1. Given $F(x) = \ln\left(x + \sqrt{x^2 - 9}\right)$:
   
   a. Show that $F'(x) = \frac{1}{\sqrt{x^2 - 9}}$.

   b. Use the result of part a. and the Fundamental Theorem of Calculus to evaluate $\int_{5}^{7} \frac{1}{\sqrt{x^2 - 9}} \, dx$

2. In this integral, start by making a substitution; then use integration by parts to evaluate:
   
   $\int_{1}^{9} e^{\sqrt{x}} \, dx$

3. Evaluate:
   
   $\int \frac{x^2 + 16}{x^2 - 16} \, dx$

4. Determine whether the integral converges or diverges. If it converges, find its value.
   
   $\int_{1}^{\infty} x^3 e^{-x^4} \, dx$

5. A patient receives a drug at a constant rate of 20 mg/h. The drug is eliminated from the bloodstream over time so that the fraction $e^{-0.2t}$ remains after $t$ hours. The patient currently has 60 mg of the drug present in the bloodstream. How much of the drug will be present in 10 hours? Write your answer as either an exact value or correct to one decimal place.

   Recall that if the renewal rate is $R(t)$ and the survival function is $S(t)$, then the amount of the drug remaining after $T$ hours is given by the equation
   
   $A(T) = S(T) \cdot A_0 + \int_{0}^{T} S(T - t) R(t) \, dt$

6. The region inside the circle $x^2 + y^2 = 36$ and above the line $y = 3$ is rotated around the $x$-axis. Find the volume of the solid that is generated.
7. Given the function \( f(x) = \sqrt{4x + 1} \):

a. Compute the Taylor polynomial of degree 2 about \( x = 2 \) for \( f(x) \).

b. Use the polynomial \( T_2(x) \) from part a. to find a single fraction, to estimate \( \sqrt{13} \).

[Note: you must use the same value for \( x \) in the polynomial as the value needed in \( f(x) \) to yield \( \sqrt{13} \)]

c. Use the error estimate \( |R_{n+1}(x)| \leq \frac{M|x-a|^{n+1}}{(n+1)!} \) to determine the size of the error when the fraction in part a. is used to estimate \( \sqrt{13} \). Find the smallest such bound. Recall that \( M = \max|f^{(n+1)}(c)| \) for \( a \leq c \leq x \).

8. Given the differential equation: \( \frac{dN}{dt} = 3N \left(1 - \frac{N}{48}\right) - 20 \)

a. Find all the equilibria for this differential equation.

b. Determine whether each equilibrium is (locally) stable or unstable. Justify your answer.

9. Use Euler's method with step size 0.5 to compute the approximate \( y \)-values \( y_1, y_2, y_3, \) and \( y_4 \) of the solution of the initial-value problem \( \frac{dy}{dx} = y' = 4x + 2y, \, y(0) = 1 \). Recall that the formula for finding successive points in an initial-value problem \( y' = F(x, y) \) with a step size \( h \) is \( y_{n+1} = y_n + h \cdot F(x_n, y_n) \).

[if you have to write an estimate, make it correct to three decimal places]

10. Solve the following differential equation with the given initial condition (if possible find \( y \) as an explicit function of \( x \)): \( \frac{dy}{dx} = x + xy^2 \) where \( y = 1 \) when \( x = 0 \)
1. a. \( F'(x) = \frac{1}{x + \sqrt{x^2 - 9}} \left( 1 + \frac{1}{2} (x^2 - 9)^{-\frac{1}{2}} \right) \cdot 2x = \frac{1}{x + \sqrt{x^2 - 9}} \left[ 1 + \frac{x}{\sqrt{x^2 - 9}} \right] \)

\[ = \frac{1}{x + \sqrt{x^2 - 9}} \frac{\sqrt{x^2 - 9} + x}{\sqrt{x^2 - 9}} = \frac{1}{\sqrt{x^2 - 9}} \]

b. \( \int_5^7 \frac{1}{\sqrt{x^2 - 9}} \, dx = \ln \left( x + \sqrt{x^2 - 9} \right) \bigg|_5^7 = \ldots = \ln \left( 7 + \sqrt{40} \right) - \ln(9) = \ln \left( \frac{7 + \sqrt{40}}{9} \right) \)

2. \( \int_1^9 e^{\sqrt{x}} \, dx = \left( \text{sub } w = \sqrt{x} \right) = \int_1^3 e^w \cdot 2w \, dw = (IBP: u = 2w, dv = e^w \, dw) \)

\[ = \left( 2we^w - 2e^w \right) \bigg|_1^3 = \ldots = 4e^3 \]

3. \( \int \frac{x^2 + 16}{x^2 - 16} \, dx = \int \left( 1 - \frac{4}{x+4} + \frac{4}{x-4} \right) \, dx = x - 4 \ln |x + 4| + 4 \ln |x - 4| + C \)

4. \( \int_1^\infty x^3 e^{-x^4} \, dx = \lim_{T \to \infty} \int_1^T x^3 e^{-x^4} \, dx = (\text{sub } u = -x^4) = \ldots = \lim_{T \to \infty} \left( -\frac{1}{4} e^{-x^4} \right) \bigg|_1^T \)

\[ = \lim_{T \to \infty} \left( -\frac{1}{4e} + \frac{1}{4e} \right) = \frac{1}{4e} \]

5. \( A(10) = e^{-2} [60] + \int_0^{10} e^{-2(10-t)} \, [20] \, dt = \frac{60}{e^2} + 20 \int_0^{10} e^{2t-2} \, dt = \ldots = 100 - \frac{40}{e^2} \approx 94.6 \)

6. \( V = \int_{-\sqrt{27}}^{\sqrt{27}} \pi \left( \sqrt{36 - x^2} \right)^2 - 3^2 \, dx = (\text{symmetry}) = \int_0^{3\sqrt{3}} \pi \left[ 27 - x^2 \right] \, dx = \ldots = 108\pi \sqrt{3} \)

7. a. \( T_2(x) = 3 + \frac{2}{3} (x - 2) - \frac{2}{27} (x - 2)^2 \)
   b. \( \sqrt{13} = f(3) \approx T_2(3) = \frac{97}{27} \)
   c. \( |R_{n+1}(x)| \leq \frac{4}{243} \)

8. a. \( N = 8; \) \( N = 40 \)
   b. \( N = 8 \) is unstable; \( N = 40 \) is stable

9. \( y_1 = 2 \quad y_2 = 5 \quad y_3 = 12 \quad y_4 = 27 \)

10. \( y = \tan \left( \frac{2x^2 + \pi}{4} \right) \)