Tuning By Spectra:

Computer tools for crafting tuning systems from Korean musical timbres and other sounds.

John Seales

— *If ratios appear to be a new language, let it be said that it is in actual fact a language so old that its beginnings as an expression of the essential nature of musical sound can only be conjectured. In learning any new language, the results are more immediate if a total plunge is made, so that the new medium surrounds and permeates the thinking …*[1]

– Harry Partch

Though we say that twelve-tone equal temperament is our standard tuning, we rarely, if ever, hear it in musical practice. Instruments that produce pitch flexibly – the woodwinds, strings, and voice – tune to other instruments in the ensemble, not to a pure equal temperament. Even the piano is not strictly 12-ET; its tuning is stretched, and the amount of stretch is gracefully varied along the range of the instrument *(cite)*. What the above examples have in common is the mutual adapting of pitches so that their combination sounds in tune.

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[1] Harry Partch. *Genesis of a Music: An Account of a Creative Work, Its Roots and Fulfillments* (New York: Da Capo Press, 1974), 77.

The inspiration for this article, and the software described herein, is a vision of a collection of notes tuning to each other democratically, with no relationship between two notes a priori more important than any other. Tuning systems emerge in a bottom-up fashion from a process of mutual adaption, resulting in myriad surprising and beautiful structures. Furthermore, the structures hold interesting implications for modes, scales, melodies, harmonies, and other musical parameters.

I begin with a look at the Pythagorean and just diatonics to introduce ways of representing and evaluating tuning systems. William Sethares' method of measuring sensory dissonance then provides a tool for selecting desirable intervals. I investigate and expand upon the principles behind Partch's monophony and Lou Harrison's collection of "slendro" pentatonics in his *Music Primer* to find a point of departure for new exploration, after which I explain in detail how my tuning algorithm works, and present a few examples of tunings generated by it. Making sense of more complex tunings requires a method of analysis, which I present through a deeper look at the just diatonic. I then use this method to reveal the intriguing structures of tunings based on the spectra of Korean percussion instruments, and of the chromatic scale for a harmonic timbre, each produced by my algorithm. I finish by introducing ways of using tuning structures as compositional resources, generating melody and harmony, and other musical parameters.

For a reasonably large set of target intervals, their possible arrangements in a tuning are astronomical in number. In the past, this profusion of possibilities virtually necessitated a top-down approach because of the time and energy required to design and implement a novel tuning system. Tuning systems from the Pythagorean/Chinese system, to just intonation, to the tempered tunings and equal temperament, each derive from the application of a small number of controlling principles that make the system easier both to reproduce and to defend to colleagues as "rational." Newer inventions such as Partch's monophony, non-12 equal temperaments, and Michael Harrison's Revelation tuning[2] are likewise top-down designs.

Computer algorithms today offer new possibilities for tuning system design. With synthesizers, tunings can be created and realized with ease, and discarded without regret. The bewildering variety of structures that arise from any possible arrangement of intervals can be managed by software. In short, it is now feasible to journey into a vast forest of tuning systems that emerge in a bottom-up fashion without getting lost

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[2] Michael Harrison, "Music In Just Intonation," http://michaelharrison.com/web/pure_intonation.htm (accessed April 22, 2009).

in burdensome practical concerns. The beauty of the structures that result from such methods make the journey well worthwhile.

Pythagorean and just diatonics

(Include rationale for the inclusion of this section)

The "Pythagorean" tuning system is both ancient and widespread. In the West, it was first described by Boethius[3] but evidence exists of its use 3500 years ago by the Babylonians[4]. It has been used throughout recorded history (and probably long into prehistory) by the Chinese[5].

In this system the "good[6]" intervals are those that can be expressed as ratios between integers from one to four. These ratios include only

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[3] Anicius Manlius Severinus Boethius, *Fundamentals of Music,* Calvin M. Bower, trans., ed. (New York: Yale University Press, 1989).
[4] M.L. West "The Babylonian Musical Notation and the Hurrian Melodic Texts," *Music & Letters* 75:2, 161-179.
[5] Ernest McClain and Ming Shui Hung, "Chinese Cyclic Tunings in Late Antiquity," *Ethnomusicology*, 23:2, 205-224.
[6] In this context, I'm using the word "good" in a technical sense: that the interval in question is a goal in the current design process. They are not necessarily good or bad in any absolute sense.

the octave and 3/2 perfect fifth, along with their complements and octave equivalents. A Pythagorean scale is generated by stacking 3/2 perfect fifths or 4/3 perfect fourths, duplicating the resulting pitches in each octave. The tuning creates pure fifths, but the tuning of seconds, sevenths, thirds, sixths and tritones are unintended by-products of the design process.

Graph theoretic notation is useful to visualize the structure of this tuning – as it will be for analyzing more complex examples. Depicting notes as vertices and the desired intervals as edges, the Pythagorean diatonic is a simple chain.

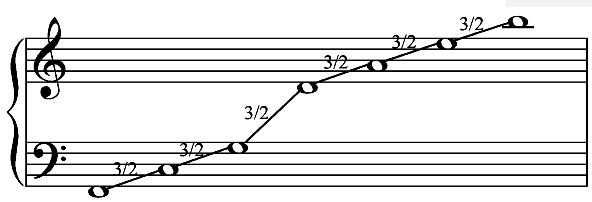

Figure 1

With seven entities, there are twenty-one possible pairings; so a tuning system of seven notes contains twenty-one distinct intervals. In the Pythagorean tuning, only six of them are "good." This poverty of relationships is a consequence of having only one kind of target interval; with n notes there are a maximum of n-1 instances of a single target interval[7].

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[7] This constraint holds unless the good interval is the kth root of an interval of equivalence. For instance, the circle of fifths in equal temperament includes 12 perfect fifths from 12 notes: n

Zarlino, in 1558, was the first to propose the deliberate use of purer-sounding thirds than those in the Pythagorean system[8]. His innovation was to increase the range of integers for use in musical ratios from four to six. Crucially, the inclusion of five among the generating numbers dramatically enriches the available good relationships between notes. A graph of the just diatonic scale (Figure 2) is more complexly connected than that of the Pythagorean; out of the twenty-one intervals between seven notes, eleven are good. The just diatonic is richer in ideal intervals than the Pythagorean.

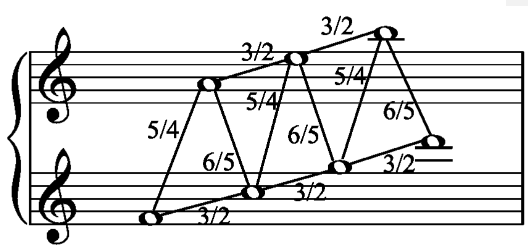

Figure 2

In general, as we broaden the palette of desirable intervals, we increase the potential connections within a group of notes. However, this expansion has psychoacoustical limits. It is generally recognized that for harmonic timbres, ratio intervals sound in tune when their numerator and denominator are small integers, though the

instances of the target interval from n notes.

[8] Zarlino Gioseffo, *Le istitutioni harmoniche* (Reprinted New York: Broude Bros.) 1965.

precise cutoff is highly contested. An exploration of the concept of sensory dissonance can help to understand why this is so, and what limits for the numerator and denominator are appropriate. In addition, calibration of sensory dissonance is one of several fruitful bases for the bottom-up construction of tuning systems.

Sensory Dissonance

James Tenney identifies five distinct concepts of dissonance and consonance that are associated with overlapping periods of history[9]. The last of these, cdc-5, is the conception of dissonance as sensory roughness[10], caused by beats between sinusoidal components of a complex sound.

An oft-raised objection to CDC-5 as a relevant concept of dissonance in music is the "fact" that intervals between simple tones (sine waves) are perceived to be relatively consonant at small integer ratios. Since upper partials are not

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9 James Tenney, *A History of "Consonance" and "Dissonance"* (New York: Excelsior, 1988).

10 This conception of consonance and dissonance was introduced by Helmholz in *On the Sensations of Tone*. Helmholz's ideas have proved a major source of inspiration to subsequent thinkers about tuning systems, including Harry Partch and William Sethares.

present in simple tones, consonance and dissonance cannot be caused by their interaction. However, Plomp and Levelt[11] showed that ratio intervals between simple tones were perceived as consonant only when experienced musicians served as test subjects, responding in accordance with habit and training. Tests on non-musicians show no preference for ratio intervals between simple tones.

Hermann von Helmholz reported that the maximum sensory dissonance occurs between simple tones about 32 Hz apart in frequency[12]. The resulting beats at 32 Hz are too fast to be perceived individually, and blur into an unpleasant roughness. Plomp and Levelt refined Helmholz's observation, showing that 32 Hz was a good estimate for maximum roughness only for tones between about 500-1000 Hz. In fact, the interval of maximum sensory roughness correlates to the critical band[13] throughout the range of hearing. Plomp and Levelt's dissonance curve (Figure 3)

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[11] R. Plomp and W. J. M. Levelt, "Tonal consonance and critical bandwidth," *Journal of the Acoustical Society of America* 38 (1965): 548-560.
[12] Hermann von Helmholz, *On The Sensations Of Tone As A Physiological Basis For The Theory Of Music,* New York: Dover Publications, 1954.
[13] R. Plomp and W. J. M. Levelt, "Tonal consonance and critical bandwidth," *Journal of the Acoustical Society of America* 38 (1965): 548-560.

shows a minimum at the unison, a maximum at about .25 of the critical band, and a rapid falling off as the two simple tones further diverge. The vertical scale is ordinal; it corresponds to subjective dissonance as perceived by test subjects.

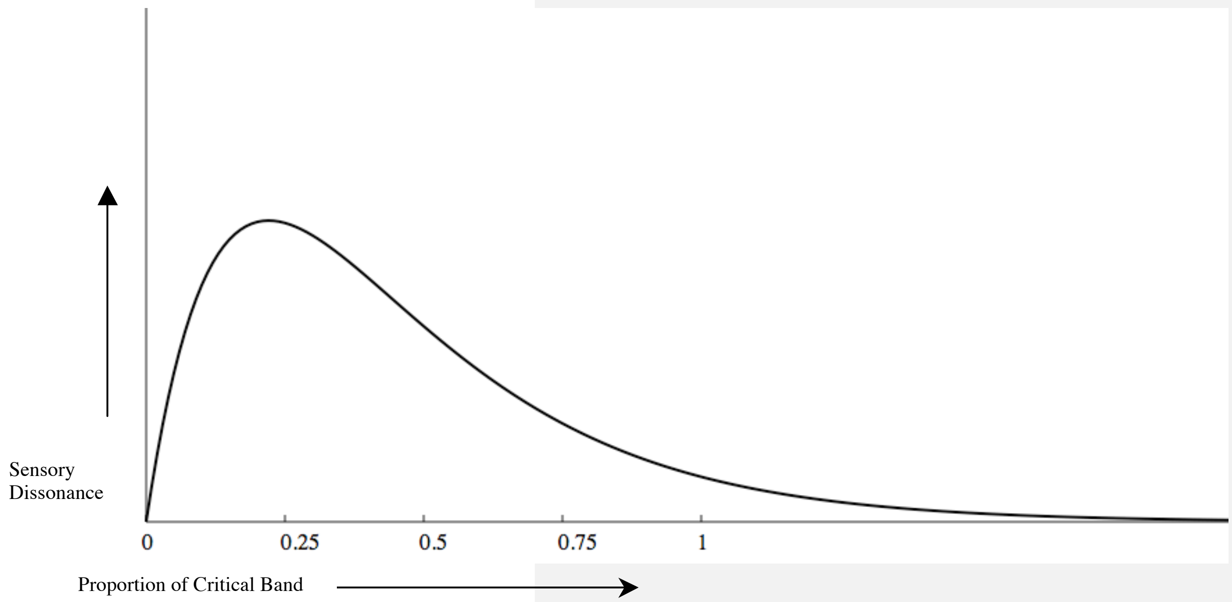

Figure 3[14]

Using this curve, it is possible to calculate sensory dissonance between any pair of simple tones. Because complex tones are composed of simple tones, the dissonance between complex tones can be measured by summing the interactions of all the constituent partials. This method of measuring the sensory dissonance

[14] The curve in Figure 3 is an approximation of Plomp and Levelt's results parameterized by William Sethares in *Tuning,Timbre, Spectrum, Scale,* Appendix E.

between tones is discussed in William Sethares' 1998 book *Tuning, Timbre, Spectrum, Scale*[15]. Sethares shows how to calculate a "dissonance curve," a graph whose x-axis is th interval between the fundamentals of two tones, and whose y-axis is the sensory dissonance of the two tones at that interval.

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Figure 4 shows a dissonance curve between two sawtooth waves with eight harmonic partials.

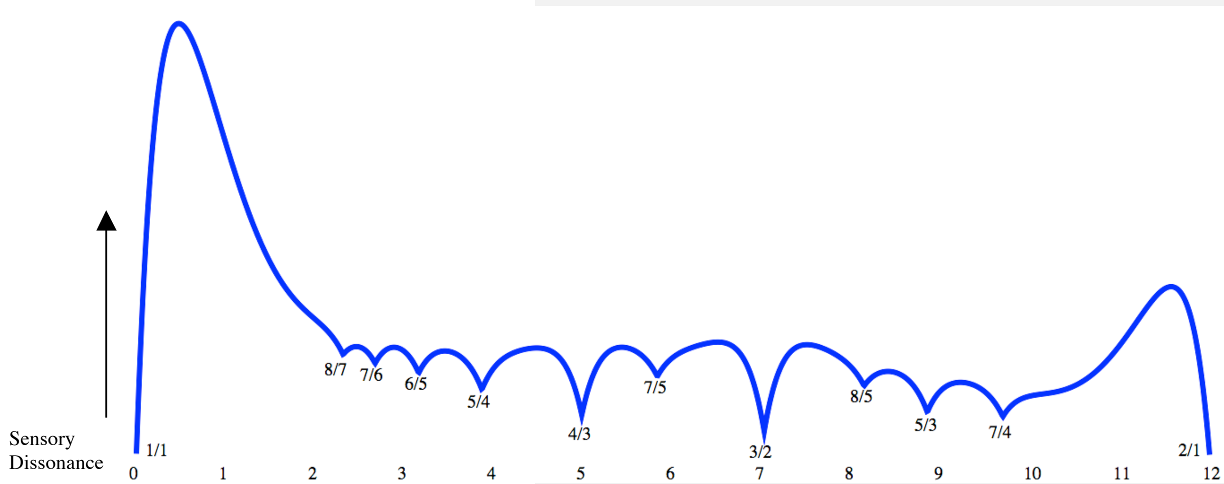

Figure 4

Semitones →

Those familiar with just intonation will be unsurprised to see that minima in the curve occur at intervals at ratios between the first eight integers. The depths of the minima lessen with the magnitude of the numbers in the numerator and

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[15] William A. Sethares, *Tuning, Timbre, Spectrum, Scale*, London: Springer, 1998.

denominator, a result of the decreasing amplitude of the partials.

If we increase the number of partials to sixteen, we see the dissonance curve in Figure 5. Even though partials 11-16 are present, their amplitudes are insufficient in the sawtooth wave to show up as a minimum in the dissonance curve.

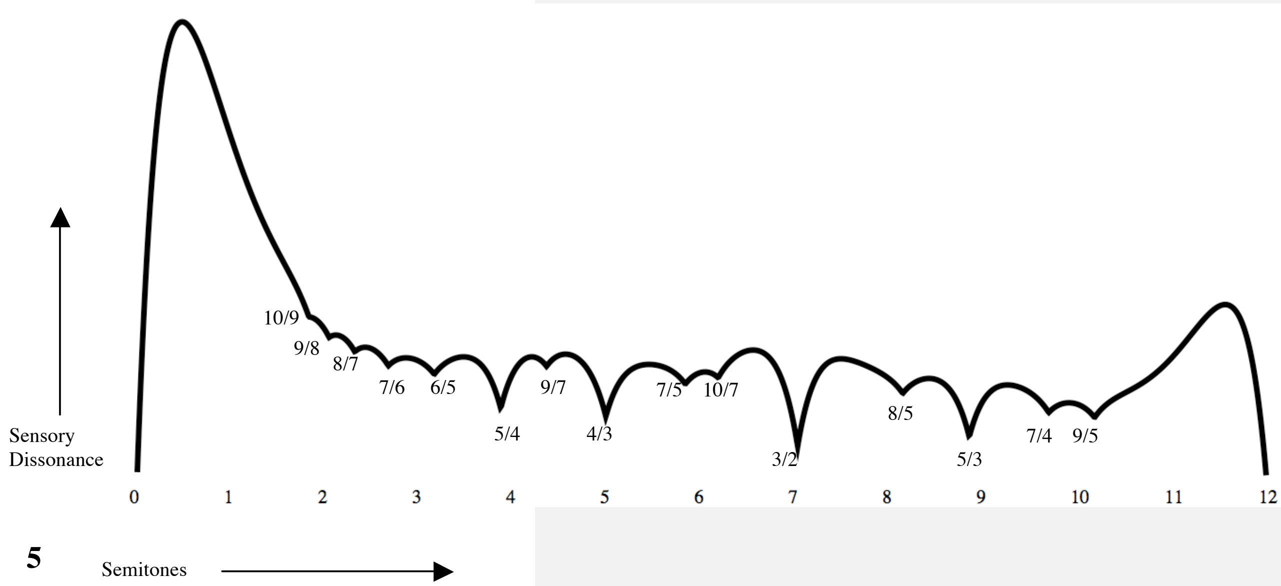

Figure 5: dissonance curve: the first 16 partials of a sawtooth wave

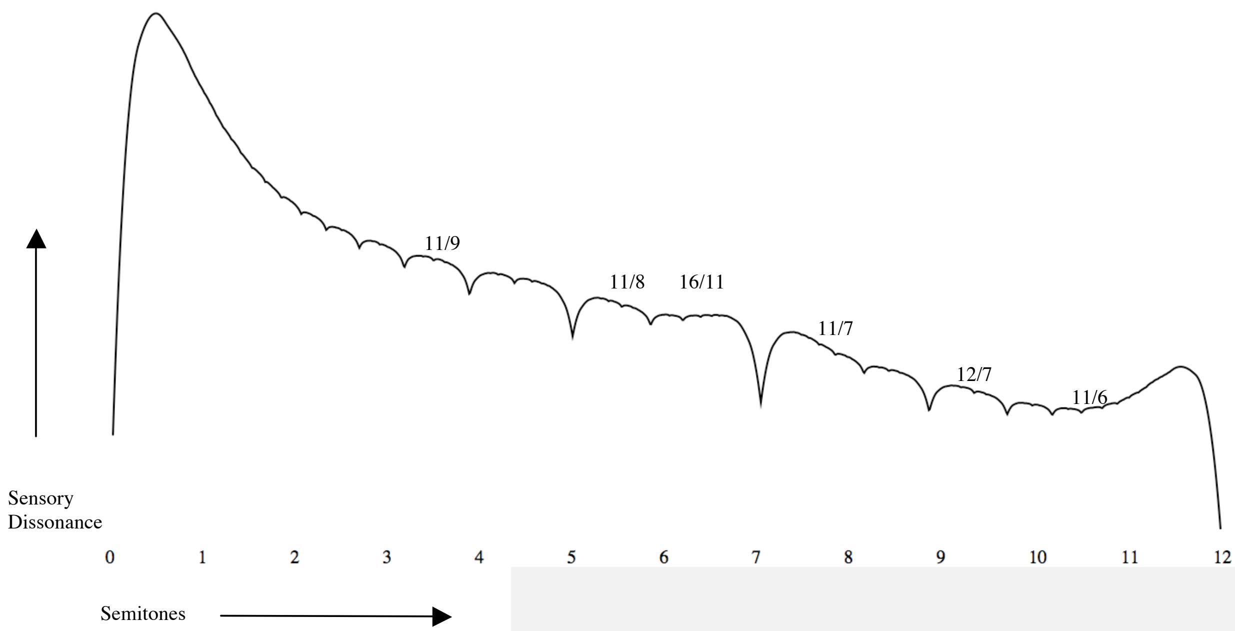

Figure 6: dissonance curve for the first 64 partials of a sawtooth wave.

Increasing the number of partials to 64 leads to a dissonance curve with a few more visible minima (Figure 6) but none of these minima are at ratios involving the number 13. Partch used 11 as the limit in his system; an examination of dissonance curves confirms the validity of his judgment. It may still make sense to use ratios involving 13, as long as the tuning designer is conscious that they are likely to be quite dissonant. In fact, including the 13 (or higher) limit chords is one way of deliberately including dissonant intervals in a tuning.

Dissonance curves can also be generated for inharmonic timbres, or between two different timbres. Figure 7c shows the dissonance as a harmonic timbre (Figure 7a) is moved in relation that of the ideal circular membrane (Figure 7b).

Figure 7a (Harmonic partials)

Figure 7b (Ideal membrane partials)

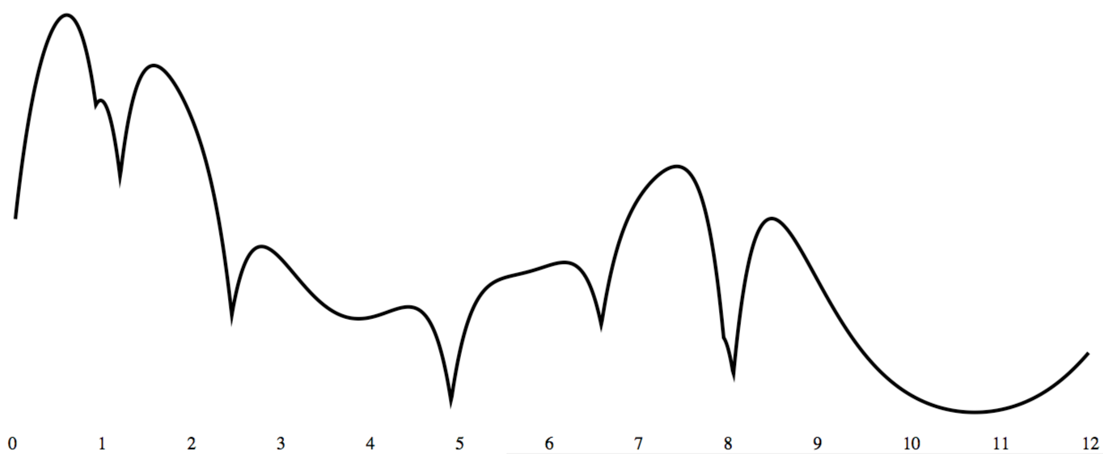

Figure 7c

Semitones:	0.90	1.17	2.42	4.87	6.55	7.92	8.03
Fractions:	3.16/3	2.14/ 2	2.3/2	2.65/2	2.92/2	3.16/2	1.59/1

Figure 7d Decimal semitone heights for ratios between the harmonic and membrane spectra falling between unison and octave.

Dissonance curves can inform our choice limit for ratio intervals, and are one method of scoring the relative desirability of intervals in a tuning algorithm. Before discussing such algorithms, however, I will examine Harrry Partch’s monophony, in the process developing several tools for designing and analyzing tuning systems.

Partch's tonality diamond

In this section I see the tonality diamond as a representation of all intervals between partials of a harmonic timbre (with 11 partials.) This bolsters the case for the software I introduced which favors target intervals based on coinciding partials. Also I introduce and broaden the concept of otonalities and utonalities, which proves useful later in analyzing algorithmically generated tunings.

In the field of new music's experimentation with tuning systems, Harry Partch's influence is extensive. His work directly influenced that of Lou Harrison,[16] Ben Johnston,[17] and many others including myself. Partch's seminal Genesis of a Music explained the rationale and construction of his tuning system, monophony, named for the fact that all tones in the system are related to a single,

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[16] Leta E. Miller and Frederic Lieberman, *Composing A World: Lou Harrison, Musical Wayfarer*, Urbana: University of Illinois Press, 2004. 44-45
[17] Heidi Von Gunden, *The Music of Ben Johnston*. Metuchen, Scarecrow, 1986. p11-13

fundamental tone. The "tonality diamond[18]" (Figure 8), originally invented by Max Meyer[19] but mainly associated with Partch and his followers, represents the frequency ratios of the tones of the system to the fundamental (designated as '1/1'.) These tones are supplemented by several more to fill relatively large gaps in the scale and make a limited amount of modulation possible, but the tonality diamond is the main organizing principle of monophony.

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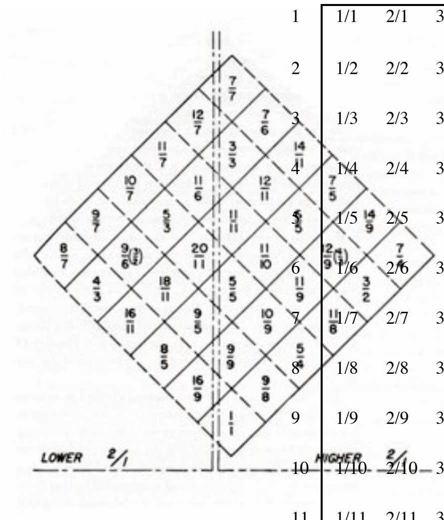
[18] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots and Fulfillments*, New York: Da Capo Press, 1974.
[19] Max, F. Meyer, *The Musician's Arithmetic*. Columbia: University of Missouri, 1929.



,	1	2	3	4	5	6	7	8	9	10	11
1	1/1	2/1	3/1	4/1	5/1	6/1	7/1	8/1	9/1	10/1	11/1
2	1/2	2/2	3/2	4/2	5/2	6/2	7/2	8/2	9/2	10/2	11/2
3	1/3	2/3	3/3	4/3	5/3	6/3	7/3	8/3	9/3	10/3	11/3
4	1/4	2/4	3/4	4/4	5/4	6/4	7/4	8/4	9/4	10/4	11/4
5	1/5	2/5	3/5	4/5	5/5	6/5	7/5	8/5	9/5	10/5	11/5
6	1/6	2/6	3/6	4/6	5/6	6/6	7/6	8/6	9/6	10/6	11/6
7	1/7	2/7	3/7	4/7	5/7	6/7	7/7	8/7	9/7	10/7	11/7
8	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8	9/8	10/8	11/8
9	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	9/9	10/9	11/9
10	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	10/10	11/10
11	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11	11/11

Figure 8[20]

The division table on the right in Figure 8 is equivalent to Partch's tonality diamond, by reorienting, normalizing to [1,2), reducing, and removing redundancies[21]. The tonality diamond is

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[20] Diagram 9. – The Expanded Tonality Diamond. In: Partch, Harry. *Genesis of a Music*, 159.

[21] To transform the raw table to Partch's representation involves three operations. First, normalize all fractions to between 1 and 2, multiplying by 2 if they fall below this range or dividing by 2 if they fall above. Then reduce the fractions. Finally, flip the table top to bottom and rotate 45 degrees counterclockwise.

Partch kept 9/6 and 12/9 unreduced, with their reduced equivalents 3/2 and 4/3 included in smaller print. The n/n fractions are partly reduced, with just powers of 2 cancelled—helpfully so, because the n a compact representation of all the fractions between integers 1 through 11, which are also the first 11 partials of a harmonic sound. And they coincide with the most salient sharp minima in a dissonance curve between to sawtooth waves.

Rows of the division table are called ‘Otonalities’; they are proportional to harmonic overtone series. Columns, proportional to an undertone series, are ‘Utonalities.’ A row and column that share a square on the n/n diagonal are inversions of one another. Together I call them an “otonal-utonal pair.”

Division tables can be generated by nonintegers representing the partials of an inharmonic spectrum. Figure 11’s division table uses the experimentally measured first nine partials of a G5 on a piano.

Because normalization involves the use of 2 as an interval of equivalence, it only applies to inharmonic-timbre tonality diamonds if one of the partials is equal to 2 (or at least very close.) Generally speaking, then, the concept of normalization does not apply to division tables from inharmonic spectra. One might use the data in Figure 11 to create slightly modified Otonalities and Utonalities specifically designed for the sound of a piano.

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is the generating number for its row and column.

	1.0	1.9	2.9	4.0	5.0	6.1	7.2	8.3	9.4
1.0	1.0	1.9	2.9	4.0	5.0	6.1	7.2	8.3	9.4
1.9	0.5	1.0	1.5	2.0	2.5	3.0	3.6	4.2	4.7
2.9	0.3	0.6	1.0	1.3	1.7	2.0	2.4	2.7	3.1
4.0	0.2	0.4	0.7	1.0	1.2	1.5	1.7	2.0	2.3
5.0	0.2	0.3	0.5	0.7	1.0	1.2	1.4	1.6	1.8
6.1	0.1	0.3	0.4	0.6	0.8	1.0	1.1	1.3	1.5
7.2	0.1	0.2	0.4	0.5	0.7	0.8	1.0	1.1	1.3
8.3	0.1	0.2	0.3	0.4	0.6	0.7	0.8	1.0	1.1
9.5	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0

Figure 11

Lou Harrison's pentatonics

In this section I revisit the inspiration I received from Lou Harrison's Music Primer. His playfulness in reordering the step-intervals of ratio tunings – and pondering the unstated reasons for his choices of what to present – inspired my vision of tunings that favor ratio relationships without following a top-down design.

In contrast to Partch's top-down design, Lou Harrison took a more relaxed and less rigorous approach to the exploration of tunings. The list of pentatonic modes in his *Music Primer* (Figure 12) are presented "at random"[22] But analysis of this list

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[22] Lou Harrison, *Music Primer; Various Items About Music To 1970*. New York, C. F. Peters, c1971. 29.

reveals a no less active interest in the profusion of desirable intervals they contain.

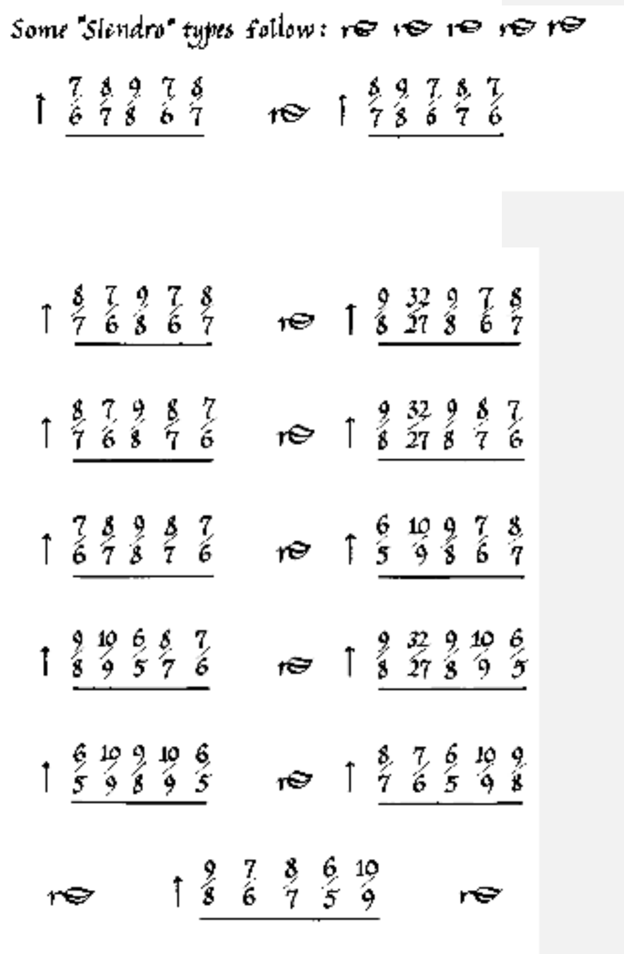

Figure 12: pages 29-30 of the *Music Primer* showing the "slendro" modes.

Now I examine further the pentatonics that reorder the intervals 7/6, 7/6, 8/7, 8/7, and 9/8. Harrison wasn't explicit about his criteria for including or excluding orderings of these intervals; perhaps a thorough examination of possible orderings and their total interval content can suggest a possible motivation.

For each distinct permutation of the intervals[23], there is an interval table in Figure 13 showing the total intervallic content of the pentatonic.

a) pentatonic "7/6 8/7 9/8 7/6 8/7"

1	7/6	4/3	3/2	7/4
	1	8/7	9/7	3/2
		1	9/8	(21/1(
			1	7/6
				1

b) pentatonic "8/7 7/6 9/8 7/6 8/7"

1	8/7	4/3	3/2	7/4
	1	7/6	(21/1(	(49/3:
		1	9/8	(21/1(
			1	7/6
				1

c) pentatonic "7/6 8/7 9/8 8/7 7/6"

1	7/6	4/3	3/2	12/7
	1	8/7	9/7	(72/4(
		1	9/8	9/7
			1	8/7
				1

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[23] Because there is only one 9/8, it can be fixed in the middle position, eliminating duplicates by rotation without loss of generality. Fitting the 8/7 intervals in two of the remaining four slots, (and the 7/6's in what's left) gives us four choose two, or six permutations. These six modes include two inversionally symmetrical pairs; eliminating these duplicates leaves us four distinct orderings with respect to rotation and inversion.

d) pentatonic "8/7 8/7 9/8 7/6 7/6"

1	8/7	(64/49	(72/49	12/7
	1	8/7	9/7	3/2
		1	9/8	(21/16
			1	7/6
				1

Figure 13

The ratios in parentheses, with their relatively large numerators and denominators, are not sensory consonances[24] and are unlikely to have been target intervals. From this standpoint, permutations a) and c) are preferable in that they are more suffused with “good” small integer ratios. Indeed, of the five pentatonics using these intervals in the music primer, three are of type a, one of type b, one of type c, and none of type d. Harrison’s relaxed approach favors tunings with more small ratios in four of five cases.

Pentatonics being a relatively simple case, an exhaustive working out of possibilities is not prohibitive, while an intuitive approach is certainly viable. For a more complex case, say a chromatic

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[24] It is possible for fractions with large integer numerator and denominator to be sensory consonances if they are extremely close in value to a small integer ratio, but these parenthesized ratios are not of this type. 64/49 is a very sharp M3, about 28 cents sharp of 9/7. 72/49 is 36 cents flat of 3/2. 21/16 is 28 cents flat of 4/3. 49/32 is 28 cents flat of 14/9. They are accidental by-products of the arrangement of the target small-number fractions.

tuning with intervals from a 11-limit division table, the myriad possibilities make a rigorous approach impractical, while an intuitive approach is liable to miss many good solutions.

Using computer programs to explore possible arrangements of target intervals is a way to manage the complexity involved. A computer can exhaustively work through a very large number of cases, presenting to the user only the results that best fit the musical material at hand. The method presented below has proved useful in the exploration and construction of a variety of novel tunings.

Insert harmonic 11-limit example.

"Spectral Tuner": software designed for exploring tuning systems, with examples.

The "Spectral Tuner" is an application for generating tunings based on sound spectra. Given a simplified representation of a sound spectrum, (several partial values and their associated amplitudes,) and a starting position for each of the notes in the tuning, the Spectral tuner retunes the notes (within an allowed range,) optimizing for maximal coincidence of partials the number of partials. It has tools to analyze the structure of the resultant tuning, which is usually a closely packed subset of a Tonnetz, as the following examples show. Though outside the scope of this article to dicuss, the Spectral Tuner also generates melodies and chords using the weighted graph of probabilities that is its structure.

Below is a tour of the program's features by way an example using a harmonic spectrum with 11 partials. Following that, more examples flesh out the program's further capabilities.

Upon starting the program, the user sees the "Tuning Frame" as shown in Figure 14.

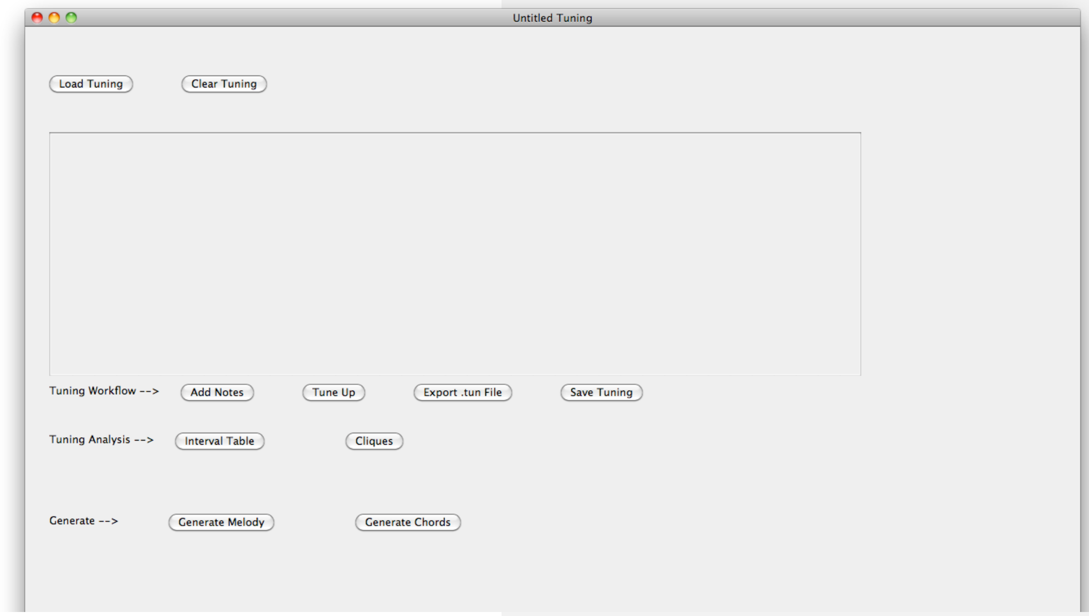

Figure 14

In the top row are the "Load Tuning" and "Clear Tuning" buttons. With the "Load Tuning" button, the user can recall a previously saved tuning, in order to alter it or explore it analytically.

“Clear Tuning” discards the current tuning in order to start afresh.

In the center of the frame is a large rectangular space, initially empty. In this space, “Note Gauges” give visual feedback about the state of the tuning.

Below the gauge frame there is a row of four buttons labeled “Tuning Workflow.” By working with these buttons in order, you may add notes to the tuning by defining their characteristics, use the Spectral Tuners’ algorithm to tune the notes, export the resulting tuning to a ‘.tun’ file[25], and save the tuning in a proprietary format so that you can reload and work on it later.

At the bottom of the frame, there are two amenities for “Tuning Analysis.” The “Interval Table” button displays a half-matrix of intervals in the tuning, labeling them with the appropriate ratio when the interval is a ratio between partial values, and a dash when it isn’t.

The “Cliques” button shows graphically how the notes in the tuning are related to each other by ratio intervals, and elucidates the structure of the tuning. On top of the stack of windows is a graph of the entire tuning, underneath which are windows for each of the cliques–fully connected subgraphs–in the tuning. The graphs’ appearance and interpretation are detailed in the tuning

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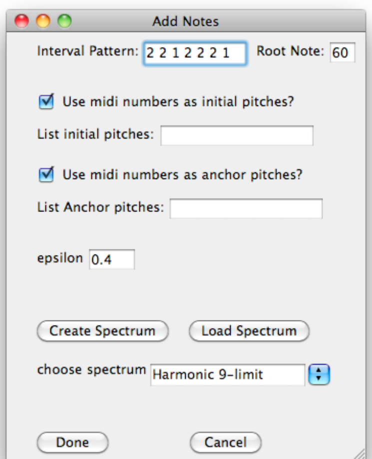
[25] “.tun” is a common file format for synthesizers to realize non-standard tunings.

example below.

The “Generate” row is on the bottom. Pressing these buttons sends midi information to a software synthesizer that explores some possibilities of the tuning. Melodic, and harmonic, and rhythmic features of the output are derived from a representation of the tuning as a weighted graph.



Tuning Example 1: Harmonic spectrum with 11 partials.

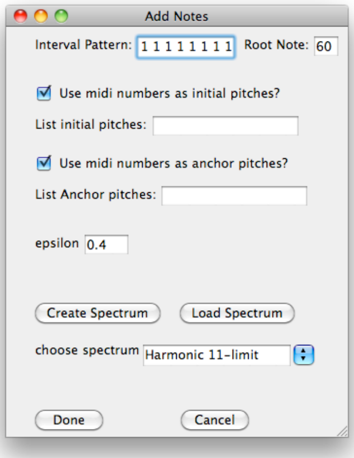
The “Add Notes” button produces the “Add Notes” frame, which offers a broad set of options for the notes’ characteristics. The Add Notes frame is shown in its initial state in Figure 15.

Figure 15

Together, the “Interval Pattern” and “Root Note” fields determine the midi numbers to which notes will be assigned. In the initial state, the notes would be 60,62,64,65,67,69,71,72–the C major scale. The differences between adjacent midi numbers are the sequence shown in the “Interval Pattern” field.

In this case, I’m going to tune up an octave-sized chromatic scale, so my interval pattern is: 1 1 1 1 1 1 1 1 1 1 1 1 – 12 ones. I could define initial pitches of any value whatever to associate with these midi numbers. However, in this instance, I want their initial pitches to be identical to the equal temperament tuning of these midi numbers, so I leave the checkbox “Use midi numbers as initial pitches?” checked.

“Anchor pitches” are the center of the range of pitches that is allowed for a particular note. Like the initial pitches, you can define them as any numbers at all, but in this case I will use their midi numbers as anchor pitches.



"Epsilon" is the pitch interval that is allowed for the notes above and below the anchor pitch. The default value is 0.4 semitones, slightly less than a quarter tone. It's important to set the value of epsilon less than half the smallest distance between anchor pitches so that the notes cannot tune themselves into a unison (unless you wish them to.)

The "Create Spectrum" button is explained below; for this example, we use the default "Harmonic 11-limit" spectrum. Clicking "Done" finalizes the choices and returns focus to the Tuning Frame.

'igure 16

Back in Tuning Frame, fields for note gauges, spectrum, "Aligned Partials," and "Consonance Score" now appear. (See Figure 17.) The "gauge frame" is now populated with "note gauges," showing the tuning of each note in reference to its anchor pitch – the center of the note's permitted pitch range. Note gauges are labeled above with the midi number to which they are assigned; below the note gauge is the decimal midi number that designates the exact pitch of the note.

Even before tuning, there are 5 aligned partials[26] and a "consonance score" of 2.832. For each pair of coinciding partials, the minimum of the two amplitudes is the consonance score. Summing these values for all aligned partials gives you the consonance score of the tuning.

---

[26] Partials 1,2,3,4,5 of note 72 align with partials 2,4,6,8,10 of note 60.

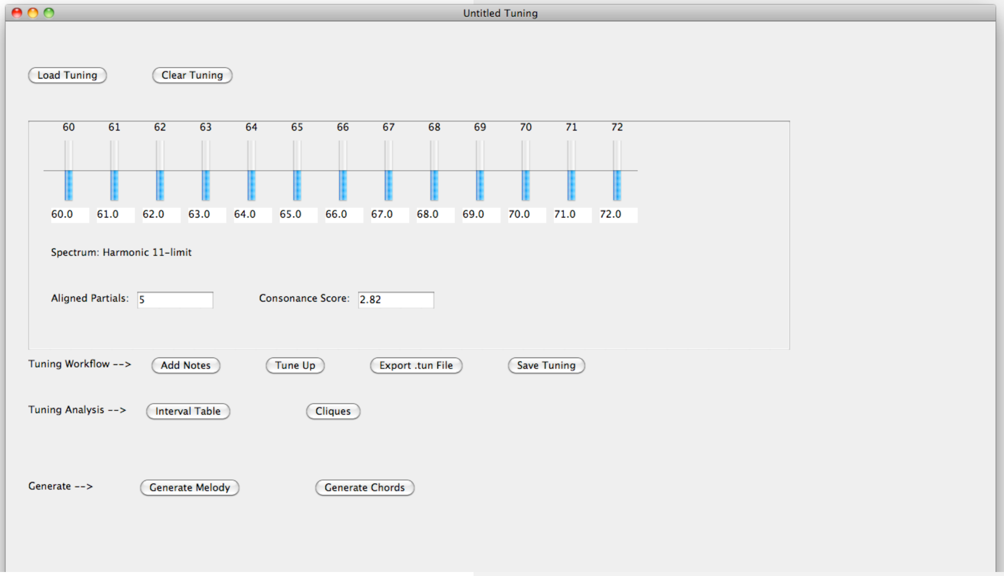


Figure 17

The next step in the process is to press "Tune Up." The program may work for several minutes or even hours depending on the number of notes and the complexity of spectra involved. This tuning took about three minutes.

In the middle of the tuning process, the Tuning Frame may look something like Figure 18:

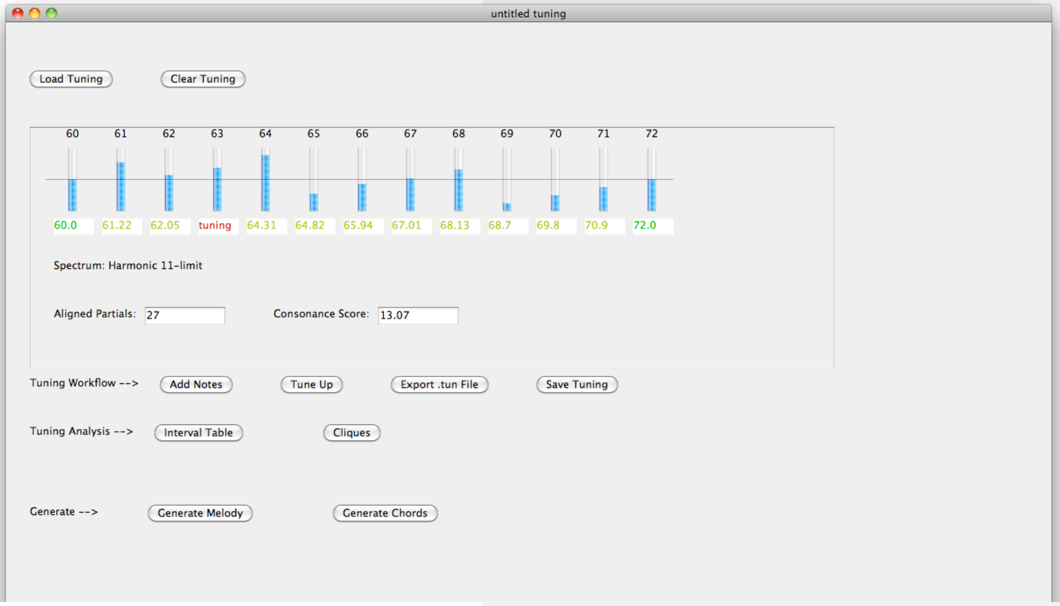


Figure 18

The word "tuning" appears in red under the gauge whose position is currently being optimized against the rest. Pitches that have already moved from their initial state are shown in yellow-green, and notes for which no improved tuning was found appear in green.

Figure 19 shows the Tuning Frame at the completion of the tuning process.

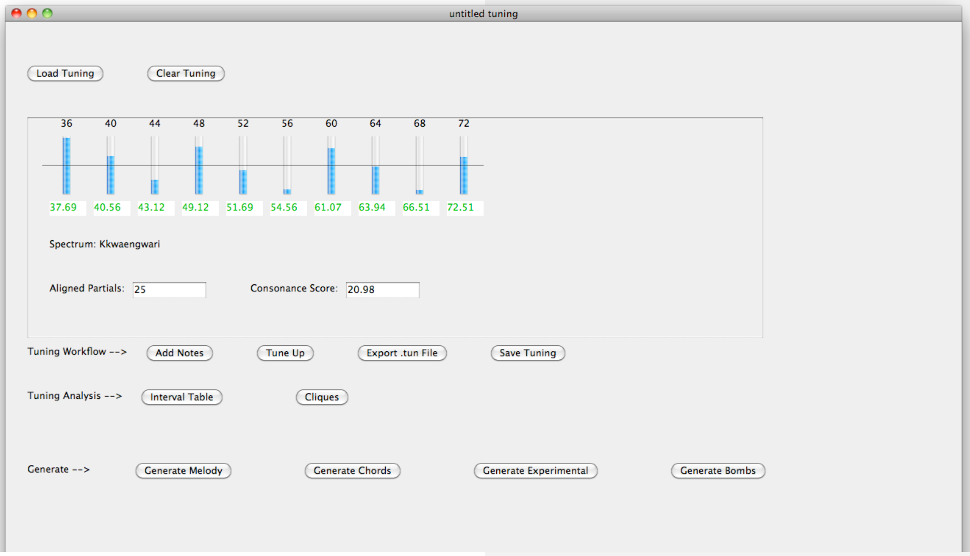
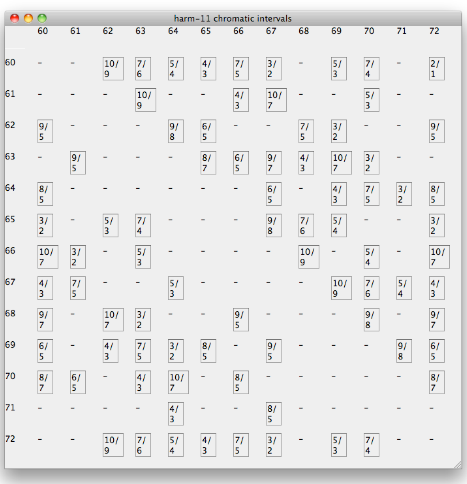


Figure 19


harm-11 chromatic intervals

	60	61	62	63	64	65	66	67	68	69	70	71	72
60	-	-	10/9	7/6	5/4	4/3	7/5	3/2	-	5/3	7/4	-	2/1
61	-	-	-	10/9	-	-	4/3	10/7	-	-	5/3	-	-
62	9/5	-	-	-	9/8	6/5	-	-	7/5	3/2	-	-	9/5
63	-	9/5	-	-	-	8/7	6/5	9/7	4/3	10/7	3/2	-	-
64	8/5	-	-	-	-	-	-	6/5	-	4/3	7/5	3/2	8/5
65	3/2	-	5/3	7/4	-	-	-	9/8	7/6	5/4	-	-	3/2
66	10/7	3/2	-	5/3	-	-	-	-	10/9	-	5/4	-	10/7
67	4/3	7/5	-	-	5/3	-	-	-	-	10/9	7/6	5/4	4/3
68	9/7	-	10/7	3/2	-	-	9/5	-	-	-	9/8	-	9/7
69	6/5	-	4/3	7/5	3/2	8/5	-	9/5	-	-	-	9/8	6/5
70	8/7	6/5	-	4/3	10/7	-	8/5	-	-	-	-	-	8/7
71	-	-	-	-	4/3	-	-	8/5	-	-	-	-	-
72	-	-	10/9	7/6	5/4	4/3	7/5	3/2	-	5/3	7/4	-	-

Now a .tun file can be exported so that the tuning can be realized on a synthesizer, and the tuning saved so it can be reloaded in a future session; the tuning workflow is finished.

Pressing the “Interval Table” button displays a table the shows intervals that are ratios between the values of the partials of the spectrum. The upper-right half matrix (where the row midi numbers are lower than the column midi numbers)

Figure 20

displays the intervals between the row note and the column note. The lower-left half matrix shows the interval between the row note and the column note raised by the size of the cycle. In this case the cycle size is the octave, so the entries in the lower left half matrix are the expected complementary intervals to those in the upper right. In general, however, the size of the cycle may be something other than an octave.

Clicking the “Cliques” button generates a series of graphs that further illustrate the structure

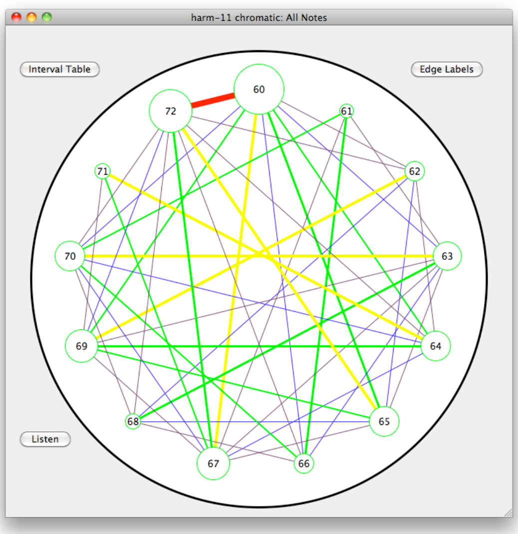


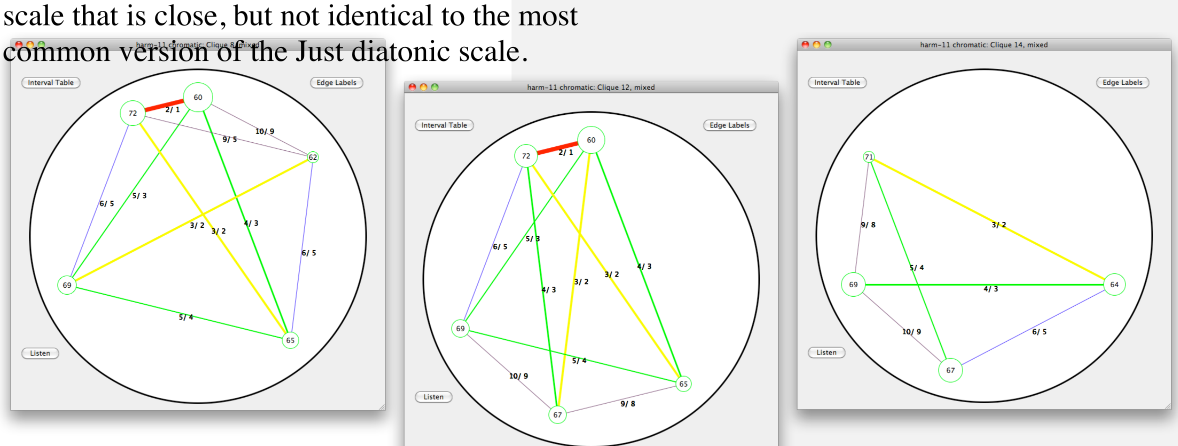
Figure 21

of the tuning. Thicker lines denote higher consonance scores between the two notes, and they are color coded with red lines for the strongest consonances, violet the weakest, and gradations in the order of the rainbow. The size of the circle around a midi number is proportional to its centrality – the sum of the weights of edges that connect to it. The top frame (Figure 21) has all the notes in the tuning, presented for reference. It is not generally a clique.

Each clique frame has a button to show the interval matrix for the notes that appear on it, and the ability to label the edges with the ratio interval they represent. Also, pressing the “listen” button, plays notes on the clique frame arpeggiated, then as a chord. (To hear the notes, there must be a synthesizer programmed with the tuning’s .tun file and set up to receive the Spectral Tuner’s midi output.)

The value of the cliques is twofold. These note collections, fully connected with relative sensory consonances, are good candidates for use as chords or scales. Furthermore, patterns in the cliques enable an understanding of the structure of the tuning as a whole. In general, tunings generated by the spectral tuner can be modeled as closely packed subsets of a Tonnetz, as this, and following examples show.

The five cliques shown in Figure 22 are subsets of the C major scale, and together show a scale that is close, but not identical to the most common version of the Just diatonic scale.



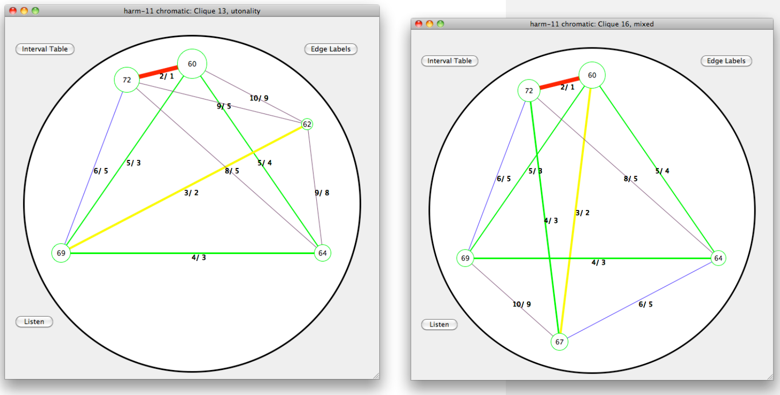
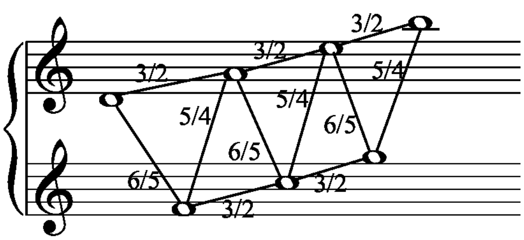


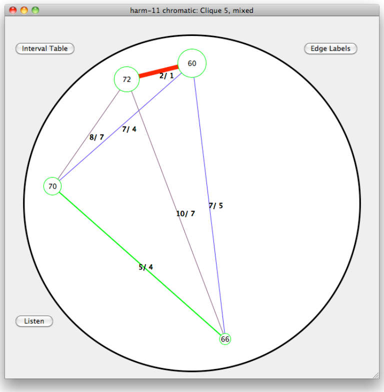
Figure 22

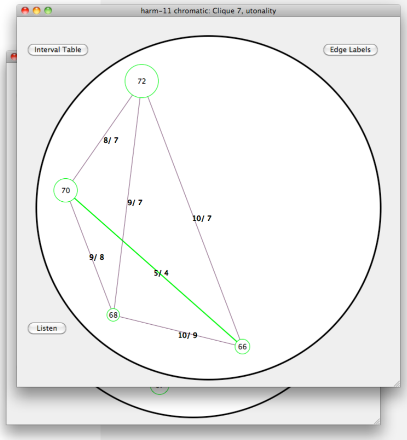


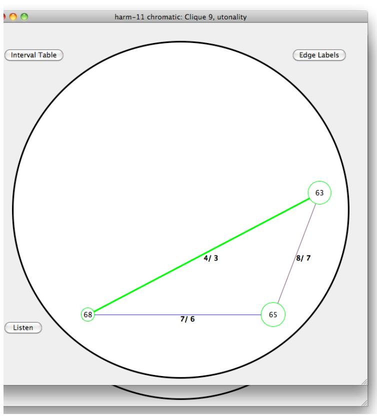
Using a hybrid Tonnetz/staff notation that mirrors that was used in Figure 2 to demonstrate the structure of the standard Just Diatonic, Figure 23 that the

algorithmically generated tuning uses the same intervals in a slightly different arrangement. The new tuning adds a justly tuned d minor triad at the expense of the G major triad of the Just Diatonic. To my ear, the triads represented by each of the five triangles in Figure 23, and seventh chords represented by adjacent triangles are compellingly clean and stirring.

Six more cliques in Figure 24 constitute another diatonic: D-flat major.







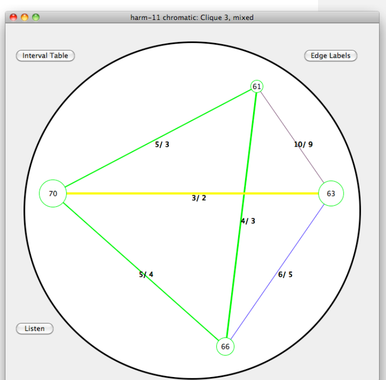


Figure 24

The cliques of Figure 24 can be synthesized into the diatonic shown in Tonnetz notation in Figure 25. As before, triangles and rhombi represent well tuned triads and seventh chords. While the wide 9/7 major third sounds odd on its own, in a triad it blends very well to my ear.

The four justly tuned seventh chords in the diagram below are interesting both for their sonic and conceptual variety: a septimal m7, a dominant with a septimal Major triad and quinatal m7, septimal half-diminished, and quintal m7.

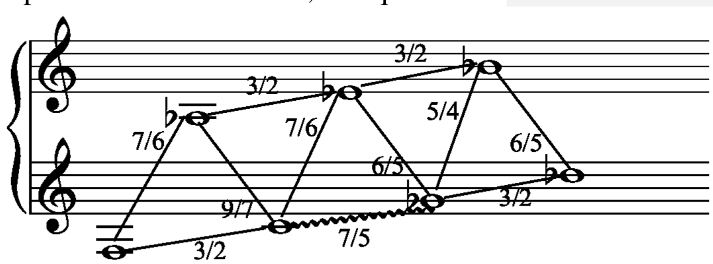


Figure 25

Further cliques point toward more diatonics that fill in the circle of

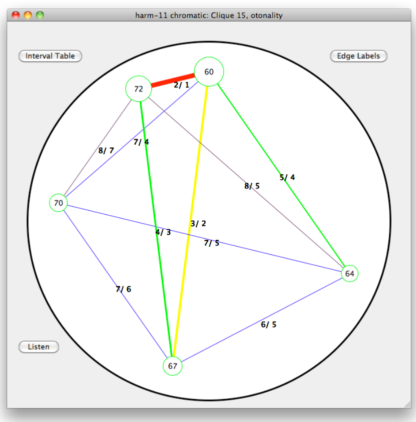


Figure 26

fifths between C and D-flat major. Figure 26 shows a clique that includes a C dominant seventh. This points to a diatonic scale in the key of F, as shown in the Tonnetz of Figure 27. The justly tuned triads sound very clean and distinctive, not least of which the exotic septimal diminished triad and dominant seventh.

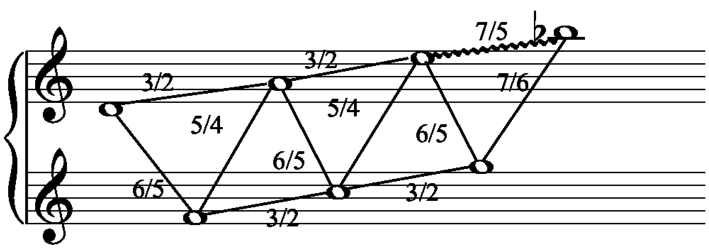
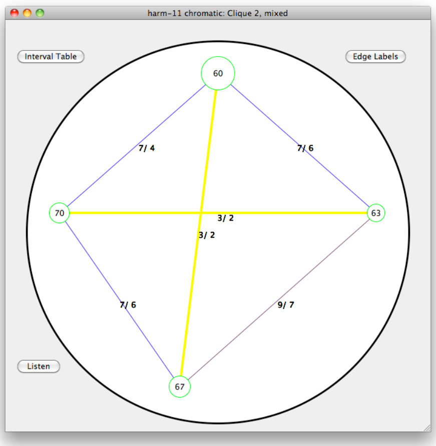


Figure 27

The clique frame in Figure 28 includes e-flat, and points toward a B-flat major diatonic



diagrammed in Figure 29. To the palette of triads and seventh chords is added the septimal-major E-flat triad, the septimal-minor c triad and seventh chord, and the septimal a diminished triad and half-diminished seventh chord.

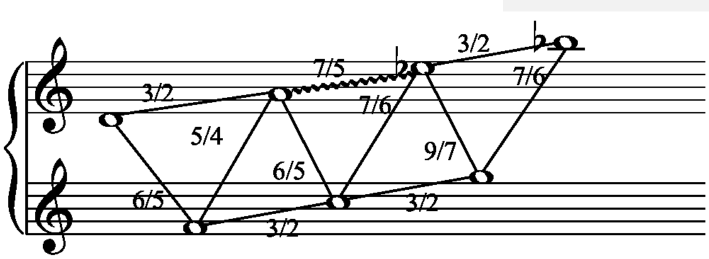
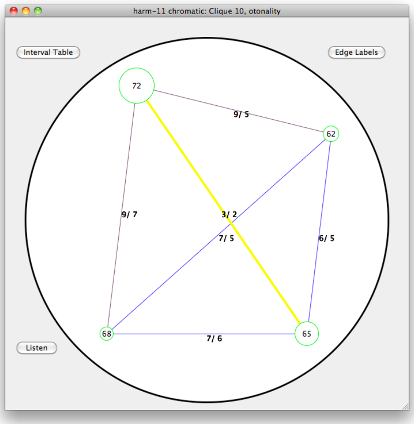


Figure 29

a

Figure 31. This diatonic adds to the diversity of seventh chords with its septimal A-flat major seventh.



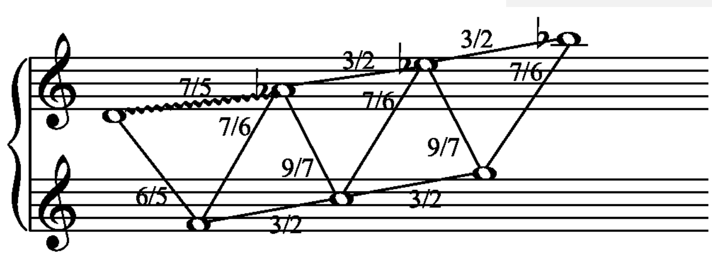
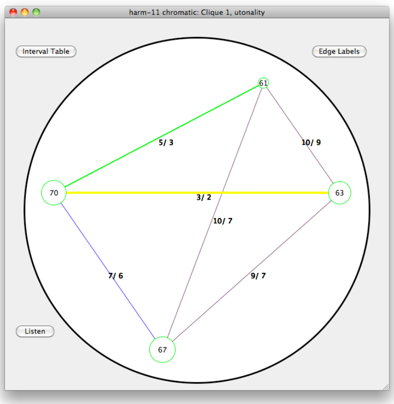


Figure 31

Figure 30

Finally, the D-flat major diatonic of Figure 32 has a variant suggested by the clique of Figure 33. Indeed, the diatonic of A-flat major is diagrammed in Figure X. A further exotic form is the E-flat dominant with a septimal major triad.



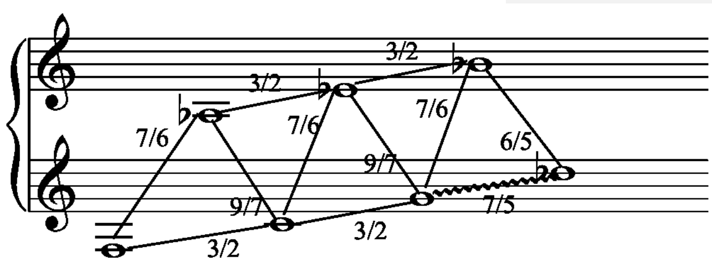


Figure 33

The diagrams for diatonic scales can be merged into a single graph, shown in Figure X. The 7/5 diminished fifths (d-a flat, a-e flat, e-b flat, c-g flat, g-d flat) are difficult to show on the two-dimensional images, but are part of justly tuned diminished triads. With these triads taken into account, there are 16 triads and 15 seventh chords and 5 diatonic scales.

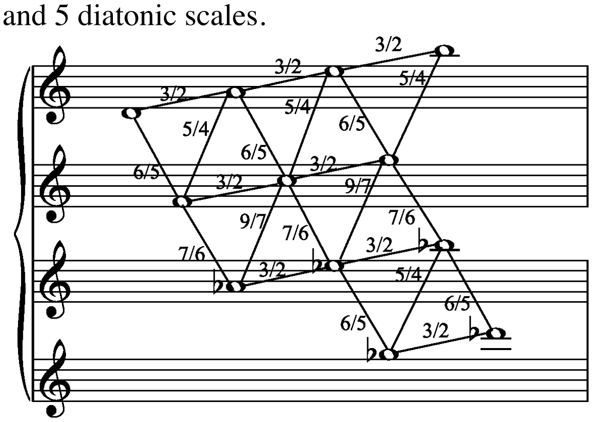
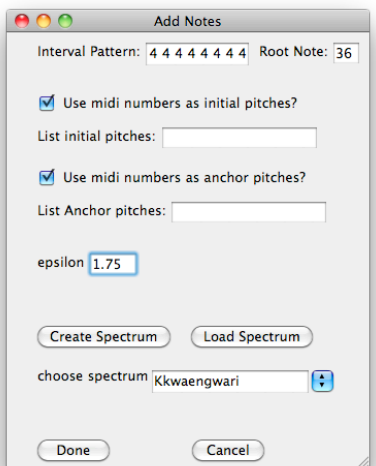


Figure 34

Tuning Example 2: Kkwaengwari augmented triads

The Kkwaengwari is a small Korean metallaphone, with a dense raucous sound, used in Korean "Nong-ak" (farmer's music) and its concert music descendant Samulnori. In this example, the Spectral Tuner will use its inharmonic spectrum tune three octaves of an augmented triad. By showing the process of the tuning's creation, I visit further features of the program. Also I analyze the result by looking for patterns in the cliques, and find that the tuning is a closely packed collection of notes in a Tonnetz.



In the Add Notes frame (Figure 35) I input an interval pattern of 4 4 4 4 4 4 4 4 4 – three octaves of stacked major thirds, and a root note of 36, to explore a deeper than normal kkwaengwari

'igure 35

sound[27]. So, this tuning concerns midi numbers 36,40,44,48,52,56,60, 64,68,72. Again, I will use midi numbers as initial and anchor pitches.

The default value for epsilon, 0.4 semitones, makes sense for a relatively dense tuning that includes half steps (like the default major scale.) In this sparser tuning, I will allow a wider range for the notes, 1.75 semitones.

The "Create Spectrum" button allows the user to specify a new spectrum for tuning. My process for measuring and inputting the kkwaengwari spectrum is detailed below.

I recorded a sample of a kkwaengwari from a Samulnori percussion ensemble class. Using the "Sonic Visualizer[28]" software tool (screenshot in Figure 36) I measured the partials at frequencies 294, 482.1, 588, 828.8, 960, and 1133.8 Hz. By hovering the mouse over one of the colored lines that indicates a strong partial, I find an estimate of its frequency. By taking several measurements along its temporal length, I can obtain a fairly reliable estimate of its pitch. Doing this for each of the partials gets a reasonably full representation of the sound. Making measurements this way takes

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[27] The original kkwaengwari's base frequency is close to midi note 62.
[28] Chris Cannam, Christian Landone, and Mark Sandler, Sonic Visualiser: An Open Source Application for Viewing, Analysing, and Annotating Music Audio Files, in Proceedings of the ACM Multimedia 2010 International Conference.

judgment; experience is required to determine which partials are important to the sound and which are too faint or too masked by other partials to be significant.

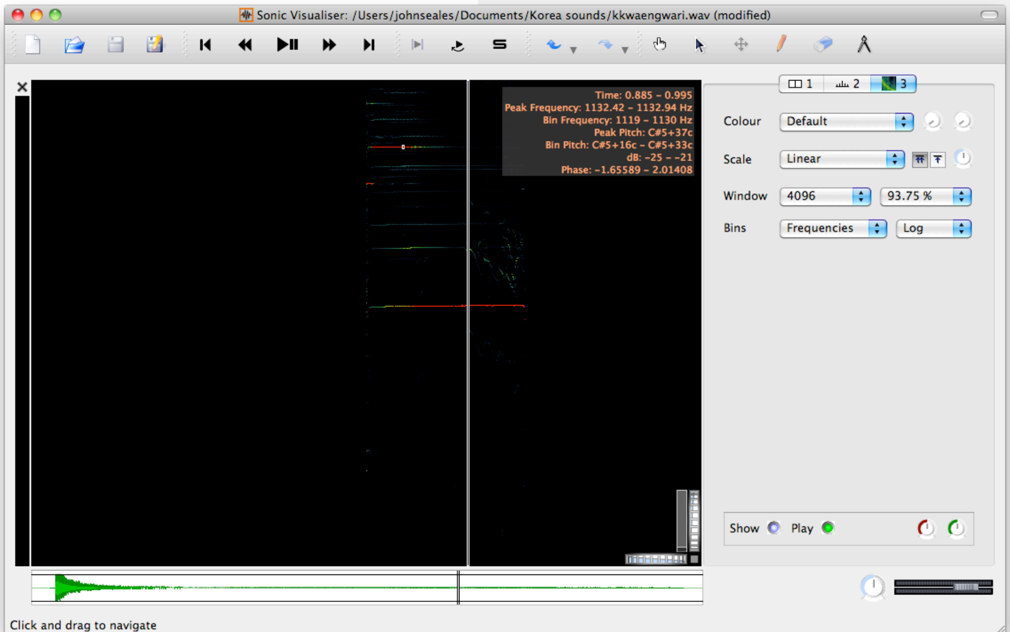
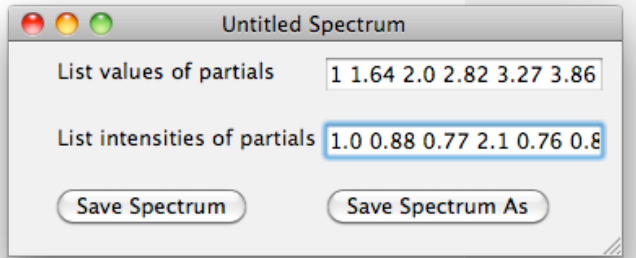


Figure 36

I then divide each frequency by that of the fundamental (the lowest frequency.) The resulting list of ratios between partials and the fundamental, along with their relative strengths, is termed its "spectrum" in the Spectral Tuner. For the kkwaengwari, the spectrum's partials are: 1,1.64,2.0,2.82,3.27,3.86. The amplitudes I approximated at 1.0,0.88,0.77,2.1,0.76,0.86.

When I click the "Create Spectrum" button, I see the frame in Figure 37, into which I type the



values for partials and amplitudes. I then click "Save Spectrum," so that I can reuse it for later tunings.

After saving the spectrum, Spectral Tuner returns focus to the Add Notes frame, (Figure 38) whose choices I finalize by clicking "Done." Focus returns to the Tuning Frame, with note gauges, spectrum, the "Aligned Partials" and "Consonance

'igure 37

Score." The numbers above the note gauges are the midi numbers to which the notes are assigned; below are their exact pitches. Even before tuning, there are 7 aligned partials, because one of the aligned partials is 2, so each of the octaves will initially be in tune. The 'consonance score' is the sum, for every coincidence of partials, of the minimum amplitude of the two partials.

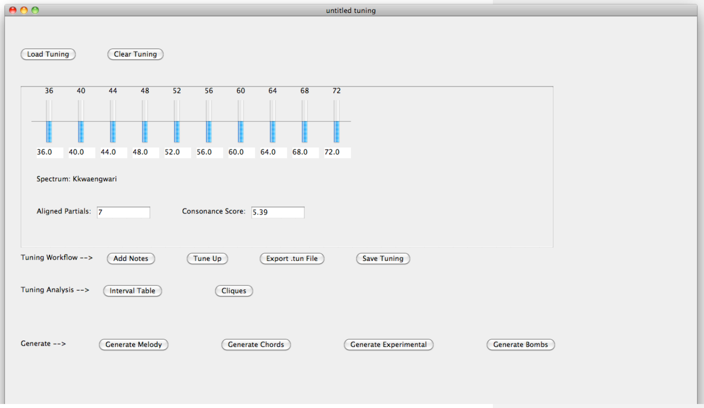


Figure 38

After pressing “Tune Up,” the program worked for about five minutes. Figure 39 shows the Tuning Frame when the tuning process is finished:

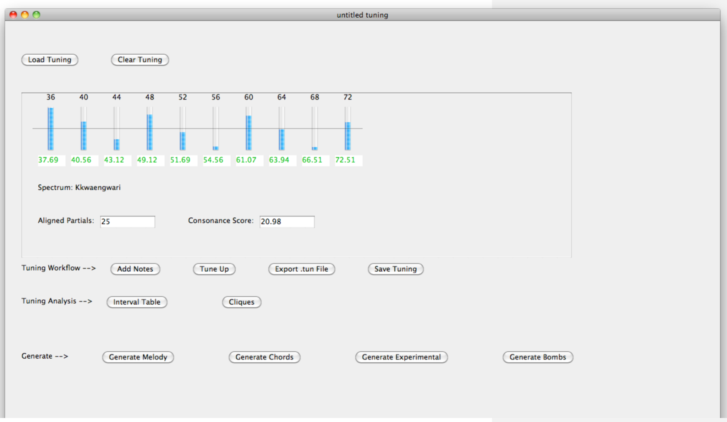


Figure 39

kkwaengwari M3: All Notes intervals

	36	40	44	48	52	56	60	64	68	72
36	-	3.86/ 3.27	3.86/ 2.82	-	-	-	3.86/ 1	-	-	-
40	-	-	3.27/ 2.82	1.64/ 1	-	-	3.27/ 1	3.86/ 1	-	-
44	-	-	-	-	1.64/ 1	-	2.82/ 1	-	3.86/ 1	-
48	3.86/ 1	-	-	-	3.27/ 2.82	3.86/ 2.82	3.27/ 1.64	3.86/ 1.64	-	3.86/ 1
52	-	-	-	-	-	3.86/ 3.27	2.82/ 1.64	-	3.86/ 1.64	-
56	2.82/ 1	-	3.86/ 1	-	-	-	-	2.82/ 1.64	3.27/ 1.64	2.82/ 1
60	-	-	-	-	-	-	-	3.86/ 3.27	3.86/ 2.82	-
64	1.64/ 1	-	-	-	-	-	-	-	3.27/ 2.82	1.64/ 1
68	-	-	-	-	-	-	-	-	-	-
72	-	3.86/ 3.27	3.86/ 2.82	-	-	-	3.86/ 1	-	-	-

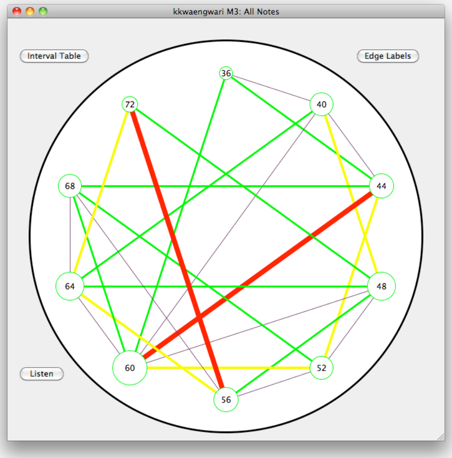


Figure 5

Figure 41

The interval table in figure 40 and the graph for all notes in figure 41 give the first opportunities for analysis of the tuning, after which we look for patterns in the cliques. The two largest cliques have four notes, and their graphs are shown in Figure 42.

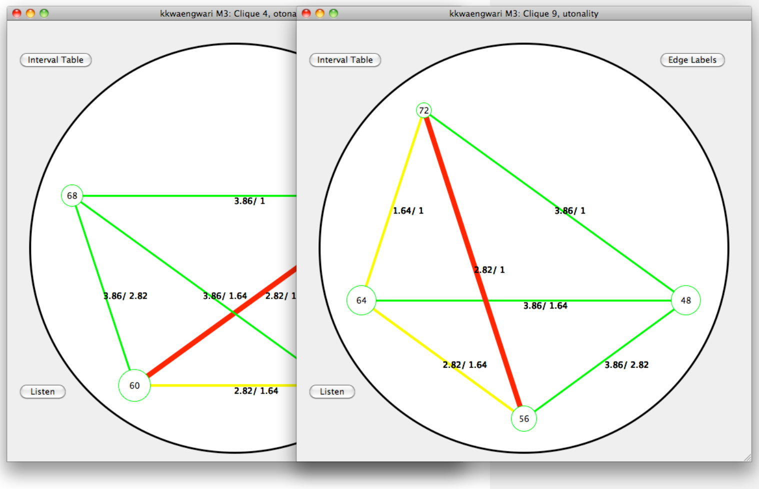


Figure 42

A look at the ratios labeling the edges of the two graphs shows that they are mirror images of each other. In fact they are the inverse of each other. Clique 4 is labeled an otonality, which can be seen by the ratios incident to midi number 44. Those same intervals are incident upon midi number 72 in Clique 9, but they are descending intervals so they are in fact the inverse. Cliques 4 and 9 are an otonal-utonal pair.

Cliques 3 and 6 (Figure 43) form another otonal-utonal pair, as do Cliques 7 and 2. (Figure 44) Otonal/utonal pairs are typical features of the cliques within tunings created by the Spectral Tuner, and can be an important compositional impetus. The three remaining cliques, 1, 3, and 8, are singletons.

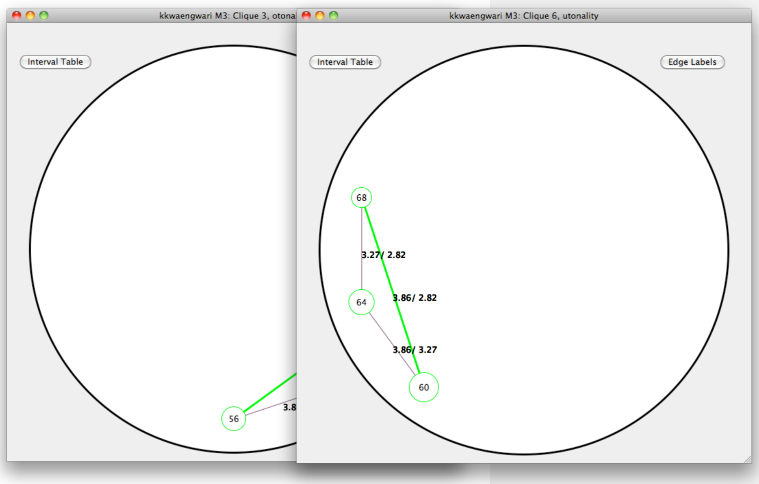


Figure 43

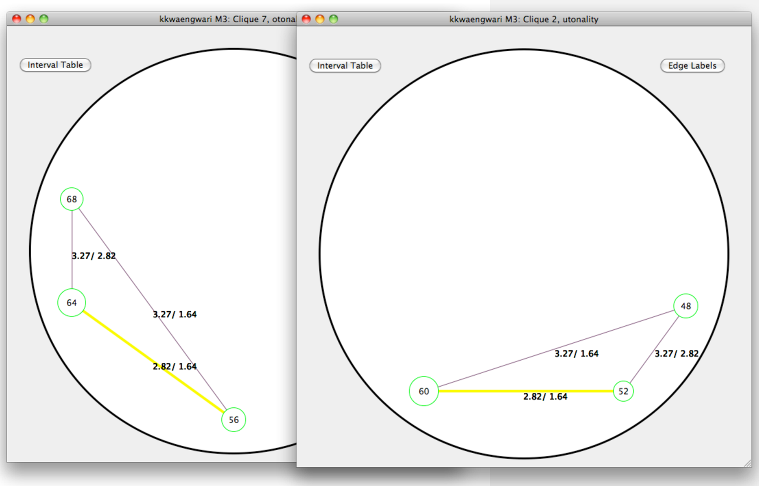


Figure 44

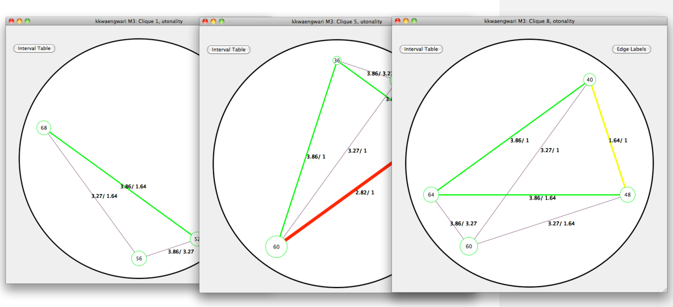


Figure 45

A synthesis of the tunings relationships in a single Tonnetz is shown in Figure 46. The edges shown are the minimum to show the tuning’s structure; the rest of the intervals in the tuning could consistently be included in the Tonnetz, but Figure 46 has been simplified for readability.

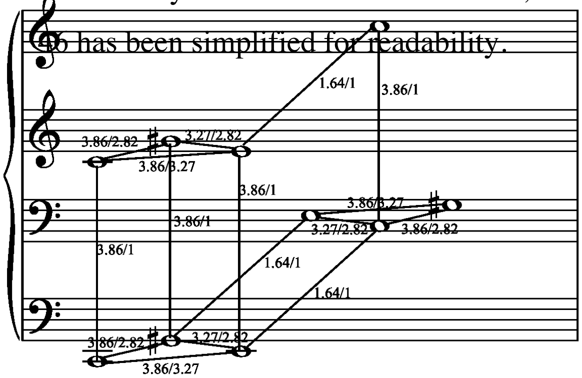


Figure 46

Tuning Example 3: Pyeon-Gyeong and Harmonic pentatonic.

The pyeon-gyeong is a court music instrument used in the Royal Ancestor Shrine Ceremony, its sixteen stones tuned approximately to 12ET midi numbers 72 through 87. I combined its spectrum with a harmonic spectrum with 7 partials, and used the hybrid spectrum to tune a two octave pentatonic scale. The interval table and cliques, as before, are key to an analysis of the tuning.

Because 1.52 is very close to 3/2 and 2.33 is very close to 7/3, the pyeon-gyeong and harmonic timbers mesh together very well to create strong consonances. Figure 47 lists approximate congruencies between the intervals in the tuning and whole number ratios. Figures 48 and 49 show the interval table and graph for the finished tuning.

1.52/1.00 ~ 3/2

2/1.52 ~ 4/3

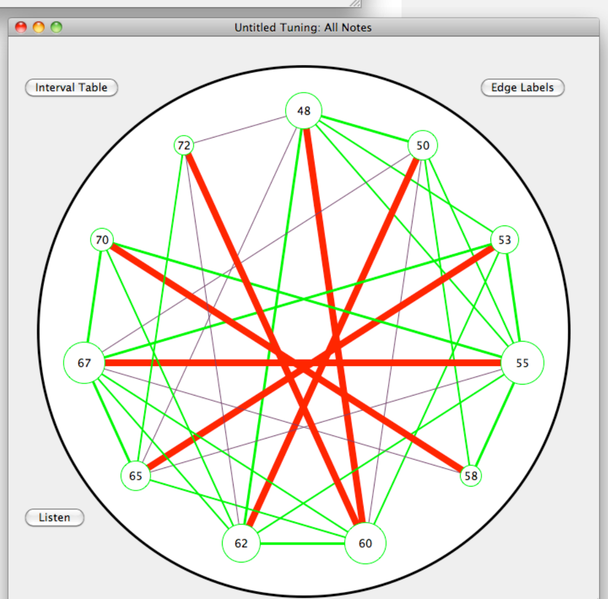
7/2.33 ~ 3/1

4/1.52 ~ 8/3

Figure 47

pyeon-harm intervals

	48	50	53	55	58	60	62	65	67	70	72
48	-	2.33/ 2	2/ 1.52	2.33/ 1.52	-	2/ 1.00	2.33/ 1.00	4/ 1.52	-	-	4/ 1.00
50	-	-	-	2/ 1.52	2.33/ 1.52	4/ 2.33	2/ 1.00	-	4/ 1.52	-	-
53	-	-	-	2.33/ 2	-	1.52/ 1.00	-	2/ 1.00	2.33/ 1.00	-	-
55	-	-	-	-	2.33/ 2	-	1.52/ 1.00	4/ 2.33	2/ 1.00	2.33/ 1.00	-
58	-	-	-	-	-	-	-	-	4/ 2.33	2/ 1.00	-
60	2/ 1.00	2.33/ 1.00	4/ 1.52	-	-	-	2.33/ 2	2/ 1.52	2.33/ 1.52	-	2/ 1.00
62	4/ 2.33	2/ 1.00	-	4/ 1.52	-	-	-	-	2/ 1.52	2.33/ 1.52	4/ 2.33
65	1.52/ 1.00	-	2/ 1.00	2.33/ 1.00	-	-	-	-	2.33/ 2	-	1.52/ 1.00
67	-	1.52/ 1.00	4/ 2.33	2/ 1.00	2.33/ 1.00	-	-	-	-	2.33/ 2	-
70	-	-	-	4/ 2.33	2/ 1.00	-	-	-	-	-	-
72	-	2.33/ 2	2/ 1.52	2.33/ 1.52	-	2/ 1.00	2.33/ 1.00	4/ 1.52	-	-	-



'igure 48 Figure 49

As in the previous examples, looking for patterns in the cliques helps to conceptualize the structure of the tuning. In Figure 50, six cliques with similar structures together compose 10 of the 11 tones in the tuning.

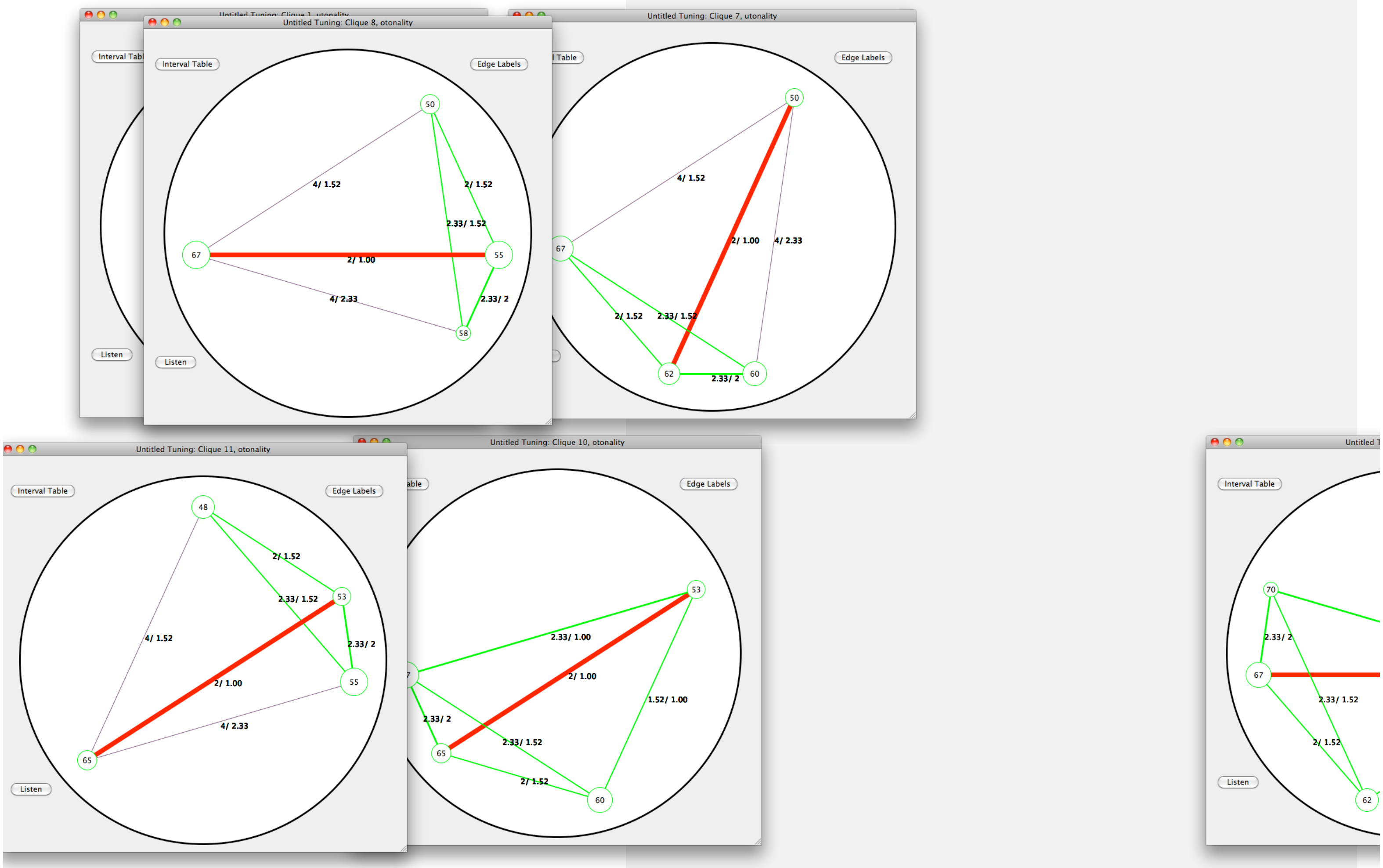
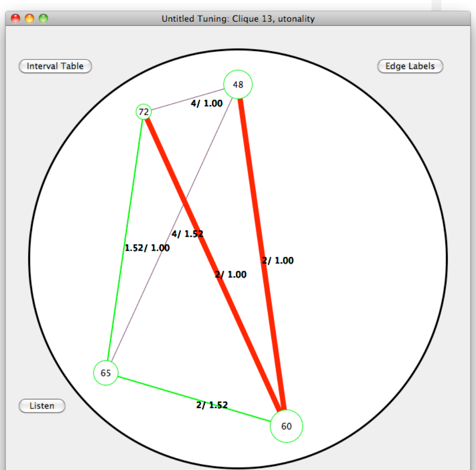
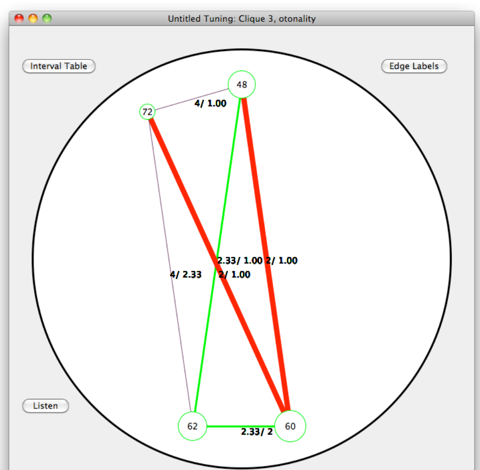


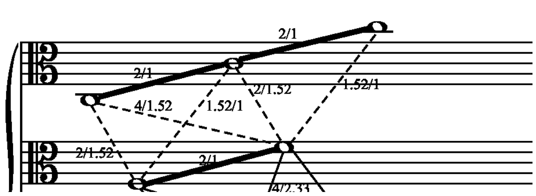
Figure 50

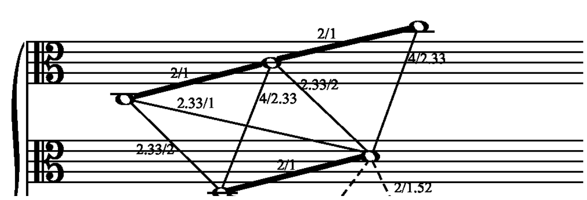
Synthesizing the relationships between the ten notes represented in the previous cliques, the beginnings of a Tonnetz appears. Two cliques





containing the remaning midi note 72 (Figure 51) allow us to assemble the Tonnetz completely.





Because of the density of relationships, I use three figures. Figure 52 contains two views that together show most of the relationships, and Figure 53 is a skeleton containing all notes of the tuning in a sparser, more readable form.

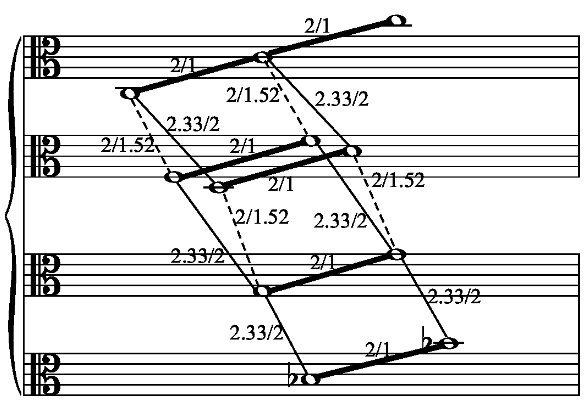


Figure 53

In this article I've presented a method of designing tuning systems in which the arrangement of pitches is a result of saturating the tuning with a set of target intervals. I've also developed a method of analyzing the structure of ratio-based tunings that is effective both for historical tunings and those that have been algorithmically generated. Finally, I've touche

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Paul Nauert 4/30/09 5:19 PM
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ways of using the structure of tunings to generate music.

One area that merits further exploration is the phenomenon of beats. Some tunings, such as Michael Harrison's "revelation" tuning, deliberately employ beats as part of the structure of the tuning, and they are a potential parameter for creating new tunings algorithmically. Also, the dissonance curve model can be developed further. The model I've used for this article can be augmented to take into account psychoacoustic effects of the critical band and of masking. The use of tunings to generate melodies and other parameters of music is a field with much room for exploration.

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Beyond that, there is a universe to explore in the many types of ratio-based tunings: their structures, their affect, their suitability to various kinds of music. It promises to be a delightful and bountiful process of discovery.

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