Chapter 2 - Limits

2.1 Limits, Rates of Change, and Tangent Lines

Rates of change: Consider the change in one quantity \((q)\) in relation to another quantity \((x)\): rate of change \(= \frac{\Delta q}{\Delta x} = \frac{q_2 - q_1}{x_2 - x_1}\)

Velocity: Note that average velocity is given by \(v_{avg} = \frac{\text{change in position}}{\text{change in time}} = \frac{\Delta s}{\Delta t}\)

Position (feet) \(s(t) = -16t^2 + 80t\). Find the (instantaneous) velocity at \(t = 2\).

<table>
<thead>
<tr>
<th>(t)</th>
<th>3</th>
<th>2.5</th>
<th>2.1</th>
<th>2.01</th>
<th>1.99</th>
<th>1.9</th>
<th>1.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s(t))</td>
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<tr>
<td>(v = \Delta s/\Delta t)</td>
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velocity (at \(t = 2\)) = (guess)

**Average rate of change** of \(y = f(x)\) on an interval \([x_0, x_1]\) is \(= \frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}\)

Note that the **instantaneous rate of change** is the limit of the average rate of change as \(\Delta x \to 0\)

ex: Tangent line to a curve: \(y = x^2\) at \((3, 9)\)

\[
\begin{array}{c|cccccc}
  x & 4 & 3.5 & 3.1 & 3.01 & 2.99 & 2.5 & 2 \\
  \hline
  (x, x^2) & \hline
  \hline
  m_{sec} = \frac{\Delta y}{\Delta x} & & & & & & & \\
\end{array}
\]

\(m_{tan} = \) (guess)