Chapter 6 - Applications of Integrals

6.1 Areas Between Curves

Area bounded by two curves \( y = f(x) \) and \( y = g(x) \) is:

\[
A(x) = \int_a^b [f(x) - g(x)] \, dx = \int_a^b f(x) \, dx - \int_a^b g(x) \, dx
\]

[Here \( f(x) \geq g(x) \) on \([a, b]\) ]

example: \( y = e^x, \quad y = \sqrt{x} \) from \( x = 0 \) to \( x = 4 \).

element: \( y = x^2, \quad y = x + 2 \) (Note: find points of intersection)

If \( f(x) \) and \( g(x) \) cross several times in the interval \([a, b]\), the area is

\[
A(x) = \int_a^b \left| f(x) - g(x) \right| \, dx
\]

example: Find the area bounded by \( y = \sin x \) and \( y = \cos x \) on the interval \([0, \frac{3\pi}{4}]\).

Cerebral Blood Flow (explanation in text pp. 390-391)

If \( A(t) \) represents the arterial concentration (units: \( mL \) of \( N_2O \) per \( mL \) of blood) of \( N_2O \) entering the brain, and \( V(t) \) represents the concentration of \( N_2O \) flowing out of the brain in the jugular vein, one can calculate the cerebral blood flow \( F \) by calculating the area between curves \( A \) and \( V \). The quantity of \( N_2O \) taken up by the whole brain in the first \( t \) minutes is

\[
Q(t) = F \int_0^t (A(t) - V(t)) \, dt
\]

So if \( Q(t) \) is known, we find that

\[
F = \frac{Q(t)}{\int_0^t (A(t) - V(t)) \, dt}
\]

Horizontal strips: 'width' = \( dy \), so area is \( \int_c^d \left| x_r - x_l \right| \, dy \) (formula needed for 6.1.16)

example: \( x + y = 6 \)

\( x - y^2 = 0 \)