1. a. Find \( \frac{dy}{dx} \), given \( y = \arctan(x) \cdot e^{(x^3)} \)

b. Find \( f''(x) \) [the second derivative] of \( f(x) = \sec\left(2x - \frac{\pi}{3}\right) \)

2. Find the critical points for the function \( f(x) = \sqrt{x} \cdot (x - 9)^2 \).

3. Use the technique of logarithmic differentiation to find \( \frac{dy}{dx} \), if \( y = (x^3 + 1)^x \). Write your answer explicitly in terms of \( x \).

4. The graph of the equation \( x^2 + 3y^2 = 6x + 6y + 7 \) is an ellipse (graph is shown below):

a. Find the point on the graph of this equation that crosses the negative \( x \)-axis.

b. Find the general formula for \( \frac{dy}{dx} \) (in terms of both \( x \) and \( y \)).

c. Find the equation, in slope-intercept form, of the line tangent to the graph of this equation at the point from part a.

5. Evaluate the following limits, if they exist (if a limit does not exist, write DNE): justify your answers using algebraic techniques and limit laws. Numerical calculations will receive only partial credit.

a. \( \lim_{x \to \infty} \frac{9e^x - 5e^{-x}}{4e^x + 7e^x} \)

b. \( \lim_{x \to 0} \frac{\sin(4x) - 4x}{7x^3} \)

c. \( \lim_{x \to 0} \frac{\cos 5x}{x^2} \)

6. Find the value of the constants \( a \) and \( b \) that makes \( g \) a continuous function for all real numbers. Explain your reasoning using the definition of continuity at a point.

\[
g(x) = \begin{cases} 
ax^2 + 6 & x \leq 3 \\
bx - 1 & 3 < x < 7 \\
x + b & x \geq 7 
\end{cases}
\]
7. Let \( f(x) = 4x + \cos(x) \). Use the formula \( \frac{d}{dx} \left( f^{-1}(x) \right) = \frac{1}{f'(f^{-1}(x))} \) to find \( \frac{d}{dx} \left( f^{-1}(2\pi) \right) \).

[Note that you will need to find a value \( c \) such that \( f(c) = 2\pi \)]

8. Find the absolute maximum and minimum values of the function \( f(x) = x^3 - 12x^2 + 36x - 20 \) on the interval \([1, 9]\). Justify your answers using calculus.

9. A motorist leaves a town at noon traveling north at 60 mph. At 1 pm, another motorist leaves the same town traveling east at 50 mph. How fast is the distance between the two motorists changing at 2 pm?

10. Given the function \( f(x) = x^4 - 11x^2 + 28 \), find:

a. the coordinates of all intercepts.
b. any critical value(s).
c. the inflection point(s).

11. Given the function \( f(x) = \frac{2x^3-6x^2}{x^2+x-12} \), find the equation of any asymptote(s) (vertical, horizontal and/or oblique). Justify your answers with appropriate algebra and calculus techniques.

12. A box with a square base and an open top must have a volume of 4000 cubic inches. Find the dimensions of the box that minimize the amount of material used to construct the bottom and sides of the box. What is the minimum amount of material used? Be sure to label your answers.
1. a. \( \frac{e^{(x^3)}}{1+x^2} + 3x^2 \arctan(x) \cdot [e^{(x^3)}] \)

b. \( 4\left(\sec\left(2x - \frac{\pi}{3}\right)\tan^2\left(2x - \frac{\pi}{3}\right) + \sec^3\left(2x - \frac{\pi}{3}\right)\right) \)

2. \( f'(x) = 0 \) at \( x = 1, 9 \) : \( f'(x) \text{ DNE at } x = 0 \) : critical numbers are 0, 1, and 9.

3. \( \frac{dy}{dx} = (x^3 + 1)^{(x^2)} \left(2x \ln(x^3 + 1) + \frac{3x^4}{x^3+1}\right) \)

4. a. \((-1, 0)\)  
   b. \( \frac{dy}{dx} = \frac{x-3}{3-3y} \)  
   c. \( y = -\frac{4}{3}x - \frac{4}{3} \)

5. a. \( \frac{9}{7} \)  
   b. \(-\frac{32}{21}\)  
   c. \( \infty \) (DNE)

6. \( a = -\frac{1}{3} \)  
   \( b = \frac{4}{3} \)

7. \( f\left(\frac{\pi}{2}\right) = 2\pi \); so \( f^{-1}(2\pi) = \frac{\pi}{2} \)  
   \( \frac{d}{dx}(f^{-1}(2\pi)) = \frac{1}{3} \)

8. \( f_{\max} = 61\) at \( x = 9 \) \( f_{\min} = -20\) at \( x = 6 \)

9. \( \frac{970}{13} \approx 74.6 \text{ mph} \)

10. a. \((0,28); (\pm 2, 0); (\pm \sqrt{7}, 0)\)  
    b. \( x = 0, \pm \sqrt{\frac{\pi}{2}} \)  
    c. \( (\pm \sqrt{\frac{\pi}{6}}, \pm \frac{403}{36}) \)

11. \( x = -4 \): \( y = 2x - 8 \)

12. base length = 20 inches  
   height = 10 inches  
   material used = 1200 sq. in.