1. a. The graph of a functions \( f \) and \( g \) are at end of exam. Find the domain and range of the functions. Each square on the graph represents 1 square unit. Write your answer using interval notation.
   
   domain of \( f \)  \hspace{1cm} \text{domain of} \ g \hspace{1cm} \text{range of} \ f \hspace{1cm} \text{range of} \ g
   
   b. Find the following values if they exist (if not write DNE): note that all answers are integer values.
   
   \[ f(4) \hspace{1cm} g(3) \hspace{1cm} f \circ g(2) \]

2. Given the sequence \( a_{n+1} = \sqrt{12 + a_n} \)
   
   a. If \( a_1 = 13 \), find and label the next three terms of the sequence, (i.e. find \( a_2 \), \( a_3 \), and \( a_4 \)).
   
   b. Assume that this sequence converges. Find the exact limit of the sequence. Use appropriate justification.

3. Evaluate the following limits, if they exist (if the limit does not exist, write DNE). Justify your answers with algebraic techniques.
   
   a. \[ \lim_{x \to 2} \frac{x-2}{\sqrt{3x+10}-4} \]  
   b. \[ \lim_{x \to 6} \left( \frac{3}{x-6} \right) \]  
   c. \[ \lim_{x \to 0} \left( \frac{\sin^2(3x)}{2x^2} \right) \]

4. Find the value(s) of \( a \) and \( b \) for which \( f(x) \) is continuous for all \( x \). Justify your answers by using the definition of continuity.
   
   \[ f(x) = \begin{cases} 
   x^2 + a & \text{if } x < 3 \\
   bx & \text{if } 3 \leq x \leq 5 \\
   4x + a & \text{if } x > 5 
   \end{cases} \]

5. Given the function \( f(x) = \frac{3x}{x-6} \):
   
   a. Use the formal definition of the derivative as the limit of a difference quotient to find \( f'(x) \). No points will be given for finding the derivative without using the definition.
   
   b. Find the equation of the tangent line (in slope-intercept form) at the point on the graph of \( f(x) \) where \( x = 4 \).

6. Differentiate the following functions. Do not simplify unless you are asked to simplify:
   
   a. \( f(x) = 4^x \cdot \cot(2x) \)  
   b. \( y = e^{\csc(x^2)} \)  
   c. \( f(x) = \frac{\sin x}{1-\cos x} \)
   
   [simplify your answer for part c.]

7. Find the second derivative of the function:
   
   \( y = \csc(4x) \)
1. a. domain of \( f = [ -5, 5) \)  
   range of \( f = [ -9, 5] \)  
   domain of \( g = ( -4, 3) \cup (3, 5] \)  
   range of \( g = ( -3, 4) \)  
   b. \( f(4) = -3 \)  
   \( g(3) \) DNE  
   \( f \circ g(2) = f(0) = 5 \)

2. a. \( a_2 = 5 \)  
   \( a_3 = \sqrt{17} \)  
   \( a_4 = \sqrt{12 + \sqrt{17}} \)  
   b. Let \( \lim_{n \to \infty} = L \) then \( \lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} \sqrt{12 + a_n} \to L = \sqrt{12 + L} \)  
   Solving for \( L \) in this equation yields \( L = 4 \) so \( \lim_{n \to \infty} = 4 \)

3. a. \( \frac{8}{3} \)  
   b. \( -\frac{1}{12} \)  
   c. \( \frac{9}{2} \)

4. \( \lim_{x \to 3} (x^2 + a) = \lim_{x \to 3^+} (bx) \to 9 + a = 3b \)  
   \( \lim_{x \to 5^-} (bx) = \lim_{x \to 5^+} (4x + a) \to 5b = 20 + a \)  
   Solving the system of equations yields \( a = \frac{15}{2} \) \( b = \frac{11}{2} \)

5. a. \( f'(x) = \lim_{h \to 0} \frac{3(x+h) - 3x}{h} = \cdots = \frac{-18}{(x-6)^2} \)  
   b. \( f(4) = -6; \ f'(4) = -\frac{9}{2} \to y = -\frac{9}{2}x + 12 \)

6. a. \( f'(x) = [4^x \ln 4 \csc(2x) + 4^x[-2 \csc^2(2x)] \)  
   b. \( \frac{dy}{dx} = e^{csc(x^3)}[-csc(x^3) \cot(x^3) \cdot 3x^2] \)  
   c. \( f'(x) = \frac{[\cos x](1-\cos x) - \sin x \sin x}{(1-\cos x)^2} = \cdots = \frac{-1}{1-\cos x} \)

7. \( \frac{d^2y}{dx^2} = 9 \sec(3x) \tan^2(3x) + 9 \sec^3(3x) \)
1. a. The graph of a function $f$ and $g$ are given. Find the domain and range of the functions. Each square on the graph represents 1 square unit. Write your answer using interval notation.

\[
\text{domain of } f = \ldots \quad \text{domain of } g = \ldots
\]

\[
\text{range of } f = \ldots \quad \text{range of } g = \ldots
\]

b. Find the following values if they exist (if not write DNE): note that all answers are integer values.

\[
f(4) = \ldots
\]

\[
g(3) = \ldots
\]

\[
f \circ g(2) = \ldots
\]