5.6 Partial Fractions

This section will 'deconstruct' rational functions into sums of 'partial fractions'

example: since \( \frac{2}{x} - \frac{1}{x+4} = \frac{2(x+4)}{x(x+4)} - \frac{(x)1}{(x)(x+4)} = \frac{x+8}{x^2+4x} \)

so we can write \( \int \frac{x+8}{x^2+4x} \, dx = \int \left( \frac{2}{x} - \frac{1}{x+4} \right) \, dx \)

example: \( \int \frac{x^3+2x^2-5x}{x+2} \, dx \); divide first.

Need integrand \( f(x) = \frac{P(x)}{Q(x)} \), where \( \deg P < \deg Q \) (divide if necessary)

Goal: Express \( f(x) \) as a sum of terms of form \( \frac{A}{(ax+b)^n} \).

Procedure: to find constants in numerator, add deconstructed fractions and compare numerators; will need to solve a system of equations.

example: \( \int \frac{x+3}{x^2-6x+5} \, dx = \int \left( \frac{2}{x-5} - \frac{1}{x-1} \right) \, dx \)

example: \( \int \frac{2x+5}{x^2+2x-8} \, dx = \int \left( \frac{\frac{1}{2}}{x+4} + \frac{\frac{3}{2}}{x-2} \right) \, dx \)

example: \( \int \frac{x^3-7x+4}{x^2+3x} \, dx \) (divide first)

The algebra gets more difficult with more factors in the denominator:

example: \( \int \frac{3x+10}{2x^3-3x^2-5x} \, dx \)
5.6 Partial Fractions (continued)

If a linear factor in the denominator is raised to a power, then the partial fraction decomposition must contain fractions with possible powers less than or equal to the power of the factor.

example: Write the partial fraction decomposition of \( \frac{p(x)}{x^4(x-3)^2} \)

example: \[ \int \frac{x^2+13x+12}{x(x+2)^2} \, dx = \int \left( \frac{3}{x} - \frac{2}{x+2} + \frac{5}{(x+2)^2} \right) \, dx \]

Making a substitution can sometimes create a problem where the partial fraction method can be used.

example (5.6.16): Make a substitution, then evaluate using the partial fraction method

\[ \int \frac{dx}{2\sqrt{x+3} + x} \, dx \]

The following example will be helpful in doing problems 21, 22 in text.

example: \[ \int \frac{3x-2}{x(x^2+1)} \, dx = \int \left( \frac{A}{x} + \frac{Bx+C}{x^2+1} \right) \, dx \]