4.7 Applied Optimization

Steps in Solving Optimizations problems

1. Understand the problem.

2. Draw a diagram.

3. Introduce notation for the quantity $Q$ to be maximized or minimized, and for other values that can possibly vary in the problem.

4. Express $Q$ in terms of some other symbols.

5. Find a formula for $Q$ in terms of a single variable: $Q = f(x)$.

6. Use calculus techniques to find the extreme value(s) of $Q$.

example: A rancher with 300 feet of fencing material wants to enclose two rectangular corrals next to a river. The livestock don't swim, so no fence is needed along the river. Find the dimensions of the corrals that maximize the total area of the two corrals.

example: (4.7.8) A rectangle is inscribed in a right triangle with legs 8 and 5. The sides of the rectangle are parallel to the legs of the right triangle. Find the rectangle of largest possible area.

example: An oil rig is located 4 kilometers offshore. The oil refinery is located 10 kilometers along a straight shoreline from the point onshore closest to the refinery (distance is measured at a 90° angle to the shore). The oil company wants to build a pipeline from the rig to the refinery. If the cost of building a pipeline in water is $5 million/kilometer and on land is $2 million/kilometer, how should the pipeline be constructed to minimize the cost?

example: A box with a square base and open top must have a volume of 32,000 cm$^3$. Find the dimensions of the box that minimizes the amount of material used.

example: Find the point(s) on the graph of $y = x^2$ that is closest to the point $(0, 3)$.

example: A right circular cylinder is inscribed in a cone with height $H$ and base radius $R$. Find the largest possible volume of such a cylinder.