2.3 Basic Limit Laws

Theorem - Basic Limit Laws: If \( \lim_{x \to c} f(x) \) and \( \lim_{x \to c} g(x) \) exist, then:

i) \( \lim_{x \to c} \left( f(x) + g(x) \right) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) \)

ii) \( \lim_{x \to c} \left( k \cdot f(x) \right) = k \cdot \lim_{x \to c} f(x) \)

iii) \( \lim_{x \to c} \left( f(x) \cdot g(x) \right) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) \)

iv) If \( \lim_{x \to c} g(x) = M \neq 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} \)

v) If \( p \) and \( q \) are integers with \( q \neq 0 \), then \( \lim_{x \to c} [f(x)]^{p/q} = \left( \lim_{x \to c} f(x) \right)^{p/q} \)

In particular, for \( n \in \mathbb{Z}^+ \), \( \lim_{x \to c} [f(x)]^n = \left[ \lim_{x \to c} f(x) \right]^n \) and

\( \lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} \)

Notes: 1) If \( \lim_{x \to c} f(x) \) or \( \lim_{x \to c} g(x) \) does not exist, then the Basic Limit Laws cannot be applied.

2) Some of the Basic Limit Laws can be extended. For example,

\( \lim_{x \to c} (f(x) + g(x) + h(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) + \lim_{x \to c} h(x) \) and

\( \lim_{x \to c} (f(x) \cdot g(x) \cdot h(x)) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) \cdot \lim_{x \to c} h(x) \)

\( \lim_{x \to c} (f(x) - g(x)) = \lim_{x \to c} f(x) - \lim_{x \to c} g(x) \)
2.3 Basic Limit Laws (p. 2)

Note: we have already discussed the following basic limits:

\[
\lim_{x \to c} k = k \quad \lim_{x \to c} x = x \quad \lim_{x \to c} x^{p/q} = c^{p/q}
\]

Examples:

\[
\lim_{x \to c} \text{(polynomials)} \quad \lim_{x \to c} \left( \frac{p(x)}{q(x)} \right) \quad \lim_{x \to 4} \sqrt{f(x)}
\]

Note that assumptions matter (see example 7 in text):

Suppose that \( f(x) = x \) and \( g(x) = \frac{1}{x} \). Since \( f(x) \cdot g(x) = x \cdot \frac{1}{x} = 1 \), we see that \( \lim_{x \to 0} (f(x) \cdot g(x)) = 1 \). But \( \lim_{x \to 0} g(x) \) does not exist, so the product rule cannot be applied: \( \lim_{x \to 0} (f(x) \cdot g(x)) \neq \lim_{x \to c} f(x) \cdot \lim_{x \to 0} g(x) \)