4.8 Newton's Method

For equation: $f(x) = 0$

Find a solution, i.e., find a root of $f(x)$

Start with a value $x = x_1$, "near" root,

Slope $= \frac{\Delta y}{\Delta x} = \frac{0 - f(x_1)}{x_2 - x_1} = \frac{f'(x_1)}{x_2 - x_1}$

Solve for $x_2$

$-f(x_1) = f(x_1)(x_2 - x_1)$

$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1 \Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

Hopefully, $x_2$ is closer to root.

Repeat process with tangent at $(x_2, f(x_2))$
\[ X_3 = X_2 \frac{f(x_2)}{f'(x_2)} \]

In general: Start w/ \( x_1 \) to find:

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

Hopefully: \( x_1, x_2, x_3, x_4, \ldots \rightarrow \text{root} \)

Ex: Find an approximation for \( \sqrt{5} \): \[ \approx 2.236067 \ldots \]

Let \( f(x) = x^2 - 5 \)
\[ f'(x) = 2x \]

Start w/ \( x_1 = 3 \).

\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{9}{6} = 3 - \frac{3}{2} = \frac{3}{2} = 2.33333333 \ldots \]
\[ x_3 = x_2 - \frac{f'(x_2)}{f''(x_2)} \]

\[ = \left( \frac{7}{3} \right) - \frac{f'(\frac{7}{3})}{f''(\frac{7}{3})} \]

\[ = \frac{7}{3} - \frac{\frac{49}{9}}{2(\frac{7}{3})} \]

\[ = \frac{7}{3} - \frac{49}{14} = \frac{7}{3} - \frac{7}{2} = \frac{47}{14} = 3.357142857 \]

\[ x_4 = \left( \frac{47}{21} \right) - \frac{\left( \frac{47}{21} \right)^2 - 5}{2 \left( \frac{47}{21} \right)} \]

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**EX:** Find solution to equation \( \cos x = x \) (using Newton's method) \( \text{NEED } f(x) = 0 \)

**LIST:**

\( f(x) = x - \cos x \)

**START:** \( x_1 = 1 \)

\( f'(x) = 1 + \sin x \)

\[ x_2 = x_1 - \frac{x_1 - \cos(x_1)}{1 + \sin(x_1)} \approx 0.75036 \ldots \]

\[ x_3 = x_2 - \frac{x_2 - \cos(x_2)}{1 + \sin(x_2)} \approx 0.7391128 \]
FINAL EXAM  10 PROBLEMS ON
9 PAGES. ~30 RESPONSES  150 Pts

2 PROBLEMS: LIMITS. I CAN'T USE L'HOSPITAL

I MUST USE L'H.

2 PROBLEMS: DERIVATIVE SKILLS.

FORMULAS: \( x^n, e^x, \) (TRIG: \( b^x, \log_b x, \) INVERSE TRIG, HYPERBOLIC

RULES: PRODUCT; QUOTIENT; CHAIN

HIGHER DERIVATIVES

TECHNIQUES: IMPLICIT DIFF'N; LOGARITHMIC, DIFF'N

2 PROBLEMS: CURVE SKETCHING SKILLS.

\[ y = f(x) \] POINTS ON CURVE
\[ \frac{dy}{dx} = f'(x) \] SLOPE (\( f \) INCREASING/DECREASING)
CRITICAL NUMBERS, LOCAL EXTREMES
POLYNOMIAL FUNCTIONS

4 PROBLEMS: APPLICATIONS.

LIST INCLUDES:
- Equation of a tangent line.
- Intermediate value thm. (Finding c, f'(c) = N)
- Extreme value thm. (Find extrema of f on [a,b])
- Linearization; approximation

★ - Related rates
★ - Optimization

NO: Definition of f'(x)
Marginal cost
Newton's method.