3.7 The Chain Rule

example: \( f(x) = e^{2x} = e^x \cdot e^x \)  
example: \( y = (x^2 + 7x)^3 \)

Theorem 1 (The Chain Rule): If \( f \) and \( g \) are both differentiable and \( f \circ g \) is the composite function defined by \( f \circ g(x) = f(g(x)) \), then \( f \circ g \) is differentiable and \( (f \circ g)' \) is given by the product

\[
(f \circ g)'(x) = f'(g(x)) \cdot g'(x)
\]

In Leibniz notation, if \( y = f(u) \) and \( u = g(x) \) are both differentiable,

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]

example: \( y = (x^3 - 4x^2 + 11)^5 \)  
example: \( F(x) = \tan(5x^2 - \frac{\pi}{6}) \)

example: \( G(x) = \left(\frac{2x-3}{5x+4}\right)^7 \)  
examples Text #86, #87 (overhead)

Theorem 2 (General Power and Exponential Rules): If \( n \) is any real number and \( u = g(x) \) is differentiable, then

\[
\frac{d}{dx} (u^n) = n u^{n-1} \frac{du}{dx} \quad \text{or} \quad \frac{d}{dx} (g(x)^n) = n (g(x))^{n-1} \cdot g'(x)
\]

\[
\frac{d}{dx} (e^u) = e^u \frac{du}{dx} \quad \text{or} \quad \frac{d}{dx} (e^{g(x)}) = g'(x) e^{g(x)}
\]

example: \( H(x) = \frac{1}{\sqrt[3]{x^2 - 5x + 7}} \)  
example: \( y = e^{\tan x} \)

Theorem 3: If \( f(x) \) is differentiable, then for any constants \( k \) and \( b \),

\[
\frac{d}{dx} f(kx + b) = k \cdot f'(kx + b)
\]

examples: a) \( y = \cos \left(3x - \frac{\pi}{2}\right) \)  
b) \( f(x) = e^{5-3x} \)