{n=2}^{\infty} \frac{1}{n \cdot \sqrt{\ln(n)}} \]

8. Find the interval of convergence of the series

   \[ \sum_{n=1}^{\infty} \frac{(x-4)^n}{7^n \sqrt{n}} \]

9. Use (known) Taylor Series to find the sums of the following series:

   a. \[ \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \sqrt{2(2n+1)}} \]

   b. \[ \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n \cdot n!} \]

10. Find the 3rd degree Taylor polynomial for \( f(x) = \sqrt{x} \) centered at \(c = 4\).

11. a. Find the Maclaurin series for \( f(x) = \cos(\sqrt{x}) \) by using the (known) Maclaurin series for \( \cos x \). Write your answer using \( \sum \) notation.

   b. Use the series to estimate \( \int_{0}^{1} \cos(\sqrt{x}) \, dx \) with a fraction or a sum and/or difference of fractions. Your estimate should be correct to within \( 0.001 = \frac{1}{1000} \), using the fewest number of terms necessary.
1.  a. $\frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$  
   b. $6 \ln 9 - 8$

2. $2 \ln |x| + \ln |x - 3| - 3 \ln |x + 3| + C = \ln \left| \frac{x^2(x-3)}{(x+3)^3} \right| + C$

3. $\frac{1}{18 \sqrt{3}} = \frac{\sqrt{3}}{54}$

4. $\frac{\pi - 3 \sqrt{3} + 6}{6}$

5.  a. area $= \frac{\pi}{8}$  
   b. volume diverges to $\infty$

6. $\frac{2\pi}{27} \left( 37 \sqrt{37} - 1 \right)$

7.  a. converges (absolutely) -- tests used: direct comparison or limit comparison with $\sum \left( \frac{3}{4} \right)^n$
   
   b. diverges -- using the integral test.

8. $[-3, 11)$

9.  a. $\tan^{-1} \left( \frac{1}{\sqrt{2}} \right)$  
   b. $e^{-\frac{1}{4}} = \frac{1}{\sqrt[e]{e}}$

10. $2 + \frac{1}{4} (x - 4) - \frac{1}{64} (x - 4)^2 + \frac{1}{512} (x - 4)^3$

11.  a. $\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$  
   b. $\int_0^1 \cos(\sqrt{x}) \, dx \approx 1 - \frac{1}{4} + \frac{1}{72} = \frac{55}{72}$