1.1 Four Ways to Represent a Function (cont'd)

Graph of \( f(x) \): set of points

\[ \{(x, f(x)) \mid x \in \text{domain}\} \]

Domain of \( f(x) \): unless specified

Domain is largest set of values for which \( f(x) \) is defined.

Q. Find domain:

a) \( f(x) = \frac{3x-9}{x^2-9} \)

\[ \text{Need } x^2 - 9 \neq 0 \]

\[ \frac{\sqrt{x^2 - 9}}{x^2 - 9} \]

\[ x^2 - 9 = 0 \]

\[ (x+3)(x-3) = 0 \]

\[ x = -3, 3 \]

Interval notation: \(-3 \; \cup \; 3\)

\[ (-\infty, -3) \cup (-3, 3) \cup (3, \infty) \]
Note: \[
\frac{3x-9}{x^2-9} = \frac{3(x-3)}{(x+3)(x-3)} = \frac{3}{x+3} \text{ \textcolor{red}{\checkmark}}
\]
still \(x \neq 3\)

\[f(x) = \sqrt{x^2-3x-10}\]

**Find Domain:**

\[\text{Need } x^2-3x-10 \geq 0\]
\[\text{Factor: } (x+2)(x-5) \geq 0\]

Domain: \((-\infty, -2] \cup [5, \infty)\)

**Piece-wise Defined Functions:** Different rules for different inputs.

\[f(x) = \begin{cases} 
2x+1 & x < 3 \\
\sqrt{x^2-4} & x > 3
\end{cases}\]

\[f(2) = 2(2)+1 \quad \Rightarrow \quad 5, \quad f(4) = (4)^2-4 \quad \Rightarrow \quad 12.\]
LINEAR FUNCTIONS

GRAPH IS A LINE:

\[ f(x) = mx + b \]

\[ m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ b = y\text{-intercept} \]

\[ (x_1, y_1) \]

\[ y - y_1 = m(x - x_1) \]

From \[ \frac{y - y_1}{x - x_1} = m \] .

EX) FIND EQUATION OF LINE THROUGH \((-3, 1)\) AND \((6, 7)\)

1) FIND SLOPE

\[ m = \frac{\Delta y}{\Delta x} = \frac{7 - 1}{6 - (-3)} = \frac{6}{9} = \frac{2}{3} \]

\[ m = \frac{2}{3} \]

\[ y - 1 = \frac{2}{3}(x + 3) \]

\[ y - 7 = \frac{2}{3}(x - 6) \]

\[ y = \frac{2}{3}x + 2 \]

SLOPE-INT FORM
POLYNOMIAL FUNCTIONS of FORM
\[ f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 \]
\( a_0, a_1, \ldots, a_n \in \mathbb{R} \).

\( f(x) = 3x^3 - 7x^2 + 11x - 9x + 3 \); DEGREE 5

DEGREE OF A TERM -> POWER OF X
LARGEST OF POLYNOMIAL -> DEGREE OF ANY TERM,

LEADING COEFFICIENT OF TERM OF LARGEST DEGREE.

\[ f(x) = 7x^4 - 8x^3 + 11x^2 - 5 + 6x \]
LEAD COEFF: -8

QUADRATIC FUNCTIONS \( f(x) = ax^2 + bx + c \)

GRAPH IS A PARABOLA, \( a \neq 0 \)

ZEROS (ROOTS) W/ QUADRATIC FORMULA:
\[ f(x) = 0 \text{ if } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
POWER FUNCTIONS: of form \( f(x) = x^r \) (or \( f(x) = ax^r \))

- \( f(x) = x^1 \)
  - \((0,0)\)

- \( f(x) = x^2 \)
  - \((0,0)\)
  - \( y = \sqrt{x} \), \( x \geq 0 \)

- \( f(x) = x^3 \)
  - \( y = \sqrt[3]{x} \)

NOTE \( f(x) = x^{\frac{3}{2}} = \sqrt{x^3} \) (or \( \sqrt[3]{x}^2 \))

- \( f(x) = x^{-2} = \frac{1}{x^2} \), \( x \neq 0 \)

- \( f(x) = x^{-3/2} = \frac{1}{x^{3/2}} \)
  - \( = \frac{1}{\sqrt{x^3}} \)
**RATIONAL FUNCTIONS**

Form \( f(x) = \frac{p(x)}{q(x)} \)

\( p, q \) POLYNOMIALS \( q(x) \neq 0 \)

\[ f(x) = \frac{x^2 - 2x - 15}{2x^2 - 18} = \frac{(x + 3)(x - 5)}{2(x + 3)(x - 3)} = \frac{x - 5}{2(x - 3)} \text{ [SINGULARITY] } \]

**ALGEBRAIC FUNCTIONS**: Functions constructed using **ALGEBRAIC OPERATIONS**: +, −, ×, ÷

AND TAKING ROOTS, BASED ON POLYNOMIALS

\[ f(x) = \frac{\sqrt{x^3 - 3x - 10}}{\sqrt[3]{x^2}} \]

\[ g(x) = x \Rightarrow g(x) = \frac{3}{x^2} \]

**NON-ALGEBRAIC FUNCTIONS**:

\( f(x) = 2^x \); \( g(x) = \log_3(x) \)

\( h(x) = \cos(x) \)

**TO BE CONTINUED.**