6.4 Volumes

Definition: A **cylinder** is a solid with congruent flat ends (called **bases**) and line segments joining corresponding points of the two bases. Hence every cross-section parallel to the bases is congruent to the bases.

Note: The volume of a cylinder is the area of the base times the 'height' of the cylinder.

Volume of a solid (not a cylinder): cut into 'slices' (each a cylinder)

\[
\text{Volume } \approx \sum \text{volume of cylindrical 'slice'} = \sum A(x) \, [\Delta x]
\]

\[
\text{Volume } = \lim_{\Delta x \to 0} \sum A(x^*) \, [\Delta x] = \int_a^b A(x) \, dx
\]

example: Volume of a sphere of radius \( R \).

Volume of rotation: A 'strip' of area perpendicular to axis of rotation yields a 'disk' whose volume is \( \pi \cdot (\text{radius of disk})^2 \cdot (\text{thickness}) \).

Hence volume \( \approx \sum \pi(r)^2 \Delta x \rightarrow \int_a^b \pi ("r")^2 \, dx \)

example: Find the volume generated when the area bounded by \( y = x^2, \ y = 0 \) and \( x = 3 \) is rotated around \( x \)-axis.

example: The area bounded by \( x = y - y^2, \ \ x = 0 \) is rotated around the \( y \)-axis. Use horizontal 'slices' to find the volume that is generated.

Note: If strip does not touch the axis of rotation, a 'washer' is formed, with volume \( = \pi (R^2 - r^2)(\text{thickness}) \)

example (6.4.5): Find the volume if the region bounded by \( y = x^3 \) and \( y = x \) for \( x \geq 0 \) is rotated around the \( x \)-axis.

example: Area bounded by \( y = \frac{1}{x}, \ y = 0, \ x = 1, \ x = 3 \) rotated around line \( y = -1 \).
6.4 Volumes (continued)

example (6.4.14): Find the volume of the solid whose base is a circular disk of radius $r$ and parallel cross-sections perpendicular to the base are squares.

element: Find the volume of the solid object whose base is the ellipse $4x^2 + 16y^2 = 64$ and where cross-sections perpendicular to the $x$-axis are equilateral triangles.

element: The area bounded by $y = \frac{1}{x}$, $y = 0$, $x \geq 1$ is rotated around the $x$-axis. Determine if the volume generated is finite; if so, find the volume.