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A causality-in-variance test and its application to financial market prices

Yin-Wong Cheung*[,a], Lilian K. Ng[b]

[a] *Department of Economics, University of California, Santa Cruz, CA 95064, USA*
[b] *Department of Economics, City Polytechnic of Hong Kong, Kowloon, Hong Kong*

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Abstract

This paper develops a test for causality in variance. The test is based on the residual cross-correlation function (CCF) and is robust to distributional assumptions. Asymptotic normal and asymptotic χ^2 statistics are derived under the null hypothesis of no causality in variance. Monte Carlo results indicate that the proposed CCF test has good empirical size and power properties. Two empirical examples illustrate that the causality test yields useful information on the temporal dynamics and the interaction between two time series.

Key words: Causality; Cross-correlation function; GARCH; Stock price; Volatility spillover
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1. Introduction

Recently, there has been increasing interest in the causation in conditional variance across various financial asset price movements.[1] The study of causality in variance is of interest to both academics and practitioners because of its

* Corresponding author.

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[1] Some recent studies are Baillie and Bollerslev (1991), Barclay, Litzenberger, and Warner (1990), Cheung and Ng (1990), Engle, Ito, and Lin (1990), Hamao, Masulis, and Ng (1990), and King and Wadhwani (1990).

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economic and statistical significance. First, changes in variance are said to reflect the arrival of information and the extent to which the market evaluates and assimilates new information. For example, Ross (1989) shows that in a no-arbitrage economy the variance of price changes is directly related to the rate of information flow to the market. Engle, Ito, and Lin (1990), however, attribute movements in variance to the time required by market participants in processing new information or in policy coordination. Thus, the relation between information flow and volatility gives an interesting perspective to interpret the causation in variance between a pair of economic time series. Second, the causation pattern in variance provides an insight concerning the characteristics and dynamics of economic and financial prices, and such information can be used to construct better econometric models describing the temporal dynamics of the time series.

This paper develops a two-stage procedure to test for causality in variance, which is asymptotically robust to distributional assumptions. The first stage involves the estimation of univariate time-series models that allows for time variation in both conditional means and conditional variances. In the second stage the resulting series of squared residuals standardized by conditional variances are constructed. The cross-correlation function (CCF) of these squared-standardized residuals is then used to test the null hypothesis of no causality in variance. This two-stage method extends the procedures developed in Haugh (1976) and McLeod and Li (1983).

This study also discusses the effect of causality in mean, if any, on tests for causality in variance and the interaction between the tests for causality in mean and variance. Depending on model specifications, causation in mean can exist with or without the presence of causality in variance and vice versa. This observation motivates us to use empirical examples to investigate the test performance when there exists causation in both the mean and variance.

The next section introduces the two-stage CCF test for causality in variance. Some remarks on the test and its finite-sample performance are offered in Section 3. Section 4 presents two examples drawn from financial data. These examples also illustrate how to handle the existence of causation in both the mean and variance. Section 5 summarizes the paper.

2. A two-stage procedure

The concept of causation in the second moment can be viewed as a natural extension of the well-known Wiener–Granger causality in mean (Granger, Robins, and Engle, 1986).[2] Consider two stationary and egordic time series,

[2] The notion of Wiener–Granger causality in mean is introduced by Granger (1969) and is extensively discussed in, for example, Granger (1980), Geweke (1984), and Pierce and Haugh (1977).

X_t and Y_t. Let I_t and J_t be two information sets defined by $I_t = \{X_{t-j}; j \geqslant 0\}$ and $J_t = \{X_{t-j}, Y_{t-j}; j \geqslant 0\}$. Y_t is said to cause X_{t+1} in variance if

$$\mathrm{E}\{(X_{t+1} - \mu_{x,t+1})^2 \,|\, I_t\} \neq \mathrm{E}\{(X_{t+1} - \mu_{x,t+1})^2 \,|\, J_t\}, \tag{1}$$

where $\mu_{x,t+1}$ is the mean of X_{t+1} conditioned on I_t. Feedback in variance occurs if X causes Y and Y causes X. There is instantaneous causality in variance if

$$\mathrm{E}\{(X_{t-1} - \mu_{x,t+1})^2 \,|\, J_t\} \neq \mathrm{E}\{(X_{t+1} - \mu_{x,t+1})^2 \,|\, J_t + Y_{t-1}\}. \tag{2}$$

As in the case of causality in mean, the concepts defined in relations (1) and (2) are too general to be empirically testable. Thus, additional structure is required in order to make the general causality concept applicable in practice. Suppose X_t and Y_t can be written as

$$X_t = \mu_{x,t} + h_{x,t}^{0.5}\,\varepsilon_t, \tag{3}$$

$$Y_t = \mu_{y,t} + h_{x,t}^{0.5}\,\zeta_t, \tag{4}$$

where $\{\varepsilon_t\}$ and $\{\zeta_t\}$ are two independent white noise processes with zero mean and unit variance. Their conditional means and variances are given by

$$\mu_{z,t} = \sum_{i=1}^{\infty} \phi_{z,i}(\theta_{z,\mu}) Z_{t-i}, \tag{5}$$

$$h_{z,t} = \varphi_{z,0} + \sum_{i=1}^{\infty} \varphi_{z,i}(\theta_{z,h}) \{(Z_{t-i} - \mu_{z,t-i})^2 - \varphi_{z,0}\}, \tag{6}$$

where $\theta_{z,w}$ is a $p_{z,w} \times 1$ parameter vector; $W = \mu, h$; $\phi_{z,i}(\theta_{z,\mu})$ and $\varphi_{z,i}(\theta_{z,h})$ are uniquely defined functions of $\theta_{z,\mu}$ and $\theta_{z,h}$; and $Z = X, Y$. Specifications (5) and (6) include the time-series models such as the commonly-used ARMA models and (generalized) autoregressive conditional heteroskedastic ((G)ARCH) processes (cf. Robinson, 1991). The feasible set of parameter values is implicitly defined by the stationarity assumption.[3]

Let U_t and V_t be the squares of standardized innovations,

$$U_t = ((X_t - \mu_{x,t})^2 / h_{x,t}) = \varepsilon_t^2, \tag{7}$$

$$V_t = ((Y_t - \mu_{y,t})^2 / h_{y,t}) = \zeta_t^2; \tag{8}$$

$r_{uv}(k)$ be the sample cross-correlation at lag k,

$$r_{uv}(k) = c_{uv}(k)(c_{uu}(0)c_{vv}(0))^{-1/2}, \tag{9}$$

[3] For example, the roots of an AR polynomial are outside the unit circle and the series $\{|\phi_{z,i}(\theta_{z,\mu})|\}$ is summable. For a fractionally integrated process, the series $\{\phi_{z,i}(\theta_{z,\mu})\}$ is square-summable. For a GARCH(p, q) process, $\varphi_{z,i}(\theta_{z,h})$ declines exponentially. See Box and Jenkins (1976), Engle (1982), Bollerslev (1986), Granger and Joyeux (1980), Hosking (1981), and Cheung (1993) for a more detailed discussion of these processes.

where $c_{uv}(k)$ is the kth lag sample cross covariance given by

$$c_{uv}(k) = T^{-1} \sum (U_t - \bar{U})(V_{t-k} - \bar{V}), \qquad k = 0, \pm 1, \pm 2, \dots, \tag{10}$$

and $c_{uu}(0)$ and $c_{vv}(0)$ are the sample variances of U and V, respectively. Since $\{U_t\}$ and $\{V_t\}$ are independent, the existence of their second moments implies (e.g., Hannan, 1970):

$$\begin{pmatrix} \sqrt{T} r_{uv}(k) \\ \sqrt{T} r_{uv}(k') \end{pmatrix} \to \mathrm{AN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), \quad k \neq k'. \tag{11}$$

As in the test for causality in mean (Haugh, 1976; Pierce and Haugh, 1977), expression (11) suggests that the CCF of squared standardized residuals can be used to detect causal relations and identify patterns of causation in the second moment.[4] The utility of the CCF has certain advantages over some possible alternative tests for causality in variance. For instance, compared with a multivariate method, the CCF approach does not involve simultaneous modeling of both intra- and inter-series dynamics, and hence it is relatively easy to implement. The uncertainty in both the first- and second-moment dynamics and the potential interaction between the series would further complicate the formulation of a multivariate GARCH model. This makes the task of correctly specifying an adequate multivariate model very challenging.[5] Thus, the CCF test is especially useful when the number of series under investigation is large and long lags in the causation pattern are expected. Further, the proposed test has a well-defined asymptotic distribution and is asymptotically robust to distributional assumptions.

The CCF approach, which is similar to the test of causality in mean, also has certain limitations. For instance, the CCF is not designed to detect causation patterns that yield zero cross-correlations. An example is the nonlinear causation

[4]The use of the CCF to detect causality in variance relates closely to the approach adopted by Granger, Robins, and Engle (1986). In their illustrative example, these authors focus on the relationship between $h_{x,t}$ and lagged Y^2's. Under the no-causality-in-variance assumption, the squared innovation of X, $h_{x,t}\varepsilon_t^2$, and $h_{y,s}\zeta_s^2$ are not correlated. Similarly, the hypothesis that X does not cause Y in variance implies zero correlation between $h_{x,s}\varepsilon_s^2$ and $h_{y,t}\zeta_t^2$, $t > s$. Thus, the null hypothesis that X and Y are independent implies zero cross-correlation between $h_{x,s}\varepsilon_s^2$ and $h_{y,t}\zeta_t^2$ for all s and t.

[5]The feasibility of using a multivariate approach to test causality in variance may be further affected by the uncertainty surrounding the (asymptotic) distribution of the maximum likelihood estimator for a multivariate GARCH process. In their recent study on multivariate GARCH models Engle and Kroner (1993) argue that 'the properties of MLEs in GARCH models are still open to debate' (p. 17) and conclude that '(V) ery little is currently known about the properties of maximum likelihood estimators in univariate GARCH model, let alone in multivariate GARCH-in-mean models, despite the fact that this estimator permeates the multivariate GARCH-in-mean literature' (p. 19).

that is admissible in Eq. (1). The appealing features of the approach, however, make it very useful in practice. The sample residual cross-correlation further provides information on the interaction between time-series data and also help construct a more complex multivariate model (Parzen, 1969; Pierce, 1977).

Since both U_t and V_t are unobservable, their estimators have to be used to test the hypothesis of no causality in variance. We use the sample cross-correlation coefficient $\hat{r}_{uv}(k)$ computed from the consistent estimates of the conditional means and variances of X_t and Y_t in place of $r_{uv}(k)$. Let $\hat{\theta}_z \equiv \{\hat{\theta}_{z,\mu}, \hat{\theta}_{z,h}, \hat{\varphi}_{z,0}\}$ be a consistent estimator of the true parameter vector $\theta_z^{\circ} \equiv \{\theta_{z,\mu}^{\circ}, \theta_{z,h}^{\circ}, \varphi_{z,0}^{\circ}\}$; $Z = X, Y$; $\theta^{\circ} = (\theta_x^{\circ}, \theta_y^{\circ})$; $\hat{\theta} = (\hat{\theta}_x^{\circ}, \hat{\theta}_y^{\circ})$; and $\theta = (\theta_x, \theta_y)$. Then $\hat{r}_{uv}(k)$ is defined as

$$\hat{r}_{uv}(k) = r_{uv}(k)|_{\theta = \hat{\theta}}. \tag{12}$$

The sample cross-covariance $\hat{c}_{uv}(k)$ and the sample variances $\hat{c}_{uu}(0)$ and $\hat{c}_{vv}(0)$ are similarly defined. The property of $\hat{r}_{uv}(k)$ is given by:

Theorem 1. Consider $\{X_t\}$ *and* $\{Y_t\}$ *defined by Eqs. (3), (4), (5), and (6).* $\sqrt{T}(\hat{r}_{uv}(k_1), \ldots, \hat{r}_{uv}(k_m))$ *converge to* $\mathrm{N}(0, I_m)$ *as* $T \to \infty$, *where* $k_1, \ldots, k_m$ *are m different integers, if* (i) *both* $\mathrm{E}(\varepsilon_t^8)$ *and* $\mathrm{E}(\zeta_t^8)$ *exist, and* (ii) *for all* θ *in an open convex neighborhood* $\mathcal{N}(\theta^{\circ})$ *of* θ^0 *and for all* T, $\sqrt{T}\,\partial^2 c_{AB}(k)/\partial\theta_i\partial\theta_j$ *exists and is bounded in probability for* $\theta_i, \theta_j \in \theta$ *and for* $A, B = U, V$.

Proof. See the Appendix.

Given the asymptotic behavior of $\hat{r}_{uv}(k)$, a normal test statistic or a chi-square test statistic can be constructed to test the null hypothesis of noncausality. To test for a causal relationship at a specified lag k, we can compare $\sqrt{T}\hat{r}_{uv}(k)$ with the standard normal distribution. Alternatively, a chi-square test statistic defined by

$$S = T \sum_{i=j}^{k} \hat{r}_{uv}(i)^2, \tag{13}$$

which has a chi-square distribution with $(k - j + 1)$ degrees of freedom, can be used to test the hypothesis of no causality from lag j to lag k. The choice of j and k depends on the specification of alternative hypotheses. When there is no *a priori* information on the direction of causality, we may set $-j = k = m$. The parameter m should be large enough to include the largest nonzero lag that may appear in the causation pattern. When a uni-directional causality pattern, say, Y_t does not cause X_t, is considered, we set $j = 1$ and $k = m$.

3. Discussion

Several remarks on the proposed CCF test are in order. First, causality in the mean of X_t and Y_t can be tested by examining $\hat{r}_{\varepsilon\zeta}(k)$, the univariate standardized-residual CCF. Under conditions similar to those of Theorem 1, it can be shown that $\sqrt{T}(\hat{r}_{\varepsilon\zeta}(k_1), \ldots, \hat{r}_{\varepsilon\zeta}(k_m))$ converge to $N(0, I_m)$ as $T \to \infty$, where $k_1, \ldots, k_m$ are m different integers. Information on causation patterns in both the mean and variance can be used to construct better models that describe the temporal dynamics of the time-series data.

Second, the existence of any serial autocorrelation in ε_t and ζ_t or in U_t and V_t can affect the size of the proposed tests for causality in mean and variance.[6] The time-series model specified in the first stage should 'accurately' account for serial autocorrelation in the data. The adequacy of the fitted model for explaining serial correlation in the first and second moments can be statistically determined by, for example, the Box–Pierce portmanteau statistics calculated from standardized residuals and their squares (Box and Jenkins, 1976; McLeod and Li, 1983).

Third, when the sample size T is small, the chi-square statistic S can be modified to

$$S_M = T \sum_{i=j}^{k} \omega_i \hat{r}_{uv}(i)^2, \tag{14}$$

to attain a more accurate small-sample approximation to the χ^2 distribution, where $\omega_i = T/(T - |i|)$ or $(T + 2)/(T - |i|)$ (Haugh, 1976; McLeod and Li, 1983). Note that S_M is always larger than S. Alternatively, the statistic

$$S^* = T \sum_{k=-m}^{m-j} \left[\sum_{i=0}^{j} \hat{r}_{uv}(k + i) \right]^2, \qquad i = 0, 1, \ldots, m - 1, \tag{15}$$

suggested by Koch and Yang (1986), can be used to detect certain cross-correlation patterns. See Koch and Yang for a more detailed discussion of the S^* statistic.

Fourth, the existence of causality in mean violates the independence assumption and hence may affect the CCF test. Whether the causality in mean (variance) has any potential effect on the test for causality in variance (mean) depends on the model specification. For example, in a GARCH model, the conditional variance is driven by the squared innovations. As the causality in mean is associated with causality in the innovation term, it is likely that the former can have an effect on the size of the causality-in-variance test. Its

[6] For instance, under the assumption that U_t and V_t are independent, the asymptotic variance of $r_{uv}(k)$, $k \geqslant 0$, is $T^{-1}(\sum_{i=-T}^{T} \rho_u(i)\, \rho_v(i))$, where $\rho_z(i)$ is the autocorrelation function of Z_t.

conditional mean, however, does not necessarily depend on the second moment of the process. Hence the causality in variance may have a possible, but smaller, effect on the causality-in-mean test. The conditional mean of a GARCH-in-mean model, on the other hand, is a function of the conditional variance. In this case the causality in variance is likely to have a potentially larger impact (cf. GARCH models) on the causality-in-mean test. In Section 4 below two empirical examples shall be used to illustrate how causality in both the mean and variance can be determined simultaneously.

The final remark pertains to the finite-sample properties of the proposed test. Simulated (G)ARCH processes are used to investigate the size, power, and sensitivity to deviations from the normality assumption. Sample sizes $T = 50$, 100, 300, and 500 are considered. The simulation results are based on 1,000 replications of each model and sample size combination. For brevity, we summarize the simulation results here.[7]

The empirical sizes, generally, are in accordance with the asymptotic result. When the sample size is small, say $T = 50$, the modified S_M statistic yields a more accurate empirical size.[8] The persistence in variance, other than a unit-root persistence, tends to have no effect on the size of the test. The simulation results show that the CCF has the ability to identify causality and reveal useful information on the causality pattern. It is further shown that the proposed test has considerable power against the appropriate causality-in-variance alternative and is robust to nonsymmetric and leptokurtic errors.

4. Applications of the CCF test

We apply the CCF test to investigate the causal relations between 1) the daily returns on the Japan Nikkei 225 stock index and the U.S. S&P 500 stock index, and 2) the 15-minute returns on the S&P 500 index futures and the corresponding returns on the underlying index. Daily data are obtained from Datastream Inc., while the 15-minute data are constructed from June 1986 S&P 500 futures contract provided by the Chicago Mercantile Exchange.

The widely-used MA(1) with GARCH(1, 1)-M process is employed to model the stock returns, R_t. It is given by[9]

$$R_t = \phi_0 + \phi_1 h_t + u_t + \phi_2 u_{t-1}, \qquad u_t \sim \mathcal{N}(0, h_t),$$

$$h_t = \varphi_0 + \varphi_1 h_{t-1} + \varphi_2 u_{t-1}^2, \tag{16}$$

[7] The complete simulation exercise is presented in Cheung and Ng (1994).

[8] However, for sample size 100 or larger, the S_M-statistic tends to reject the null hypothesis too often.

[9] See, for example, French, Schwert, and Stambaugh (1987) and Hamao, Masulis, and Ng (1990).

Table 1
Maximum-likelihood estimates of the GARCH(1, 1) and the MA(1)-GARCH(1, 1) models

Parameter	Daily S&P 500 index	Daily Nikkei 225 index	15-minute S&P 500 index	15-minute S&P 500 futures
ϕ_0	0.1043	0.1278	– 0.0002	0.0063
	[0.0173]	[0.0145]	[0.0040]	[0.0040]
	(0.0259)	(0.0197)	(0.0041)	(0.0038)
ϕ_2		0.1279	0.1169	
		[0.0228]	[0.0336]	
		(0.0359)	(0.0361)	
φ_0	0.0796	0.0715	0.0048	0.0066
	[0.0142]	[0.0128]	[0.0005]	[0.0011]
	(0.0511)	(0.0265)	(0.0029)	(0.0017)
φ_1	0.7915	0.6237	0.6031	0.5920
	[0.0230]	[0.0350]	[0.0369]	[0.0485]
	(0.1261)	(0.1510)	(0.1468)	(0.0652)
φ_2	0.1483	0.3534	0.2789	0.2717
	[0.0151]	[0.0317]	[0.0480]	[0.0384]
	(0.1447)	(0.2389)	(0.0827)	(0.0566)
$Q(10)$	7.3883	9.9079	10.8054	8.8185
$Q^2(10)$	2.8688	5.0374	5.5098	5.9563
Log-likelihood	– 1794.14	– 1513.24	755.43	444.52

Asymptotic standard errors computed under the normality assumption are in square parentheses and the Bollerslev–Wooldridge (1990) asymptotic standard errors are in round parentheses. $Q(10)$ and $Q^2(10)$ are the Box–Pierce portmanteau statistics for the first ten autocorrelations of standardized residuals and their squares, respectively.

where u_t is the unexpected return and h_t is the conditional variance. Preliminary data analyses, however, show that not all parameters in model (16) are significant. To avoid any spurious cross-correlation, we compare model (16) with the following alternatives: (1) a GARCH(1, 1)-M model (i.e., model (16) with $\phi_2 = 0$); (2) an MA(1) with GARCH(1, 1) model [i.e., model (16) with $\phi_1 = 0$]; and (3) a GARCH(1, 1) model [i.e., model (16) with $\phi_1 = \phi_2 = 0$]. Based on the log-likelihood and the Akaike and Bayesian information criteria, we select 1) a GARCH(1, 1) model for the returns on the daily S&P 500 index and on the 15-minute index futures, and 2) a GARCH(1, 1) with an MA(1) error for the returns on the daily Nikkei 225 index and on the 15-minute spot index.

Table 1 presents maximum-likelihood estimates and diagnostic statistics of the selected models. Standard errors based on the normality assumption are given in square parentheses. Though the CCF test results presented below

Table 2
Maximum-likelihood estimates of the GARCH(1, 1) and the MA(1)-GARCH(1, 1) models with exogenous variables

Parameter	Daily S&P 500 index	Daily Nikkei 225 index	15-minute S&P 500 index	15-minute S&P 500 futures
ϕ_0	0.0943	0.1022	0.0002	0.0018
	[0.0179]	[0.0136]	[0.0009]	[0.0013]
	(0.0256)	(0.0179)	(0.0010)	(0.0013)
ϕ_2		0.0836	− 0.4960	− 0.5392
		[0.0213]	[0.0387]	[0.0292]
		(0.0328)	(0.0509)	(0.0302)
ϕ_3			− 0.1418	
			[0.0392]	
			(0.0416)	
ξ_1	0.0948	0.1993	0.5568	1.0680
	[0.0244]	[0.0159]	[0.0275]	[0.0317]
	(0.0360)	(0.0237)	(0.0341)	(0.0346)
ξ_2			0.1329	− 0.1622
			[0.0291]	[0.0270]
			(0.0289)	(0.0505)
ξ_3			0.0473	
			[0.0206]	
			(0.0198)	
ξ_4			0.0469	
			[0.0172]	
			(0.0168)	
φ_0	0.0809	0.0360	0.0025	0.0046
	[0.0142]	[0.0128]	[0.0003]	[0.0005]
	(0.0503)	(0.0178)	(0.0030)	(0.0019)
φ_1	0.7927	0.7528	0.4454	0.3463
	[0.0226]	[0.0295]	[0.0415]	[0.0449]
	(0.1259)	(0.0498)	(0.2277)	(0.0230)
φ_2	0.1442	0.1407	0.1756	0.3267
	[0.0147]	[0.0197]	[0.0418]	[0.0448]
	(0.1454)	(0.0327)	(0.0859)	(0.2214)
δ		0.0301	0.1626	0.1622
		[0.0060]	[0.0217]	[0.0296]
		(0.0171)	(0.0393)	(0.0571)
$Q(10)$	5.1301	11.7226	10.7426	6.8108
$Q^2(10)$	2.6066	8.1087	5.0261	7.0477
Log-likelihood	− 1790.30	− 1400.86	1141.01	1051.88

See notes to Table 1.

are robust to distributional assumptions, inferences about the GARCH parameter estimates may be sensitive to deviations from normality. Hence we also report the Bollerslev and Wooldridge (1990) standard errors, shown in the round parentheses, that are robust to nonnormality in dynamic models.[10] The Bollerslev–Wooldridge standard errors are generally larger than those based on the normality assumption, but the results are qualitatively the same. The Box–Pierce portmanteau statistics for the first ten autocorrelations of the resulting standardized residuals and squared-standardized residuals, $Q(10)$ and $Q^2(10)$, are not significant at conventional levels. Thus, the selected models adequately describe the first two moments.

Sample cross-correlations of the resulting standardized and squared standardized residuals are reported in panel A of Table 3.[11] The 'lag' refers to the number of periods the Nikkei 225 index data lag the S&P 500 index or the index futures data lag the underlying spot index. The lead is given by a negative lag. Given that the Japanese and U.S. stock markets operate in different time zones, the observed returns at the same time period would not be synchronized. Thus, any significant correlation between the two indexes on the same calender day, defined as a lag-zero cross-correlation, should be interpreted as evidence of the Nikkei 225 index causing the S&P 500 index. Similar interpretation applies to the 15-minute S&P 500 futures returns and its underlying spot index. Much existing evidence has shown that, as a result of nonsynchronous trading, the spot S&P 500 index tends to slightly lag the true value of the 500 composite stocks. Thus, a significant cross-correlation at lag 0 should be appropriately interpreted as evidence of the spot S&P 500 index causing the index futures.

As seen in Table 3, the cross-correlation of standardized residuals reveals evidence of feedback in the mean of these two pairs of financial price series. The causation pattern in daily stock returns is of lag 1 in both directions. For the 15-minute return data, the current spot index return is affected by past index futures returns up to four lags. Two different causality-in-variance patterns are found in these data. Although there exists no evidence of feedback, the daily U.S. stock index causes the daily Japanese stock index in variance. In contrast, there appears feedback in variances of the 15-minute stock index and futures returns.

Based on the sample cross-correlation causation patterns, we reconstruct the respective time-series models by adding the relevant and significant exogenous variables (i.e., the other market index's lagged return or lagged squared return) to the original GARCH-type models. The augmented models are then estimated,

[10] Note that the consistency of these parameter estimates does not depend on the normality assumption in Eq. (16); see Weiss (1986).

[11] $\hat{r}_{\varepsilon\zeta}(k)$ and $\hat{r}_{uv}(k)$ with larger (absolute) values of k are not significant and, hence, are not reported. For detailed results, see Cheung and Ng (1994).

Table 3
Cross-correlation in the levels and squares of standardized residuals resulting from the models reported in Tables 1 and 2

	Daily S&P 500 and Nikkei 225 indexes		15-minute S&P 500 index and futures data	
Lag k	$\hat{r}_{\varepsilon\xi}(k)$	$\hat{r}_{uv}(k)$	$\hat{r}_{\varepsilon\xi}(k)$	$\hat{r}_{uv}(k)$
Panel A				
−5	−0.0036	−0.0129	0.0710	0.0097
−4	−0.0264	0.0360	0.0116	−0.0107
−3	0.0170	0.0113	−0.0109	−0.0178
−2	0.0292	0.0555*	0.0074	−0.0264
−1	0.2981*	0.5143*	−0.0284	0.0109
0	0.0670*	0.0430	0.6069*	0.4187*
1	−0.0023	0.0067	0.3204*	0.1916*
2	0.0202	0.0087	0.0221	−0.0055
3	0.0057	−0.0078	0.0512*	0.0347
4	0.0259	−0.0062	0.0602*	0.0218
5	−0.0317	−0.0062	−0.0079	−0.0136
Panel B				
−5	0.0053	−0.0244	0.0415	−0.0143
−4	0.0327	0.0040	−0.0585*	−0.0146
−3	0.0102	0.0084	0.0220	0.0298
−2	0.0318	0.0150	0.0263	0.0029
−1	0.0145	0.0393	0.0306	0.0242
0	−0.0177	0.0088	0.0101	−0.0088
1	0.0027	0.0130	0.0035	0.0567*
2	0.0123	0.0257	−0.0336	−0.0209
3	0.0185	−0.0207	0.0149	0.0153
4	0.0151	−0.0187	−0.0264	−0.0348
5	−0.0329	−0.0015	−0.0153	−0.0056

Notes: $\hat{r}_{\varepsilon\xi}(k)$ and $\hat{r}_{uv}(k)$ are the cross correlations of standardized residuals and squared-standardized residuals computed from the models reported in Tables 1 and 2. k is the number of periods the Nikkei 225 index lags the S&P 500 index or the stock futures lags the underlying spot index quotation. '*' indicates significance at the 5% level.

compared, and evaluated. For the daily S&P 500 index, the resulting model is

$$R_t = \phi_0 + u_t + \xi_1 R_t^*,$$
$$h_t = \varphi_0 + \varphi_1 h_{t-1} + \varphi_2 u_{t-1}^2, \tag{17}$$

where R_t and R_t^* are the returns on the daily S&P 500 index and Nikkei 225 index. The model selected for the Nikkei index is

$$R_t^* = \phi_0 + u_t + \phi_2 u_{t-1} + \xi_1 R_{t-1},$$
$$h_t = \varphi_0 + \varphi_1 h_{t-1} + \varphi_2 u_{t-1}^2 + \delta R_{t-1}^2. \tag{18}$$

The specifications for two 15-minute stock index/futures series are

$$R_t = \phi_0 + u_t + \phi_2 u_{t-1} + \phi_3 u_{t-2} + \xi_1 R^*_{t-1} + \xi_2 R^*_{t-2} + \xi_3 R^*_{t-3} + \xi_4 R^*_{t-4},$$
$$h_t = \varphi_0 + \varphi_1 h_{t-1} + \varphi_2 u^2_{t-1} + \delta R^{*2}_{t-1}, \tag{19}$$

for the spot index, and

$$R^*_t = \phi_0 + u_t + \phi_2 u_{t-1} + \xi_1 R_t + \xi_2 R_{t-1},$$
$$h_t = \varphi_0 + \varphi_1 h_{t-1} + \varphi_2 u^2_{t-1} + \delta R^2_t, \tag{20}$$

for the futures index, where R_t and R^*_t are the 15-minute returns on the spot and futures index series.[12]

Table 2 reports maximum-likelihood estimates of these models. According to both the normality-based and the Bollerslev–Wooldridge standard errors, the added explanatory variables are significant. And none of the $Q(10)$ and $Q^2(10)$ statistics are significant at conventional levels. The maximum log-likelihood values of all four cases have improved, with the most pronounced increase in the 15-minute financial price series. The estimated regressions suggest that the causal relationship between the two 15-minute return series is stronger and more intricate. This result is consistent with the notion that information is better measured using higher-frequency data.

The sample CCFs of the resulting residuals from Eqs. (17) to (20) are reported in panel B of Table 3. Unlike those in panel A, the cross-correlation in the levels and squares of the standardized residuals from the daily stock return models is weak. However, there are two cross-correlation coefficients from the 15-minute data models that are significant at the 5% level. The significance of $\hat{r}_{\varepsilon\zeta}(-4)$ may be attributable to the type I error as the same estimate is not significant in panel A. Although $\hat{r}_{uv}(1)$ in panel B is still significant, the magnitude has dramatically reduced by 70% as compared to that reported in panel A. The remaining evidence of causality may be driven by factors not already captured by these two return series. Overall, Eqs. (17) to (20) explain a large proportion of the causal relationship in these series, suggesting that the CCF method is useful in determining causality in the variance.

5. Conclusions

In this paper we propose a statistical test for causality in the variance that is robust to the distributional assumption. The test entails fitting a univariate

[12] The inclusion of additional terms in these modified specifications is determined by the sample cross-correlation reported in panel A of Table 3, the parameter significance, and the Box–Pierce diagnostic statistic. For example, for model (18), we also included the R^2_{t-2} term in the h_t equation. However, this variable turned out to be insignificant. Also note that the inclusion of additional lagged terms in models (19) and (20) was dictated by the Box–Pierce diagnostic statistics.

model that incorporates both changing variances and means to each time series and computing cross-correlations of the squares of resulting standardized residuals. Under the null hypothesis of no causality in variance, the sample cross-correlation is shown to have an asymptotic normal distribution. Monte Carlo results suggest that the proposed test has good empirical size and power properties and is robust to nonnormal errors. The two empirical examples using financial prices illustrate that the proposed causality test provides constructive information on the temporal dynamics and the interaction of two time series.

Our analysis has some implications for future research. First, although this involves a complex experimental design, it is worthwhile to compare the CCF test with a likelihood-ratio test, a Wald test, and the other distribution-free tests (e.g., Bollerslev and Wooldridge, 1990; Wooldridge, 1990) using, say, Monte Carlo methods. Second, as a natural extension of the current univariate approach, it would be of interest to develop a causality-in-variance test in a multivariate framework. This can be a very challenging task in view of the uncertainty associated with the dynamics and the potential interaction in any pair of time-series observations. Thus, the information extracted using the CCF method may be exploited to build the appropriate multivariate specification.

Appendix: Proof of Theorem 1

The argument is similar to that used in Haugh (1976) and McLeod and Li (1983). First, we establish the result

$$\sqrt{T}\left.\frac{\partial c_{uv}(k)}{\partial \theta_i}\right|_{\theta=\theta^0} = O_p(1), \qquad \forall \theta_i \in \theta. \tag{A.1}$$

Consider $\theta_{x,h,j} \in \theta_{x,h}$,

$$\left.\frac{\partial c_{uv}(k)}{\partial \theta_{x,h,j}}\right|_{\theta=\theta^0} = T^{-1}\sum_t (p_t)(q_{t-k}), \tag{A.2}$$

where

$$p_t = -\frac{U_t}{h_{x,t}}\sum_{i=1}^{\infty} \varphi_{x,i,j}\{(X_{t-i}-\mu_{x,t-1})^2 - \alpha_x^2\}, \tag{A.3}$$

$$\varphi_{x,i,j} = \frac{\partial \varphi_{x,i}(\theta_{x,h})}{\partial \theta_{x,h,j}}, \tag{A.4}$$

$$q_{t-k} = V_{t-k} - 1. \tag{A.5}$$

Note that the highest order of ε_t in p_t is 4. Under the null hypothesis of independence, p_t and q_{t-k} are independent. Then the assumptions of stationarity and existence of the eighth moments imply that (e.g., Hannan, 1970)

$$\sqrt{T}\left[\frac{\sum_t (p_t)(q_{t-k})}{T}\right] \to \mathrm{AN}(0, \sigma^2), \tag{A.6}$$

where σ^2 is a product of autocorrelations of p_t and q_{t-k}. Hence,

$$\sqrt{T}\left.\frac{\partial c_{uv}(k)}{\partial \theta_{x,h,j}}\right|_{\theta=\theta^o} = \mathcal{O}_p(1). \tag{A.7}$$

Next, consider $\theta_{x,\mu,j} \in \theta_{x,\mu}$,

$$\left.\frac{\partial c_{uv}(k)}{\partial \theta_{x,\mu,j}}\right|_{\theta=\theta^o} = T^{-1}\sum_t (g_t)(q_{t-k}), \tag{A.8}$$

where

$$g_t = \frac{-2(X_t - \mu_{x,t})}{h_{x,t}} \sum_{i=1}^{\infty} \phi_{x,i,j} X_{t-1}, \tag{A.9}$$

$$\phi_{x,i,j} = \frac{\partial \phi_{x,i}(\theta_{x,\mu})}{\partial \theta_{x,\mu,j}}. \tag{A.10}$$

The highest order of ε_t in g_t is less than 4 and g_t is independent of q_{t-k} under the null. Thus,

$$\sqrt{T}\left.\frac{\partial c_{uv}(k)}{\partial \theta_{x,\mu,j}}\right|_{\theta=\theta^o} = \mathcal{O}_p(1). \tag{A.11}$$

With the same argument

$$\sqrt{T}\left.\frac{\partial c_{uv}(k)}{\partial \theta_{y,w,j}}\right|_{\theta=\theta^o} = \mathcal{O}_p(1) \tag{A.12}$$

for $\theta_{y,w,j} \in \theta_{y,w}$, $W = \mu, h$. Hence, we have the result (A.1).

With (A.1), Theorem 1 can be proved as follows. Expanding $\hat{c}_{uv}(k)$ about the true parameter vector θ^o,

$$\begin{aligned}\sqrt{T}\,\hat{c}_{uv}(k) &= \sqrt{T}\,c_{uv}(k) + \sqrt{T}(\hat{\theta} - \theta^o)' \left.\frac{\partial c_{uv}(k)}{\partial \theta}\right|_{\theta=\theta^o} \\ &\quad + \sqrt{T}(\hat{\theta} - \theta^o)' \left.\frac{\partial c_{uv}^2(k)}{\partial \theta \partial \theta'}\right|_{\theta=\theta^*} (\hat{\theta} - \theta^o),\end{aligned} \tag{A.13}$$

where $\|\theta^o - \theta^*\| \leqslant \|\theta^o - \hat{\theta}\|$. Since $\hat{\theta}$ is consistent by assumption,

$$(\hat{\theta} - \theta^o) = o_p(1). \tag{A.14}$$

(A.14) and the boundedness of $\sqrt{T}\partial c_{uv}^2(k)/\partial\theta_i\partial\theta_j$ imply

$$\sqrt{T}\hat{c}_{uv}(k) = \sqrt{T}c_{uv}(k) + \sqrt{T}(\hat{\theta} - \theta^{\circ})' \left.\frac{\partial c_{uv}(k)}{\partial\theta}\right|_{\theta = \theta^{\circ}} + o_p(1). \tag{A.15}$$

Further, the second term on the right-hand side of (A.15) is $o_p(1)$ by (A.1) and (A.14). Hence,

$$\sqrt{T}\hat{c}_{uv}(k) = \sqrt{T}c_{uv}(k) + o_p(1). \tag{A.16}$$

Now, for $k \neq k'$,

$$\begin{aligned} T\hat{c}_{uv}(k)\hat{c}_{uv}(k') &= (\sqrt{T}c_{uv}(k) + o_p(1))(\sqrt{T}c_{uv}(k') + o_p(1)) \\ &= Tc_{uv}(k)c_{uv}(k') + o_p(1). \end{aligned} \tag{A.17}$$

Using a similar technique, we can show that

$$\hat{c}_{uu}(0) = c_{uu}(0) + o_p(1), \tag{A.18}$$

$$\hat{c}_{vv}(0) = c_{vv}(0) + o_p(1). \tag{A.19}$$

Note that $c_{uu}(0)$ and $c_{vv}(0)$ converge to the variances of U_t and V_t, respectively. Applying the Slutsky theorem and Cramer–Rao device (e.g., Rao, 1973), $\hat{r}_{uv}(k)$ and $r_{uv}(k)$ have the same asymptotic distribution and

$$\sqrt{T}(\hat{r}_{uv}(k_1), \ldots, \hat{r}_{uv}(k_m)) \to \mathrm{N}(0, I_m). \quad \blacksquare$$

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