Stock Market Volatility and Fractional Integration

Yin-Wong Cheung
Board of Studies in Economics, University of California at Santa Cruz, Santa Cruz, CA 95064, U.S.A.

A fractional integration framework and a relationship between the variability of innovations in real stock prices and real dividends implied by the present value model are used to examine the issue of stock market volatility raised by Shiller (1981) and LeRoy and Porter (1981). It is found that both stock price and dividend data are neither trend stationary nor difference stationary; they are fractionally integrated. The data also show that low interest rates and investors' myopic behaviour only have a limited role in explaining excessive market volatility. On the other hand, the evidence for excess market volatility seems substantial even after controlling for sampling uncertainty.

Keywords: ARFIMA model; long memory; present value model; stock price dynamics; variance bounds

SUMMARY

In their seminal papers, Shiller (1981) and LeRoy and Porter (1981) report that variations in stock prices appear too large to be explained by changes in the fundamental value constructed from the dividend stream. The finding of excess market volatility contradicts the notion that the stock market is efficient and the price reflects the true value of the underlying stock. In fact, results reported by Shiller and LeRoy-Porter are usually interpreted as evidence against the rational or efficient market hypothesis and as indirect evidence that stock prices are also driven by 'fads' and 'fashions.'

Since these authors published their papers, the excess market volatility result has been re-evaluated using different methodologies. Marsh and Merton (1986) provide, perhaps, the most forceful argument against the excess market volatility result. Marsh and Merton show that the original Shiller's result is driven by the model Shiller used to describe the behaviour of dividend data. Specifically, there is no evidence of excess market volatility if a different model is used to describe dividend data. Therefore, whether the market is too volatile or not depends on the characterization of dividend dynamics. If Shiller's original characterization is correct, the stock market is too volatile and not efficient. If Marsh and Merton's claim is right, then there is no evidence against market efficiency.

In this study we use a stochastic process, called a fractionally integrated autoregressive and moving average (ARFIMA) process, to model dividend dynamics. One advantage of using the ARFIMA process is that it can describe a wide range of data dynamics. In particular, the processes adopted by both Shiller and Marsh–Merton to characterize dividend data are special cases of this general stochastic process. Therefore, the use of ARFIMA processes allows us to discriminate effectively between the two previous views on dividend dynamics and provides a more accurate description of dividend data.

In contrast to the existing results, we find both the real stock price and the real dividend data are fractionally integrated, a property that is different from the characterizations adopted by either Shiller or Marsh and Merton. We use a relation between

CCC 1076–9307/96/040263–11

Received 30 January 1995
Int. J. Finance Economics, Vol. 1, 263–273

the variability of innovations in real stock prices and real dividends, which is derived under the fractional integration framework, to evaluate stock price volatility. Overall, there is substantial evidence for excess market volatility when the real interest rate is higher than 3%. Our findings support the view that the market is excessively volatile unless the market uses a low real interest rate to discount future dividend payments. This implies that price movements may not correspond to changes in the fundamental value of the underlying stock. This study, however, does not address the question of whether this mispricing behaviour represents some exploitable profit opportunities.

INTRODUCTION

In their seminal papers, Shiller (1981) and LeRoy and Porter (1981) report that variations in stock prices appear too large to be explained by changes in the fundamental value constructed from the dividend stream. Their results are based on variance bounds tests derived from the present value model. The finding of excess market volatility is interpreted as evidence against the rational or efficient market hypothesis and as indirect evidence that stock prices are also driven by 'fads' and 'fashions' (Shiller, 1984).[1] Recent studies such as Mankiw *et al.* (1991) and LeRoy and Steigerwald (1992) also report evidence of excess market volatility. LeRoy (1989) and Shiller (1989) provide an excellent review on the variance bounds test literature.

Marsh and Merton (1986) provide, perhaps, the most forceful argument against Shiller's original variance bounds test result.[2] These authors show that Shiller's (1981) result is driven by the assumption of trend stationary dividends. They argue that the dividend process is better described as difference stationary (that is, the dividend data contain a unit root and are integrated of order one) when firm managers smooth dividend payments over time. When dividend data are difference stationary, the observed stock market volatility is no longer excessive. Kleidon (1986) also attributes the observed excess volatility to the presence of a unit root but not to market inefficiency. That is, the validity of the market inefficiency interpretation depends on the assumed temporal dynamics of dividends; the presence of long- term persistence in dividends, as implied by difference stationarity, does not favour the excess volatility interpretation.

However, the issue of whether dividends are trend stationary or integrated is still unsettled. DeJong and Whiteman (1991) find that the unit root hypothesis for dividend data is rejected in three studies and not rejected in eight. However, the failure to reject the unit root hypothesis is sometimes attributed to the low power of standard unit root tests. Using Bayesian techniques both DeJong and Whiteman (1991) and Koop (1991) find little evidence for unit roots in the dividend data.

In this study we examine the stochastic process generating dividends and its implications for the relationship between stock price and dividend variations from a different perspective. Temporal dynamics are modelled by long memory, fractionally integrated autoregressive and moving average (ARFIMA) processes. The ARFIMA process is a generalized ARMA process and can describe a wide range of data persistence. For instance, a fractional process includes trend stationary and difference stationary ARMA processes as special cases. That is, the use of fractional time series models can avoid the potential bias caused by the stringent classification of trend stationarity or difference stationarity adopted in the previous studies. This aspect of fractional models is important for studying market volatility because of the crucial role played by dividend dynamics.

For instance, in the short run, dividend smoothing can introduce persistence and give rise to temporal properties not associated with sustainable earnings. Also, smoothed dividend payments may add extra noise to stock prices as they distort the true underlying present value relationship. However, in a longer horizon, dividend smoothing mechanisms will be affected by the dynamics of sustainable earnings. Thus, although firm managers can smooth dividends and induce persistence over time, it would be interesting to know if such dividend-setting behaviour implies a unit root persistence, a weaker than unit root persistence, or a stronger than unit root persistence in the data. The ARFIMA model can provide a flexible way to model data persistence induced by the dividend setting behaviour without imposing a strong prior.

The remainder of the paper is organized as follows. The ARFIMA process is introduced in the following section. In the section after we derive a relation between the variability of innovations in real stock prices and real dividends implied by the present value model. The relation holds when the data follow a trend stationary, difference stationary, or a fractionally integrated process. Preliminary data analysis and estimates of fractional models are then presented. In contrast to results reported in previous studies, both stock price and dividend data are found to be neither trend stationary nor difference stationary; they are fractionally integrated. When we use the sample information to evaluate the variability relationship derived, we find substantial evidence of excess market volatility even after controlling for data persistence and sampling uncertainty.

ARFIMA MODEL

An ARFIMA(p, d, q) representation for a time series $\{X_t\}$ is

$$\Phi(B)(1-B)^d X_t = \Theta(B)\epsilon_t, \tag{1}$$

where B is the backward-shift operator, $\Phi(B) \equiv 1 - \phi_1 B - \cdots - \phi_p B^p$, $\Theta(B) \equiv 1 + \theta_1 B + \cdots + \theta_q B^q$, all roots of $\Phi(B)$ and $\Theta(B)$ are outside the unit circle, $\varepsilon_t \sim$ i.i.d. $(0, \sigma^2)$, and $(1-B)^d$ is the fractional differencing operator defined by $(1-B)^d = \sum_{k=0}^{\infty} \Gamma(k-d)B^k/[\Gamma(k+1)\Gamma(-d)]$ with $\Gamma(.)$ being the Gamma function. $\{X_t\}$ is stationary and has an MA representation if $d < 0.5$. This property of fractional processes will be used in the next section. When d is an integer or zero, an ARFIMA process becomes a conventional ARIMA process.[3]

The ability of fractional time series models to describe long-term persistence can be seen from the autocorrelation function, $\rho(.)$. Hosking (1981) shows that the $\rho(.)$ of an ARFIMA process declines hyperbolically; $\rho(k) \propto k^{2d-1}$, $k \to \infty$. This is in contrast with the $\rho(.)$ of a stationary ARMA process which decays exponentially; $\rho(k) \propto r^k$, $0 < r < 1$, $k \to \infty$. By allowing the degree of integration d to assume non-integer values, the ARFIMA process can be used to model data dependence that is stronger than allowed in stationary ARMA processes and weaker than implied by unit root processes. Further, we can transfer a non-stationary ARFIMA process (i.e. with $d > 0.5$) to a stationary process by appropriate differencing.

Examples of applying fractional models to economic issues include Cheung (1993), Cheung and Lai (1993), Cheung and Lai (1995a), Diebold and Rudebusch (1989), and Diebold *et al.* (1991). Sowell (1992b) also points out that, in addition to its flexibility in capturing low-frequency dynamics, the ARFIMA model can be used to nest the conventional trend stationary and difference stationary models and, hence, avoid the possible bias caused by the stringent classification of data as either trend stationary or unit root processes. Consider a trend stationary model; that is, d is zero and X_t has a time trend. Then $(1-B)X_t$ has $d = -1$. For an integrated process, d equals one. The first differenced series $(1-B)X_t$ has $d = 0$. This suggests one can discriminate between these two models by testing if the estimate of d from the first differenced series is around -1 or 0.

THE PRESENT VALUE MODEL

Following Shiller (1981), we work with the present value model

$$P_t = \sum_{k=0}^{\infty} E_t D_{t+k}/(1+r)^{k+1}, \tag{2}$$

where P_t is the real stock price at the beginning of time period t, D_t is the real dividend distributed at, say, the end of period t, E_t is the expectations operator conditional on information available at t, and r is the constant real discount rate.[4] We will consider different values of r in the next section. In the literature, the variance bounds tests are usually based on the sample variances of the (detrended) P_t and P_t^*, where

$$P_t^* = \sum_{k=0}^{\infty} D_{t+k}/(1+r)^{k+1} \tag{3}$$

is the perfect foresight or *ex post* rational price.

Here we examine the issue from a different perspective. The set-up of Diebold and Rudebusch (1991a) is adopted to account for the importance of persistence in dividend data. An advantage of this approach is that it explicitly incorporates the effect of the integration property of dividend data on market volatility. We can derive the relationship between the variability of the innovations in P_t and D_t that is valid under a wide range of dividend dynamics. In addition to trend stationary and difference stationary specifications, the derived relationship also holds when the data follow a fractionally integrated process.[5]

Let u_t be the innovation in the price in response to news arriving between $t-1$ and t. Then, we have

$$u_t = P_t - E_{t-1}P_t = \sum_{k=0}^{\infty}(E_tD_{t+k} - E_{t-1}D_{t+k})/(1+r)^{k+1}. \tag{4}$$

The unpredictable component of the price is related to the revision in the expected and discounted income stream made in response to the incoming news. Suppose the real dividend series follows an ARFIMA(p, d, q) process defined by Equation (1). From earlier we know that when d is assumed to be less than 1.5, the first differenced real dividend series is stationary and has the following infinite-order moving average representation:

$$(1-B)D_t = \Psi(B)\eta_t \equiv (1+\psi_1 B + \cdots + \psi_k B^k + \cdots)\eta_t, \tag{5}$$

where $\Psi(B) \equiv (1-B)^{1-d}\Phi^{-1}(B)\Theta(B)$ and $\eta_t \sim$ i.i.d. $(0, \omega^2)$ is the innovation in the dividend series. For notational simplicity, we have omitted the constant term in the changes in real dividends. Given Equation (5), we can show that

$$E_tD_{t+k} - E_{t-1}D_{t+k} = \sum_{i=0}^{k}\psi_i\eta_t, \quad \psi_o = 1, \quad k = 0, 1, \ldots \tag{6}$$

Combining Equations (4) and (6), we obtain

$$\begin{aligned} u_t &= \left[\sum_{i=0}^{\infty}\psi_i/(1+r)^i\right](\eta_t/r) \\ &= C(r, \infty)\eta_t, \end{aligned} \tag{7}$$

where $C(r, k) = [\sum_{i=0}^{k}\psi_i/(1+r)^i]/r$ is the cumulative impulse response function, which measures the persistence in the real dividend series, discounted by the interest rate factor $(1+r)^k r$. Under the present value model, innovations in real stock prices are proportional to current innovations in real dividends. The constant of proportionality depends on both the persistence in the real dividend series and the discount factor as represented by the discounted infinite cumulative impulse response.

Before we derive the relationship between the variability of u_t and η_t from Equation (7), we would like to establish the convergence property of the discounted infinite cumulative response $C(r, \infty)$. Since the sum of coefficients in $\Phi^{-1}(B)\Theta(B)$ is finite, the convergence of $C(r, \infty)$ depends on the convergence of

$$\sum_{k=0}^{\infty}\Gamma(k+(d-1))B^k/[\Gamma(k+1)\Gamma(d-1)(1+r)^k]$$

valuated evaluated at $B = 1$. It can be shown that

$$\begin{aligned} (1-B)^{1-d} &= \sum_{k=0}^{\infty}\Gamma(k+(d-1))B^k/[\Gamma(k+1)\Gamma(d-1)] \\ &= F(d-1, 1, 1; B), \end{aligned} \tag{8}$$

where $F(.)$ is the hypergeometric function defined by $F(\alpha, \beta, \gamma; z)$

$$\begin{aligned} &= \sum_{k=0}^{\infty}\Gamma(k+\alpha)\Gamma(k+\beta)\Gamma(g)Z^k/[\Gamma(\alpha)\Gamma(\beta) \\ &\quad \times \Gamma(k+\gamma)\Gamma(k+1)]. \end{aligned} \tag{9}$$

Thus,

$$\begin{aligned} &\sum_{k=0}^{\infty}\Gamma(k+(d-1))B^k/[\Gamma(k+1)\Gamma(d-1)(1+r)^k] \\ &\quad = F(d-1, 1, 1; B/(1+r)). \end{aligned} \tag{10}$$

From Gradshteyn and Ryzhik (1980, pp. 1039–1040), we know that $F(d-1, 1, 1; 1/(1+r))$ converges and, hence, $C(r, \infty)$ is finite if $r > 0$.[6]

Therefore, assuming the real discount rate r is positive, Equation (7) implies the relationship

$$v = C(r, \infty)\omega, \tag{11}$$

where v and ω are the standard deviations of u_t and η_t. That is, the variability of real stock price innovations is a function of three elements: the variability of real dividend innovations, the persistence in the real dividend series, and the discount factor. A similar relationship between income and consumption volatility has been examined by Diebold and Rudebusch (1991a) using the fractional integration technique. Note that Equation (11) holds when the real dividend series is either trend stationary or integrated of order one. In either case, Equation (11) can be used to evaluate whether the stock price is too volatile. In addition to its ability to accommodate fractionally integrated dividend data, Equation (11) also provides a convenient way to encompass the two opposing views on dividend dynamics discussed in the literature.

Relation (11) cannot be used directly to investigate if the stock market is too volatile as the variables on both sides of (11) are not known *a priori*. However, we can use the sample information on v, σ, and the discounted cumulative response $C(r, k)$ function to examine the relative variability of these two innovations. Further the confidence intervals of the estimates can be used to assess the uncertainty associated with the estimation procedure. This is done in the following section.

EMPIRICAL RESULTS

Preliminary data analysis

Annual stock price and dividend data from 1871–1987 were obtained from Shiller (1989, Chapter 26). The price series is the Standard & Poor's Monthly Composite Stock Index for January and the dividend series is the total annual dividend accruing to the portfolio represented by the Index. These two series are converted to real data series using the January Producer Price Index and the annual average Producer Price Index available from the same source. See Shiller (1989) for more details.

The real stock price and real dividend series and their differences are plotted in Figures 1 and 2. One striking, if not surprising, feature is that real dividends are not that smooth. The pattern of variation in real dividends is similar to that of real stock prices, although the magnitude of fluctuations in the latter series is larger. The changes in real stock prices have a higher variability in the 20th century while the changes in real dividends are more volatile around World Wars I and II. We should point out that these graphs have different scales and cannot be used directly to compare the sample variances of these data. In fact, the sample variance of changes in real stock prices is higher than that of changes in real dividends.

Table 1(a) reports the descriptive statistics. The sample average of real stock prices, in both levels and first differences, is larger than the corresponding sample average of real dividends. According to the sample coefficients of variation, which is a unit-free measure of variability relative to the mean, real stock prices and real dividends have a similar degree of volatility. The level and difference of real stock prices are, respectively, 1.50 and 1.22 times more volatile than those of real dividends. However, the standard errors (not reported in the table) of the level and difference of real stock prices are 33.94 and 41.64 times larger than those of real dividends. The sample correlation coefficient of 0.92 for the levels and 0.55 for the differences suggest movements in these two real series are closely related. Overall, these descriptive statistics are in accordance with the graphs presented in Figures 1 and 2.

The sample autocorrelation and partial autocorrelation coefficients given in Table 1(b) indicate a large autoregressive root in both the real stock price and real dividend series. The results of testing for a unit root in these data are then presented in Table

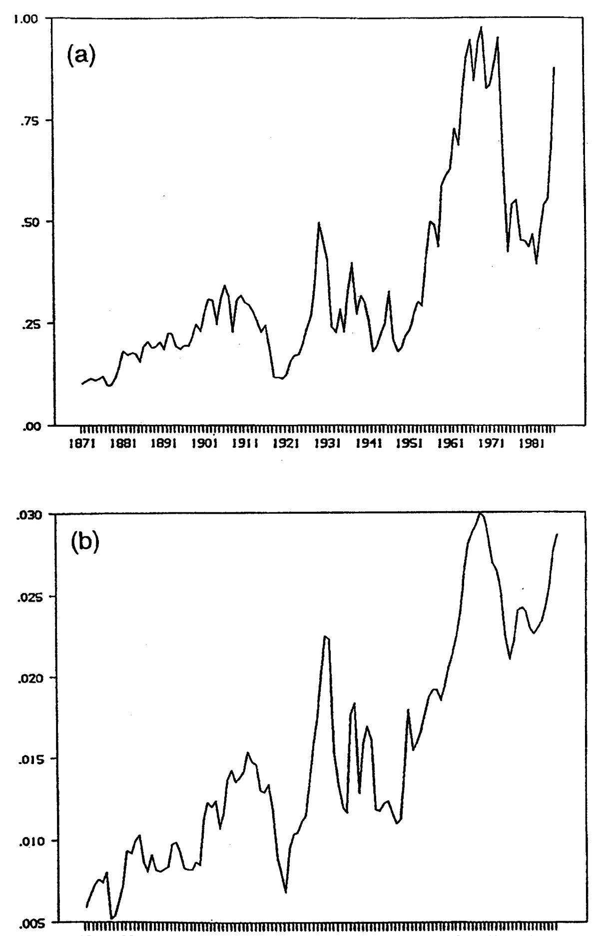

Figure 1. (a) Annual real stock price index: 1871–1987. (b) Annual real dividend series: 1871–1987.

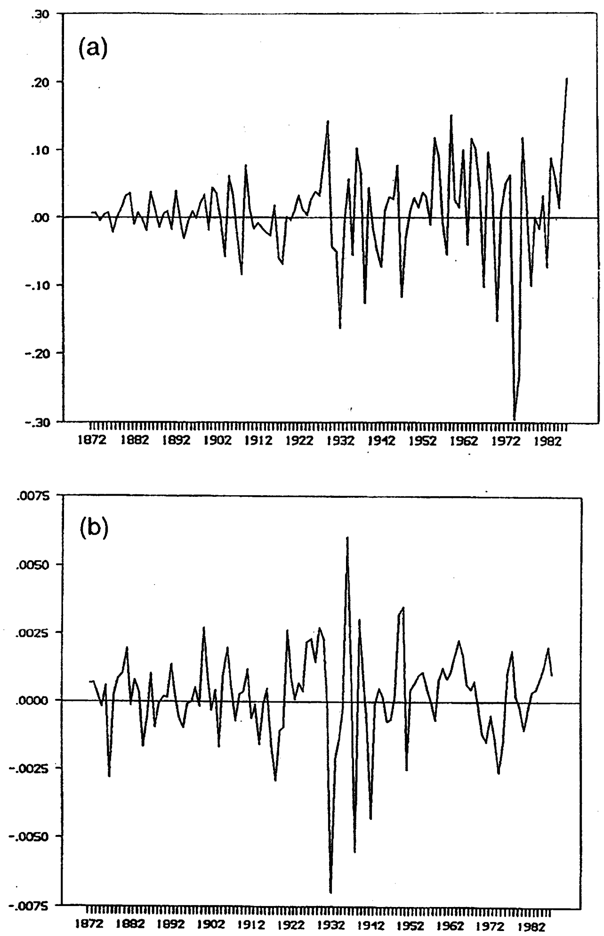

Figure 2. (a) Changes in annual real stock prices: 1872–1987. (b) Changes in annual real dividends: 1872–1987.

1(c). The test results are based on a 4-lag model specification. Because the standard augmented Dickey–Fuller (ADF) test can misinterpret a time series with structural changes as a unit root process (Perron, 1989), we also consider the recursive and sequential unit root tests proposed by Banerjee *et al.* (1992). An advantage of the Banerjee *et al.* procedure is that their tests treat the break point as unknown *a priori*. This makes their procedure not subject to the data-mining problem inherent in tests that are conditional on a pre-assigned break point.

In Table 1(c), τ is the standard augmented ADF statistic that allows for both a mean and a time trend under the stationary alternative. τ_{MN} and τ_{R} are the recursive unit root statistics given by the smallest and by the range of the ADF statistics computed from subsamples. $F_{\mathrm{A,MX}}$, $\tau_{\mathrm{A}}(F)$ and $\tau_{\mathrm{A,MN}}$ are the sequential statistics for the alternative of a shift in the trend during the sample period. $F_{\mathrm{A,MX}}$ is the largest of the F-statistics for the no-shift null hypothesis obtained under different shift-point specifications, $\tau_{\mathrm{A}}(F)$ is the ADF statistic from the specification that yields $F_{\mathrm{A,MX}}$, and $\tau_{\mathrm{A,MN}}$ is the smallest of the ADF statistics obtained from different shift point specifications. The definitions of the sequential statistics $F_{\mathrm{B,MX}}$, $\tau_{\mathrm{B}}(F)$ and $\tau_{\mathrm{B,MN}}$ are similar to those of $F_{\mathrm{A,MX}}$, $\tau_{\mathrm{A}}(F)$ and $\tau_{\mathrm{A,MN}}$. However, these statistics are for the alternative of a break, instead of a shift, in the trend. $|t_{\mathrm{MX}}|$ is the maximal t-statistic for the no- break-in-trend hypothesis computed under the null of a unit root and no time trend. See Banerjee *et al.* (1992) for more details on these test statistics and their critical values.[7]

For all the sample test statistics reported in Table 1(c), only the τ_{R} statistic for the price series is significant. Both the ADF and Banerjee *et al.* tests yield little evidence against the unit root hypothesis. Further, the unit root result is not likely to be explained by structural changes in the data. Based on these conventional unit root tests, one tends to agree that the real dividend process is better described as an integrated process, and the excess volatility result reported in, say, Shiller (1981) may be the consequence of assuming a trend stationary dividend series, as argued by Marsh and Merton (1986). However, there is a caveat. As pointed out by Diebold and Rudebusch (1991b) and Sowell (1990), the conventional unit root tests have low power against fractional integration alternatives.

Estimates of ARFIMA models

Based on the sample size and efficiency considerations, we use the time domain exact maximum likelihood (ML) estimation procedure to estimate jointly the parameters of an ARFIMA model. In general, the time domain exact ML method is more efficient under correct model specification. This procedure amounts to maximizing the likelihood function

$$\mathfrak{L}(\mathbf{X}|.) = (2\pi)^{-T/2}|\Sigma|^{-1/2}\exp(-\mathbf{X}'\Sigma^{-1}\mathbf{X}/2) \qquad (12)$$

with respect to the parameter vector $\xi = (d, \phi_i\text{'s}, \theta_i\text{'s}, \sigma)$, where $\mathbf{X} = (X_1, \ldots, X_T)$ is the vector of sample observations, Σ is the $T \times T$

Table 1. Descriptive statistics and results of unit root tests.

1(a) Descriptive statistic

	MEAN	C.V.	MIN	MAX	CORR
1(a) 1 Levels					
PRICE	0.3456	0.6595	0.9687	0.9783	0.922
DIVIDEND	0.0152	0.4398	0.0051	0.0299	
1(a) 2 Differences					
PRICE	0.0067	10.354	−0.2955	0.2050	0.545
DIVIDEND	0.0002	8.507	−0.0069	0.0060	

1(b) Correlation

	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$	$\alpha(5)$
1(b) 1 Levels										
PRICE	0.925	0.859	0.824	0.777	0.719	0.925	0.027	0.178	−0.076	−0.076
DIVIDEND	0.944	0.877	0.825	0.780	0.740	0.944	−0.125	0.110	0.010	0.033
1(b) 2 Differences										
PRICE	0.090	−0.217	0.131	0.180	−0.030	0.090	−0.227	0.187	0.099	0.033
DIVIDEND	0.221	−0.149	−0.063	−0.069	−0.007	0.221	−0.208	0.026	−0.099	0.030

1(c) Unit Root Test

	τ	τ_{MN}	τ_R	$F_{A,MX}$	$\tau_A(F)$	$\tau_{A,MN}$	$F_{B,MX}$	$\tau_B(F)$	$\tau_{B,MN}$	$\lvert t_{MX}\rvert$
PRICE	−2.807	−3.201	3.725	4.381	−3.480	−3.482	9.903	−4.288	−4.288	1.208
DIVIDEND	−2.995	−3.276	2.446	3.126	−3.497	−3.497	8.055	−4.180	−4.180	1.156

The mean (MEAN), coefficient of variation (C.V.), minimum (MIN), maximum (MAX), and the correlation coefficient of the annual real stock prices (PRICE) and real dividends (DIVIDEND), both in levels and first differences, are reported in Panel 1(a). The sample period is 1871–1987. The sample autocorrelation and partial autocorrelation at lag k ($\rho(k)$ and $\alpha(k)$) for both the original and first differenced series are given in Panel 1(b). The unit root test results are reported in Panel 1(c). τ is the standard augmented ADF statistic that allows for both a mean and a time trend under the stationary alternative. τ_{MN} and τ_R are the recursive unit root statistics. $F_{A,MX}$, $\tau_A(F)$ and $\tau_{A,MN}$ are the sequential statistics for the alternative of a shift in the trend during the sample period. $F_{B,MX}$, $\tau_B(F)$ and $\tau_{B,MN}$ are the sequential statistics for the alternative of a break, instead of a shift, in the trend. $|t_{MX}|$ is the maximal t-statistic for the no- break-in-trend hypothesis computed under the null of a unit root and no time trend. See Banerjee *et al.* (1992) for a more detailed discussion on these test statistics and their critical values. Critical values for the standard augmented ADF test are taken from Cheung and Lai (1995b). The unit root statistics are all based on a 4-lag specification and are not significant, except the τ_R for the real stock price series, at the 5% level.

covariance matrix of **X** and is a function of the parameter vector ξ. Sowell (1992a) provides a detailed discussion of the ML estimation procedure. The performance of this ML procedure in small samples is investigated in, for example, Cheung and Diebold (1994) and Sowell (1992a).

For each of the real stock price and real dividend series, ARFIMA(p, d, q) models with both p and q less than four are considered. First differenced series are used in the estimation process to ensure stationarity. Estimates are obtained via the Davidon–Fletcher–Powell algorithm. The Akaike information criterion (AIC) and the Schwartz information criterion (SIC) are used to select the model specification. For the real stock price series, the AIC chooses an ARFIMA(3, d, 0) model and the SIC selects a more parsimonious ARFIMA(0, d, 1) model. Both information criteria select an ARFIMA(0, d, 1) for the real dividend data.

ML estimates of the models selected by the AIC and SIC are reported in Table 2. The ML estimates of the differencing parameter d are of special interest. The point estimates of d are all significantly different from both zero and one. Both the

Table 2. Parameter estimates of ARFIMA models.

	Real stock price		Real dividend
	AIC	SIC	AIC & SIC
d	0.4713	0.7296	0.7040
	(0.1977)	(0.1044)	(0.1157)
ϕ_1	0.6331		
	(0.1921)		
ϕ_2	−0.2158		
	(0.1067)		
ϕ_3	0.3267		
	(0.0883)		
θ_1		0.4563	0.5157
		(0.1238)	(0.0950)
σ	0.0645	0.0670	0.0016
	(0.0042)	(0.0044)	(0.0001)

Parameter estimates of ARFIMA(p, d, q) models of the annual real stock prices and real dividends from 1871–1987, selected by the Akaike information criterion (AIC) and Schwartz information criterion (SIC), are reported. The estimated model is $\Phi(B)(1-B)^d X_t = \Theta(B)\epsilon_t$, where $\Phi(B) \equiv 1 - \phi_1 B - \cdots - \phi_p B^p$, $\Theta(B) \equiv 1 + \theta_1 B + \cdots + \theta_q B^q$, and the standard deviation of ε_t is σ.

real stock price and the real dividend series are fractionally integrated with d between zero and one. They are neither integrated of order one nor trend stationary as reported in previous studies. The persistence in these data is stronger than that allowed for by stationary processes and weaker than that implied by unit root models.

The point estimates of d are in the range of 0.47 to 0.73. These estimates imply that the data are either covariance non-stationary or near covariance non-stationary in levels but are covariance stationary in first differences. This result is another indication of the low power of the conventional unit root tests against fractional alternatives. In contrast to the conventional knife-edged $d=0$ or $d=1$ classification, the uncertainty of long-term persistence in data can be assessed using the (asymptotic) standard error of the d estimate. Comparing the two ARFIMA models for the real stock price series, we find that the ARFIMA(0, d, 1) selected by the SIC implies a stronger long- term persistence in price data than the ARFIMA(3, d, 0) model selected by the AIC.

The estimates of the ARMA parameters also provide useful information on the short-term temporal dynamics. The two ARFIMA(0, d,1) models suggest the short-term effect of a shock will last for only two periods. For both real stock prices and real dividends, the persistence beyond two years is driven by the fractional differencing component. The largest AR root in the estimated ARFIMA(3, d, 0) is 0.83. That is, in contrast to the model selected by the SIC, the model based on AIC suggests a substantial short-term persistence.

Excessive variability

The parameter estimates reported in the previous subsection allow us to compute $\hat{C}(r,\infty)\hat{\sigma}$, the sample estimate of the standard deviation of real stock price innovations implied by the present value model. Theoretically, the implied standard deviation is derived from an infinite horizon model. For real world investors, however, decisions are likely to be based on a shorter horizon consideration. In a recent study Kothari and Shanken (1992) find that, after controlling for measurement errors, dividend growth rates in the current and the next three years can account for 72% of the total variation in the current annual return on the CRSP equal-weight portfolio from 1927–1985. Further it is interesting to see if the investor's myopic behaviour can explain market volatility. Therefore we calculate $\hat{C}(r,\infty)\hat{\sigma}$ for $k = 1, 2, \ldots, 120$. For the discount rate factor, we consider $r=0.02$ to 0.10. This covers the typical range examined in the literature. For example, Shiller (1981) and Mankiw *et al.* (1985, 1991) considered real interest rates in the range of 0.04 to 0.10.

Since $\hat{C}(r,k)\hat{\sigma}$ is only a point estimate of the implied standard deviation, it is difficult to interpret the difference between this point estimate and the sample standard deviation computed from real stock price data. Since the parameters are jointly estimated, we can assess the uncertainty associated with the point estimate $\hat{C}(r,\infty)\hat{\sigma}$ that is due to sampling variability as follows. For a given k and r, $C(r,k)\sigma$ is a function of the parameter vector ξ. Hence, the asymptotic variance of $\hat{C}(r,k)\hat{\sigma}$ is given by $\nabla[C(r,k)\sigma]\Omega\nabla[C(r,k)\sigma]^{\mathrm{T}}$, where $\nabla[C(r,k)\sigma]$ is $\partial[C(r,k)\sigma]/\partial\xi$, the superscript-T denotes transpose, and Ω is the variance-covariance matrix of ξ (Campbell and Mankiw, 1987, Appendix). Given the sample information on ξ and Ω, we can evaluate the sampling uncertainty of the point estimate $\hat{C}(r,k)\hat{\sigma}$ accordingly.

Table 3. Standard errors of innovations in real stock prices implied by the present value model.

Horizon					r				
K=	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
1	0.0943†	0.0628†	0.0470†	0.0375	0.0312	0.0267	0.0234	0.0207	0.0186
	0.0153	0.0101	0.0076	0.0060	0.0050	0.0043	0.0037	0.0033	0.0030
2	0.0752†	0.0503†	0.0378	0.0303	0.0253	0.0218	0.0191	0.0170	0.0153
	0.0183	0.0121	0.0090	0.0072	0.0059	0.0050	0.0044	0.0039	0.0035
3	0.0669†	0.0449†	0.0339	0.0273	0.0229	0.0197	0.0173	0.0155	0.0140
	0.0194	0.0128	0.0095	0.0076	0.0062	0.0053	0.0046	0.0041	0.0036
4	0.0619†	0.0417†	0.0316	0.0255	0.0214	0.0185	0.0163	0.0146	0.0133
	0.0199	0.0132	0.0098	0.0077	0.0064	0.0054	0.0047	0.0041	0.0037
5	0.0583†	0.0394†	0.0300	0.0243	0.0205	0.0177	0.0157	0.0141	0.0128
	0.0202	0.0133	0.0099	0.0078	0.0065	0.0055	0.0047	0.0042	0.0037
10	0.0493†	0.0338†	0.0260	0.0214	0.0182	0.0159	0.0142	0.0128	0.0118
	0.0205	0.0135	0.0100	0.0079	0.0065	0.0055	0.0048	0.0042	0.0037
20	0.0429†	0.0301	0.0236	0.0197	0.0170	0.0150	0.0135	0.0123	0.0113
	0.0203	0.0134	0.0099	0.0079	0.0065	0.0055	0.0048	0.0042	0.0037
30	0.0403†	0.0288	0.0228	0.0192	0.0166	0.0148	0.0133	0.0122	0.0112
	0.0201	0.0133	0.0098	0.0078	0.0064	0.0055	0.0047	0.0042	0.0037
60	0.0376†	0.0276	0.0222	0.0188	0.0165	0.0147	0.0133	0.0121	0.0112
	0.0197	0.0131	0.0098	0.0078	0.0064	0.0055	0.0047	0.0042	0.0037
120	0.0367†	0.0273	0.0221	0.0188	0.0164	0.0147	0.0133	0.0121	0.0112
	0.0195	0.0130	0.0097	0.0078	0.0064	0.0055	0.0047	0.0042	0.0037

In each cell, the first entry is $\hat{C}(r, k)\hat{\sigma}$, the point estimate of the standard deviation of innovations in real stock prices implied by the present value model, and the second is its asymptotic standard error. K is the time horizon in years and r is the real discount rate. † indicates no statistically significant evidence of excess market volatility as the 95% confidence interval of $\hat{C}(r, k)\hat{\sigma}$ overlaps that of the sample standard deviation of innovations in real stock prices reported in Table 2 (also, see Endnote 8). However, in other cases, there is substantial evidence of excess market volatility as the 95% confidence interval of $\hat{C}(r, k)\hat{\sigma}$ lies below and does not overlap that of the sample standard deviation of innovations in real stock prices.

The implied standard errors $\hat{C}(r, k)\hat{\sigma}$ based on various r and k combinations and their associated asymptotic standard errors are reported in Table 3. As expected, the $\hat{C}(r, k)\hat{\sigma}$ decreases when r (or k) increases. The discounted cumulated impulse response appears very close to the convergence value at $k=120$.

In general, the observed variation in real stock price innovations is larger than that implied by the present value model. The point estimate of $\hat{C}(r, k)\hat{\sigma}$ is smaller than 0.0645 or 0.0670, the sample standard errors of the real stock price innovation reported in Table 2, with the exceptional cases given by $r=0.02$ and $k \leq 3$. Only when the real discount rate is 2% and the relevant horizon for investors is three years, is the stock market not excessively volatile. Although the investor's planning horizon seems plausible given the Kothari and Shanken (1992) study, the real discount rate of 0.02 appears too small.

The result changes slightly when we compare the 95% confidence intervals of the implied and sample standard deviations of real stock price innovations.[8] When $r=0.02$, the confidence intervals of the implied and sample standard deviations overlap for $k \leq 120$. That is, the two point estimates are not statistically different from each other and there is no significant excess market volatility. The no-excess-market-volatility result is also observed for $r=0.03$ and $k \leq 10$ and $r=0.04$ and $k=1$. However, the evidence of excessive market volatility is still very strong. For instance, the estimated variability obtained from real stock price data is significantly larger than $\hat{C}(r, k)\hat{\sigma}$ when r is larger than 4%. Even after allowing for sampling uncertainty, the variation in stock price innovations is significantly larger than that implied by the present value model in 73 out of 90 cases reported in the table and in no cases is this result reversed. It appears that the no-excess-market-volatility result relies heavily on either a very low interest rate factor or a very short horizon for investors.

CONCLUSIONS

The issue of stock market volatility raised by Shiller (1981) and LeRoy and Porter (1981) is re-examined. Previous studies suggest that market volatility test results can depend on the temporal dynamics of the dividend process. If one assumes the dividend data are trend stationary, the stock market is excessively volatile. The result is reversed if the dividend data are assumed to be difference stationary. In order to encompass the two opposing views on dividend dynamics, the fractional integration time series model, which is more general than standard time series models, is used to describe the temporal behaviour. In contrast to the existing results, we find both the real stock price and the real dividend data are fractionally integrated with an order between zero and one. They are neither trend stationary nor difference stationary.

We use a relation between the variability of innovations in real stock prices and real dividends, which is derived under the fractional integration framework, to investigate if the stock market is excessively volatile. This relation is valid when the dividend data are trend stationary, difference stationary, or fractionally integrated. Overall, we find substantial evidence for excess market volatility even after controlling for sampling uncertainty and long-term persistence represented by fractional integration. Low interest rates and investors' myopic behaviour have only a limited role in explaining excess market volatility.

ACKNOWLEDGEMENTS

The author thanks Menzie Chinn, Frank Diebold, Mike Dooley, Keith Heyen, Kon S. Lai, Stephen Leroy, Judy Newman, Carl Walsh, participants of the UCSC seminar, an anonymous referee, and the editor of this journal for helpful comments and suggestions.

ENDNOTES

1. Studies such as Fama and French (1988), Lo and MacKinlay (1988), and Poterba and Summers (1988) also cast doubt on the efficient market hypothesis by showing that stock returns over long holding horizons have predictable components.
2. For instance, Flavin (1983) shows that the original Shiller test is biased toward finding excessive volatility. Cochrane (1991) argues that excess market volatility is a description of the behaviour of discount factors in an efficient market and not an evidence against market efficiency.
3. See Granger and Joyeux (1980) and Hosking (1981) for a detailed discussion on ARFIMA processes.
4. Some studies use the discount factor $(1+r)^k$ instead of $(1+r)^{k+1}$. However, such modification does not affect the results reported below qualitatively.
5. Compared with Diebold and Rudebusch (1991a), we directly derive the relationship between the variability in innovations using the fractional time series representation. This approach gives the range of d values in which the derived relationship is valid. Also, a joint estimation method, instead of a two-stage procedure, is used. In addition to efficiency, the joint estimation method also facilitates the computation of the standard error of the implied volatility.
6. In fact, $C(r, \infty)$ is finite if $d<2$ and $r>0$. Since we assumed $d<1.5$ to write down Equation (5), the first condition is satisfied by assumption.
7. The trimming parameters used to compute the recursive and sequential statistics are the same as those used in Banerjee *et al.* (1992).
8. For the ARFIMA model selected by the AIC reported in Table 2, the 95% confidence interval of the standard error of innovations in real stock prices is (0.05627, 0.0727). The 95% confidence interval computed for the model selected by the SIC is (0.0674, 0.0846).

REFERENCES

Banerjee, A., Lumsdaine, R.L. and Stock, J.H. (1992) Recursive and sequential tests of the unit root and trend break hypotheses. *Journal of Business & Economic Statistics*, **10**, 271–287.

Campbell, J.Y. and Mankiw, N.G. (1987) Are output fluctuations transitory? *Quarterly Journal of Economics*, **102**, 857–880.

Cheung, Y.-W. (1993) Long memory in foreign exchange rates. *Journal of Business & Economic Statistics*, **11**, 93–101.

Cheung, Y.-W. and Lai, K. S. (1993) A Fractional Cointegration Analysis of Purchasing Power Parity. *Journal of Business & Economic Statistics*, **11**, 103–112.

Cheung, Y.-W. and Diebold, F.X. (1994) On maximum-likelihood estimation of the differencing parameter of fractionally integrated noise with unknown mean. *Journal of Econometrics*, **62**, 301–316.

Cheung, Y.-W. and Lai, K.S. (1995a) A search for long memory in international stock market returns. *Journal of International Money and Finance*, **14**, 597–615.

Cheung, Y.-W. and Lai, K.S. (1995b) Lag order and critical values of the augmented Dickey–Fuller test. *Journal of Business and Economic Statistics*, **13**, 277–280.

Cochrane, J.H. (1991) Volatility tests and efficient markets. *Journal of Monetary Economics*, **27**, 463–485.

DeJong, D. and Whiteman, C.H. (1991) The temporal stability of dividends and stock prices: evidence from the likelihood function. *American Economic Review*, **81**, 600–617.

Diebold, F.X., Husted, S. and Rush, M. (1991) Real exchange rates under the gold standard. *Journal of Political Economy*, **99**, 1252–1271.

Diebold, F.X. and Rudebusch, G.D. (1989) Long memory and persistence in aggregate output. *Journal of Monetary Economics*, **24**, 189–209.

Diebold, F.X. and Rudebusch, G.D. (1991a) Is consumption too smooth? Long memory and the Deaton paradox. *Review of Economics and Statistics*, **73**, 1–17.

Diebold, F.X. and Rudebusch, G.D. (1991b) On the power of Dickey–Fuller tests against fractional alternatives. *Economics Letters*, **35**, 155–160.

Fama, E.F. and French, K.R. (1988) Permanent and temporary components of stock prices. *Journal of Political Economy*, **96**, 246–273.

Flavin, M.J. (1983) Excess volatility in the financial markets: a reassessment of the empirical evidence. *Journal of Political Economy*, **91**, 929–956.

Gradshteyn, I.S. and Ryzhik, I.M. (1980) *Table of Integrals, Series, and Products*, corrected and enlarged edition, Academic Press.

Granger, C.W.J. and Joyeux, R. (1980) An introduction to long-memory time series models and fractional differencing. *Journal of Time Series Analysis*, **1**, 15–29.

Hosking, J.R.M. (1981) Fractional differencing. *Biometrika*, **68**, 165–176.

Kleidon, A.W. (1986) Variance bounds tests and stock price valuation models. *Journal of Political Economy*, **94**, 953–1001.

Koop, G. (1991) Cointegration tests in present value relationships. *Journal of Econometrics*, **49**, 105–139.

Kothari, S.P. and Shanken, J. (1992) Stock return variation and expected dividends. *Journal of Financial Economics*, **31**, 177–210.

LeRoy, S. (1989) Efficient capital markets and the martingale model of stock prices. *Journal of Economic Literature*, **XVII**, 1583–1621.

LeRoy, S. and Porter, R.D. (1981) Stock price volatility: tests based on implied variance bounds. *Econometrica*, **49**, 97–113.

LeRoy, S. and Steigerwald, D. (1992) Volatility. Manuscript, UC Santa Barbara.

Lo, A.W. and MacKinlay, A.W. (1988) Stock prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies*, **1**, 41–66.

Marsh, T.A. and Merton, R.C. (1986) Dividend variability and variance bounds tests for the rationality of stock market prices. *American Economic Review*, **76**, 483–498.

Mankiw, N.G., Romer, D. and Shapiro, M.D. (1985) An unbiased reexamination of stock market volatility. *Journal of Finance*, **XL**, 677–689.

Mankiw, N.G., Romer, D. and Shapiro, M.D. (1991) Stock market forecastability and volatility: a statistical appraisal. *The Review of Economic Studies*, **58**, 455–477.

Perron, P. (1989) The great crash, the oil price shock and the unit root hypothesis. *Econometrica*, **57**, 1361–1401.

Poterba, J. and Summers, L.H. (1988) Mean reversion in stock prices: evidence and implications. *Journal of Financial Economics*, **22**, 27–59.

Shiller, R.J. (1981) Do stock market prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, **71**, 421–436.

Shiller, R.J. (1984) Stock prices and social dynamics. *Brookings Papers on Economic Activity*, **2**, 457–498.

Shiller, R.J. (1989) *Market Volatility*, The MIT Press.

Sowell, F. (1990) Fractional unit root distribution. *Econometrica*, **50**, 495–505.

Sowell, F. (1992a) Maximum likelihood estimation of stationary univariate fractionally integrated time series models. *Journal of Econometrics*, **53**, 165–188.

Sowell, F. (1992b) Modeling long-run behaviour with the fractional ARIMA model. *Journal of Monetary Economics*, **29**, 277–302.