(0.2) AVERAGE VALUE OF A FUNCTION (WRAP-UP)

\[ f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx \]

**Theorem:** If \( f \) is continuous on \([a, b]\), then there is a \( c \) such that

\[ f(c) = f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx \]

**Example:** \((0, 2, 10)\) \( f(x) = \frac{2x}{(1+x^2)^2} \) on \([0, 2]\)

**A) Find** \( f_{\text{avg}} \) **on** \([0, 2]\) **b) Find** \( c \) **such that**

**Last Time:** \( f_{\text{avg}} = \frac{1}{2-0} \int_0^2 \frac{2x}{(1+x^2)^2} \, dx = \frac{2}{5} \)

**b) \( f(c) = \frac{\frac{2}{5}}{(1+(c^2)^2)} = \frac{2}{5} \)** Solve for \( c \).

\[ 5c = (1+c^2)^2 = 1 + 2c^2 + c^4 \]
\[ 0 = c^4 + 2c^2 - 5c + 1 = p(c) \]

\[
\begin{array}{c|cccc}
 c & 0 & 1 & 2 \\
\hline
 p(c) & 1 & -1 & 15 \\
\end{array}
\]

2 such c values
\[ \approx 0, 22 \quad \approx 1, 21 \quad \text{(using calc.)} \]

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3.3 Further Applications to Biology

\underline{Survival \div Renewal}

\[ P_0 = \text{initial population (at time } t = 0) \]

\[ R(t) = \text{rate at which new members are added to population, (renewal function)} \]

\[ S(t) = \text{survival function = proportion of pop'n that survives at least } t \text{ years} \]

Find a model to predict population

\[ P \text{ as a function of time } = P(t) \]
The population in $T$ years is given by

$$P(T) = S(T) \cdot P_0 + \int_0^T S(T-t) \cdot R(t) \, dt$$

Example (Example 1 in Text) Currently $P_0 = 5600$

Trout in a Lake. Reproduction rate is $R(t) = 720e^{0.1t}$ fish/year. The proportion of trout population that survives $T$ years is.

$$S(t) = e^{-0.2t}$$

Find the number of trout in lake in 10 years. (Set $T = 10$)

$$P(10) = S(10) \cdot 5600 + \int_0^{10} S(10-t)R(t) \, dt$$

$$= e^{-0.2(10)} \cdot 5600 + \int_0^{10} e^{-0.2(10-t)} \cdot (720e^{0.1t}) \, dt$$

$$= 5600e^{-2} + \int_0^{10} e^{-2 + 0.2t} \cdot 720e^{0.1t} \, dt$$

$$= \frac{5600}{e^2} + 720\int_0^{10} e^{0.1t} \cdot e^{0.2t} \, dt$$
\[
= \frac{5600}{e^2} + \frac{720}{e^2} \int_0^{10} e^{0.3t} \, dt \\
\quad \text{where } u = 0.3t, \quad du = 0.3 \, dt \\
\quad \frac{1}{0.3} \int e^u \, du, \quad \frac{du}{0.3} = dt \\
= \frac{5600}{e^2} + \frac{720}{e^2} \left[ \frac{1}{0.3} \left( e^{0.3t} - 1 \right) \right]_0^{10} \\
= \frac{5600}{e^2} + \frac{720}{e^2} \left[ \frac{1}{0.3} \left( e^{3} - 1 \right) \right] - \frac{7200}{3} \\
= \frac{5600}{e^2} + 2400 \left( e - \frac{1}{e^2} \right) \\
= 2400e + \frac{3200}{e^2} \approx 6960,949...
\]

In 10 years there will be about 6960 trout in Lake.
MIDTERM EXAM - MONDAY, 5 NOV.
START AT 8AM - ENDS AT 9:10AM

6 PROBLEMS: 7 RESPONSES.

1 PROBLEM: CHAP 4: ANTIDERIVATIVES
AND/OR INITIAL VALUE
PROBLEM.

1 PROBLEM: EVALUATE A DEFINITE INTEGRAL
USING AREA CALCULATIONS (BOTH ABOVE
\& BELOW X-AXIS)

2 PROBLEMS: TECHNIQUES OF INTEGRATION
- SIMPLE SUBSTITUTION
- I.B.P.
- PARTIAL FRACTIONS

1 PROBLEM: IMPROPER INTEGRAL(S)

1 PROBLEM: AREA BETWEEN CURVES.