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2.5 CONTINUITY (CONT'D)

**DEFN:** \( f \) is continuous at \( a \) if

\[
\lim_{{x \to a}} f(x) = f(a)
\]

**Example:**

\[
\hat{g}(x) = \begin{cases} 
\frac{1}{(x-3)^2} & x \neq 3 \\
100 & x = 3
\end{cases}
\]

\( \hat{g}(3) = 100 \) \( \checkmark \)

\[
\lim_{{x \to 3}} \frac{1}{(x-3)^2} = \infty \text{ (DNE)}
\]

\( g \) cont at \( x = 7 \? \)

\( g(7) = \frac{1}{(7-3)^2} = \frac{1}{16} \checkmark \)

\[
\lim_{{x \to 7}} \frac{1}{(x-3)^2} = \frac{1}{16} \checkmark
\]

**NO** \( g(3) \) **not defined**
Types of Discontinuities

Removable:

\[ f(a) \text{ D.N.E. (or } \neq L) \]

\[ \lim_{x \to a} f(x) = L \]

Jump:

\[ \lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x) \]

Infinite (Jump):

\[ \lim_{x \to a^-} f(x) = \frac{+\infty}{0} \]

\[ \lim_{x \to a^+} f(x) = \frac{-\infty}{0} \]

\[ \lim_{x \to a} f(x) \text{ is cont. from left} \]
DEF: A function \( f \) is continuous from the right at \( a \) (or from above) if 
\[ \lim_{{x \to a^+}} f(x) = f(a) \]

Continuous from the left (below): 
\[ \lim_{{x \to a^-}} f(x) = f(a) \]

\[ f(x) = \begin{cases} 
  x^2 + 3x & x \leq 2 \\
  2x + 5 & x > 2 
\end{cases} \]

\[ f(2) = (2)^2 + 3(2) = 10 \]

\[ \lim_{{x \to 2^-}} (x^2 + 3x) = (2)^2 + 3(2) = 10 \]

\[ \lim_{{x \to 2^+}} (2x + 5) = 2(2) + 5 = 9 \]

\[ \lim_{{x \to 2}} f(x) = \text{DNE.} \]

Not continuous at \( x = 2 \), but \( f(x) \) is continuous from left at \( x = 2 \)

\[ f(x) = \begin{cases} 
  x^2 + a & x < 4 \\
  b & x = 4 \\
  ax + 1 & x > 4 
\end{cases} \]

\[ f(4) = b \]

\[ \lim_{{x \to 4^-}} (x^2 + a) = (4)^2 + a = 16 + a \quad \uparrow \quad \text{EQUAL} \]

\[ \lim_{{x \to 4^+}} (ax + 1) = a(4) + 1 = 4a + 1 \]

Find values of \( a \) and \( b \) so that \( f \) is continuous at \( x = 4 \).
**Definition:** A function \( f \) is **continuous on an interval** \( I \) if it is continuous at every point in \( I \).

**Note:**
- If \( f \) is **not** continuous at \( x = -4 \) and \( x = 4 \),
- But \( f \) is continuous on \((-4, 6] \).
- \[ \lim_{{x \to 6^-}} f(x) \]
If $f$, $g$ are continuous at $x = a$, and if $c$ is a constant, then the following are continuous at $x = a$:

- $f + g$
- $f - g$
- $c \cdot f$
- $f \cdot g$
- $\frac{f}{g}$ (if $g(a) \neq 0$)

**Note:** Functions continuous where defined, polynomials, rational functions, radical functions.

- $f(x) = \frac{x^2 - 4}{x - 2}$
- $\sqrt{9 - x^2}$

**Trig. functions, inverse trig. functions, exponential functions, log functions.**

**Then** if $f$ is continuous at $x = b$ and $\lim_{x \to a} g(x) = b$

Then $\lim_{x \to a} f(g(x)) = f(b)$
So \( \lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) \)

**Theorem** If \( g \) is **continuous at** \( x = a \) and if \( f \) is **continuous** at \( b = g(a) \) then \( f \circ g \) is **continuous** at \( x = a \).

**Show** \( \lim_{x \to a} f(g(x)) = f(g(a)) \)

\[
\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) \\
\text{exists} \\
\text{f(}g(a)\text{) exists}
\]

**Ex** \( F(x) = \ln(1 + \cos x) \) **where is** \( F \) **continuous?**

**Note** \( \ln(\_\_\_) \) **is continuous for** \( (\_\_\_) > 0 \) **need** \( 1 + \cos x > 0 \)

\[
\lim_{x \to a} \ln(1 + \cos x) = \ln(\lim_{x \to a} (1 + \cos x))
\]

\[1 + \cos x = 0 \]
\[\cos x = -1 \]
\[x = \pm \pi, \pm 3\pi, \pm 5\pi, \ldots \]

\( F \) **is continuous when** \( x = \pm \pi, \pm 3\pi, \ldots \)
**THM (Intermediate Value THM) - I.V.T.**

Suppose \( f \) is cont. on \([a, b]\) and let \( N \) be a number between \( f(a) \) and \( f(b) \). There exists (\( \exists \)) a \( c \in (a, b) \) such that \( f(c) = N \).