2.5 Evaluating Limits Algebraically

Algebra can be used to rewrite $f(x)$ to evaluate $\lim_{x \to c} f(x)$

Recall: $\lim_{x \to 4} \left( \frac{x^2 - 16}{x - 4} \right)$

A function $f(x)$ has an indeterminate form at $x = c$ in case the formula for $f(x)$ yields an expression of the form $0 \cdot \infty$, $\infty$, $0 - \infty$, as $x \to c$. The strategy is to algebraically manipulate $f(x)$ so that the limit can be evaluated using the Basic Limit Laws.

Examples:

a) $\lim_{x \to 3} \left( \frac{x^2 + 2x - 15}{x^2 + 5x - 24} \right)$ [factor and reduce]

b) $\lim_{t \to 4} \left( \frac{\sqrt{t} - 2}{t - 4} \right)$ [multiply by the conjugate of the numerator]

c) $\lim_{\theta \to \frac{\pi}{2}} (\cos \theta \cdot \tan \theta)$ [use trig identities]

d) $\lim_{h \to 0} \left( \frac{(3 + h)^2 - 9}{h} \right)$ [simplify the numerator and reduce]

e) $\lim_{x \to 4} \left( \frac{1}{\sqrt{x} - 2} - \frac{4}{x - 4} \right)$ [write as a single fraction]

f) (2.5.54) $\lim_{h \to a} \left( \frac{1}{h - a} \right)$ [simplify the complex fraction]