**Midterm on:** Chap 1 (not 1.3)  
Chap 2 (all)  
Chap 3 (§1-5) - not implicit differentiating.

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**35 Chain Rule (Wrap-Up)**

If $y = f(u)$ with $u = g(x)$ so $y = f(g(x))$.

So $\frac{dy}{dx} = \left[ f(g(x)) \right]' = f'(g(x)) \cdot g'(x)$

$$= \frac{dy}{du} \cdot \frac{du}{dx}$$

**Related Rates:** Variables depend on one another will change depending on each other.

**Example Spherical Balloon:** Volume changes

- Area
- Radius

Related Rates show relationship of these changes.
START w/ EQN INVOLVING VARIABLES.
\[ V = \frac{4}{3} \pi r^3 \]

VOLUME CHANGES:
\[ \text{RADIUS IS } \frac{dV}{dt} \]

RADIUS CHANGES:
\[ \frac{dr}{dt} \]

\[ \frac{dV}{dt} = \frac{d}{dt} \left( \frac{4}{3} \pi r^3 \right) = \frac{d(\frac{4}{3})}{dt} \pi r^3 + \frac{d(r^3)}{dt} \cdot \frac{dr}{dt} = \frac{4}{3} \pi r^2 \cdot \frac{dr}{dt} \]

\[ \frac{dV}{dt} = \frac{4}{3} \pi r^2 \frac{dr}{dt} \]

EX FIXED PERIMETER LENGTH FOR A RECTANGLE:
\[ 2l + 2w = \text{CONSTANT} = \boxed{B} \]

INCREASING L WOULD REQUIRE W TO GET SMALLER.

GIVEN \( \frac{dl}{dt} \) FIND \( \frac{dA}{dt} \)

\[ A = l \cdot w = l \left[ \frac{B-2l}{2} \right] \]

\[ A = \frac{B}{2} l - l^2 \]

\[ \frac{dA}{dt} \]

\[ \frac{d(l)}{dt} \left( 2 \frac{dl}{dt} + 2 \frac{dw}{dt} \right) = 0 \]

So \( \frac{dl}{dt} = -\left( \frac{dw}{dt} \right) \)
\[ \frac{dl}{dt} = \frac{dA}{dt} = B \cdot \frac{1}{2} \cdot \frac{dl}{dt} - 2L \cdot \frac{dl}{dt} \]

**Example:**

If \( V = \frac{4}{3} \pi r^3 \) and \( \frac{dV}{dt} = 7 \text{ (cm)}^3/\text{sec} \)

Find \( \frac{dr}{dt} \) when \( r = 6 \text{ cm} \).

\[ \frac{dV}{dt} = \frac{4}{3} \pi \left[ 3r^2 \frac{dr}{dt} \right] \]

Substitute

\[ [7] = 4 \pi (6)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{7}{4 \pi (36)} = \frac{7}{144 \pi} \text{ cm/sec} \]

\[ \boxed{\text{cm/sec}} \]

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**3.6 Exponential Growth \& Decay**

**Note:** Change in quantity depends on quantity itself.

\[ \frac{dQ}{dt} = f(Q) = kQ \]

\( k = \text{growth rate (if } k > 0) \)

\( k = \text{decay (if } k < 0) \)
\[ \frac{dy}{dt} = k \cdot y \] is a solution to \[ \frac{dy}{dt} = ky \] (\( \frac{dy}{dt} = Ce^{kt} \cdot k \))

Given \( \frac{dy}{dt} = ky \), the solution is \( y = Ce^{kt} \).

Need to find \( C \) and \( k \).

\[ y(t) = Ce^{kt} \]
\[ y(0) = Ce^{0} = C \]
\[ C = y(0) = \text{initial value of } y \]

*Example 1 in text*

World pop'n in 1950: 2,560 million in 1950

\[ 1950: t=0 \]

*3,040 million in 1960

Find a model to predict world population.

Assuming exponential growth, \( \frac{dp}{dt} = kp \)

\[ p(t) = p(0) \cdot e^{kt} = 2560e^{kt} \]
\[ P = 2560e^{0.017t} \]

\[ (9000) = 2560e^{0.017t} \quad \text{Find } t \]

\[ \frac{9000}{2560} = e^{0.017t} \]

\[ \ln \left( \frac{9000}{2560} \right) = 0.017t \]

\[ t = \frac{\ln \left( \frac{9000}{2560} \right)}{0.017} \]

\[ = 74 \text{ yrs.} \]

\[ 1950 + 74 = 2024 \]

Radioactive Decay \((k < 0)\)

\[ M = \text{Amount of Material} \]

\[ M = M_0e^{kt} \quad k < 0 \]

Half-Life: Time to Decay 1/2 of Material.

\[ \left( \frac{1}{2} M_0 \right) = M_0e^{kt} \]

\[ \ln \left( \frac{1}{2} \right) = \ln(e^{kt}) = kt \]

\[ \frac{\ln \left( \frac{1}{2} \right)}{t_h} = k \]

\([t_h] = \text{Half Life}\)
MODEL IS: \( P(t) = 2560 \, e^{kt} \)

Solve for \( k \)

\[
\ln(e^{10k}) = 10k = \ln\left(\frac{304}{256}\right)
\]

\[
k = \frac{1}{10} \ln\left(\frac{304}{256}\right) \approx 0.017185...
\]

\[
P(t) = 2560 \left[ \frac{\ln\left(\frac{304}{256}\right)}{10} \right] e^{0.017185t}
\]

\[
P(t) \approx 2560 e^{0.017185t}
\]

a) WHAT IS CURRENT WORLD POPULATION IN 2018 ACCORDING TO THIS MODEL?

2018: \( t = 2018 - 1950 = 68 \)

\( P(68) = 2560 \cdot e^{0.017185(68)} \approx 8133.6 \text{ MILLION} \)

OVERESTIMATE: AS OF 7 NOV

7662 MILLION.

b) WHEN DOES MODEL SAY POPULATION WILL REACH 9 BILLION PEOPLE (9000 MILLION)
(3.6.7) \( \text{HALF-LIFE OF CESIUM-137 IS 30 YRS. START W/ 100 MG SAMPLE} \)

a) Find mass after \( t \) years.

\[
M(t) = M_0 e^{kt}
\]

\[
m = 100 e^{k \cdot 30}
\]

\[
\frac{1}{2} = e^{30k} \rightarrow \ln\left(\frac{1}{2}\right) = 30k
\]

\[
\frac{\ln\left(\frac{1}{2}\right)}{30} = k \approx -0.0231
\]

\[
M = 100 e^{\frac{-0.0231 \cdot t}{30}}
\]
MIDTERM EXAM:
7 PROBLEMS ON 7 PAGES,
20-25 RESPONSES.

CH1
1 PROB → DOMAIN/RANGE: INTERPRET GRAPHS,
1 PROB → SEQUENCES (FUNCTION LIMITS)

CH2
1 PROB → LIMITS (ALGEBRAIC CALC.)
1 PROB → CONTINUITY

CH3
1 PROB → DERIVATIVE USING DEFINITION
\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
2 PROB → DERIVATIVE "SKILLS"
FORMULAS; RULES.

ALSO TANGENT LINE TO A CURVE GIVEN BY A FUNCTION.