4.1 Maximum and Minimum Values

Definition: A function $f$ has an **absolute maximum** (or **global maximum**) at $c$ if $f(c) \geq f(x)$ for all $x$ in $D$, where $D$ is the domain of $f$. The number $f(c)$ is the **maximum value** of $f$.

Note: $f$ has an **absolute minimum** at $c$ if $f(c) \leq f(x) \ \forall x \in D$.

Definition: A function $f$ has a **local maximum** (or **relative maximum**) at $c$ if $f(c) \geq f(x)$ when $x$ is "near" $c$.

Examples: graphs

Example: $f(x) = \sin x$

Example: $f(x) = x^2$

Example: $f(x) = x^3$

Theorem (the Extreme Value Theorem): If $f$ is continuous on a closed interval $[a, b]$, then $f$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some numbers $c$ and $d$ in $[a, b]$.

(Fermat's) Theorem: If $f$ has a local extreme at $c$, and if $f'(c)$ exists, then $f'(c) = 0$. 
4.1 Maximum and Minimum Values (continued)

Definition: A **critical number** of a function $f$ is a number $c$ in the domain of $f$ such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example: $f(x) = 18x + 15x^2 - 4x^3$ on $[-3, 4]$.

Note: If $x = c$ is a critical number for $f(x)$, $f(c)$ may not be a local extreme. [consider $f(x) = x^3$]

**Finding extrema on a closed interval**

1) Find the critical numbers of $f$.

2) Evaluate the function $f$ at the critical numbers and at the endpoints of the interval.

3) Largest value is the absolute maximum, smallest the absolute minimum.

Example: $f(x) = x - 2 \cos x$ on $[0, \pi]$.

Example: (text example 4.1.6) **The Allee Effect**: Suppose that $f(N) = growth rate$ of a population, where $N$ is measured in 100's of individuals. If $f(N) = N(N - 3)(8 - N)$ where $0 \leq N \leq 9$, find the maximum and minimum values of the growth rate.

Example: (text example 4.1.7) **Blood Alcohol Concentration**: Suppose that the function $C(t) = 0.0225te^{-0.0467t}$ is a model for the average blood alcohol concentration (BAC) of a group after rapid consumption of 15ml of ethanol, and $t$ is measured in minutes and $C$ is measured in $mg/mL$. Find the maximum BAC in the first hour.