1. Decide whether each of the following statements is true or false. Write "TRUE" if the statement is true or "FALSE" if it is false. You do not have to justify your answer.

(a) (2 points) Two sides and one angle always determine a unique triangle.
   **FALSE**

(b) (2 points) The largest side of a triangle is opposite the largest angle.
   **TRUE**

(c) (2 points) The Law of Cosines can be used to solve a triangle with SSS condition.
   **TRUE**

(d) (2 points) If three angles of an oblique triangle are known, then we can solve the triangle.
   **FALSE**

(e) (2 points) Every oblique triangle has one obtuse angle.
   **FALSE**

2. (10 points) Solve the oblique triangle with $a = 8$, $c = 5$ and $B = 40^\circ$. (Approximate all answers to four decimal places.)

*Solve for $b$*

\[ b^2 = a^2 + c^2 - 2ac \cos(B) \]
\[ b = \sqrt{a^2 + c^2 - 2ac \cos(B)} \]
\[ b = \sqrt{64 + 25 - 80 \cos(40^\circ)} \]
\[ b \approx 5.2646 \]

Now we know $A$ is the largest angle since $a$ is the largest side.

*Solve for $A$*

\[ a^2 = b^2 + c^2 - 2bc \cos(A) \]
\[ \cos(A) = \frac{b^2 + c^2 - a^2}{2bc} \]
\[ \cos(A) \approx \frac{(5.2646)^2 + 5^2 - 8^2}{2(5.2646)\times 5} \]
\[ \cos(A) \approx -0.2143 \]
\[ A \approx \cos^{-1}(-0.2143) \]
\[ A \approx 102.3745^\circ \]

*Solve for $C$*

\[ A + B + C = 180^\circ \]
\[ C = 180^\circ - B - A \]
\[ C = 180^\circ - 40^\circ - 102.3745^\circ \]
\[ C \approx 37.6255^\circ \]
3. (a) (6 points) Sketch \( \vec{u} \) and \( \vec{v} \) in standard position. Are \( \vec{u} \) and \( \vec{v} \) equivalent? (Explain why or why not.)

\[
\vec{v} = \langle 2 - (-5), 25 - 1 \rangle = \langle 7, 24 \rangle
\]

\[
\|\vec{v}\| = \sqrt{7^2 + 24^2} = \sqrt{625} = 25
\]

No, \( \vec{u} \) and \( \vec{v} \) are not equivalent. They do not have the same direction.

(b) (6 points) Find the component form and magnitude of the vector \( \vec{v} \) having initial point \((-5, 1)\) and terminal point \((2, 25)\).

(c) (6 points) Sketch \( \vec{u} + \vec{v} \) and \( \vec{u} - \vec{v} \) in standard position.

(d) (6 points) Find the unit vector in the opposite direction of \( \vec{v} = (4, -3) \).

\[
\|\vec{v}\| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5
\]

\[
\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{5} \langle 4, -3 \rangle = \langle \frac{4}{5}, \frac{-3}{5} \rangle
\]

\[
\text{unit vector in direction of } \vec{v}
\]

\[
\text{unit vector in opposite direction of } \vec{v}
\]
4. (a) (6 points) Is \((\vec{u} \cdot \vec{v})\) a vector or a scalar? Is \((\vec{u} \cdot \vec{v}) - (\vec{u} \cdot \vec{u})\) a vector or a scalar? (No explanation needed.)

\[(\vec{u} \cdot \vec{v}) \hat{v} \text{ is a vector} \quad \hat{\theta} \quad (\vec{u} \cdot \vec{v}) - (\vec{u} \cdot \vec{u}) \text{ is a scalar.}\]

(b) (6 points) Find the angle \(\theta\) between \(\vec{u} = \langle 1, -1 \rangle\) and \(\vec{v} = \langle 0, 2 \rangle\). Are \(\vec{u}\) and \(\vec{v}\) orthogonal? (Explain why or why not.)

\[
\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| ||\vec{v}||} = \frac{(1)(0) + (-1)(2)}{\sqrt{1^2 + (-1)^2} \cdot \sqrt{0^2 + 2^2}} = \frac{-2}{\sqrt{2} \cdot 2} = \frac{\sqrt{2}}{2} \\
\theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}
\]

No, \(\vec{u}\) and \(\vec{v}\) are not orthogonal since \(\vec{u} \cdot \vec{v} = -2\) not 0.

(c) (6 points) Find the component form of \(\vec{v}\) given that \(||\vec{v}|| = 2\) and \(\vec{v}\) is in the direction of \(\hat{i} + \hat{j}\).

\[\hat{i} + \hat{j} = \langle 1, 0 \rangle + \langle 0, 1 \rangle = \langle 1, 1 \rangle, \]
so \(\hat{i} + \hat{j}\) is \(\boxed{\text{Q E}}\) and

\[
\tan(\theta) = \frac{1}{1} = 1 \quad \Rightarrow \quad \theta = \tan^{-1}(1) = \frac{\pi}{4}
\]

(d) (8 points) Find a vector orthogonal to \(\vec{u} = \langle \sqrt{3}, -1 \rangle\).

\[\text{direction angle } \theta \text{ of } \hat{u} \]
\[\hat{u} \text{ is in } \boxed{\text{Q IV}} \text{ and} \]
\[\tan(\theta) = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}
\]

(e) (10 points) Find the projection of \(\vec{u} = \langle 2, 2 \rangle\) onto \(\vec{v} = \langle 6, 1 \rangle\), then write \(\vec{u}\) as the sum of two vectors, one of which is \(\text{proj}_v \vec{u}\).

\[\text{proj}_v \vec{u} = \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2} \vec{v} \]
\[= \frac{2 \cdot 6 + 2 \cdot 1}{(\sqrt{6^2 + 1^2})^2} \langle 6, 1 \rangle \]
\[= \frac{14}{37} \langle 6, 1 \rangle \]
\[= \left\langle \frac{64}{37}, \frac{1}{37} \right\rangle
\]

\[\hat{u} - \text{proj}_v \vec{u} = \langle 2, 2 \rangle - \left\langle \frac{64}{37}, \frac{1}{37} \right\rangle \]
\[= \left\langle 2 - \frac{64}{37}, 2 - \frac{1}{37} \right\rangle \]
\[= \left\langle \frac{22}{37}, \frac{73}{37} \right\rangle
\]

\[\hat{u} = \left\langle \frac{64}{37}, \frac{1}{37} \right\rangle + \left\langle \frac{-10}{37}, \frac{73}{37} \right\rangle
\]
5. (10 points) A plane flies 500 kilometers with a bearing of $316^\circ$ from Naples to Elgin. The plane then flies 720 kilometers from Elgin to Canton (Canton is due west from Naples.) Find the bearing of the flight from Elgin to Canton. *Remember that for airplanes, bearing is measured clockwise from North.* (Approximate your answer to 2 decimal places.)

\[ \frac{a}{b} = \frac{\sin(A)}{\sin(B)} \]

\[ \sin(A) = \frac{a}{b} \sin(B) = \frac{500}{720} \sin(46^\circ) \]

Two possible angles $A \approx 0.4995$.

\[ A = \sin^{-1}(0.4995) \]

\[ A \approx 29.670^\circ \]

\[ A \approx 180^\circ - \sin^{-1}(0.4995) \]

\[ A \approx 150.0265^\circ \]

\[ \text{Too Big} \]

\[ A + B \approx 196.0265^\circ \]

\[ \geq 180^\circ \]

\[ \boxed{29.670^\circ} \]
6. (10 points) Find the magnitude of the tension in each of the cables supporting the load. In the free-body diagram, $\vec{T}_1$ is the tension in the first cable, $\vec{T}_2$ is the tension in the second cable and $\vec{G}$ is the gravitational force on the load. (Approximate your answers to 2 decimal places.)

\[ \vec{G} = \langle 0, -200 \rangle \]
\[ \vec{T}_1 = \langle \|\vec{T}_1\| \cos(60^\circ), \|\vec{T}_1\| \sin(60^\circ) \rangle = \langle \frac{1}{2}\|\vec{T}_1\|, \frac{\sqrt{3}}{2}\|\vec{T}_1\| \rangle \]
\[ \vec{T}_2 = \langle \|\vec{T}_2\| \cos(135^\circ), \|\vec{T}_2\| \sin(135^\circ) \rangle = \langle -\frac{1}{\sqrt{2}}\|\vec{T}_2\|, \frac{1}{\sqrt{2}}\|\vec{T}_2\| \rangle \]

The vector sum $\vec{G} + \vec{T}_1 + \vec{T}_2 = \vec{0}$

\[ \begin{cases} 
\frac{1}{2}\|\vec{T}_1\| - \frac{1}{\sqrt{2}}\|\vec{T}_2\| = 0 \\
-200 + \frac{\sqrt{3}}{2}\|\vec{T}_1\| + \frac{1}{\sqrt{2}}\|\vec{T}_2\| = 0
\end{cases} \]

\[ \begin{align*}
\|\vec{T}_2\| &= \frac{\|\vec{T}_1\|}{2} \implies \|\vec{T}_1\| = \frac{\|\vec{T}_2\|}{2} \\
\|\vec{T}_2\| &= \frac{\sqrt{3}}{2}\|\vec{T}_1\| + \frac{1}{\sqrt{2}}\|\vec{T}_1\| = 200
\end{align*} \]

\[ \|\vec{T}_1\| = \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \left( \frac{\|\vec{T}_1\|}{2} \right) = 200 \]

\[ \|\vec{T}_2\| = 103.53 \text{ lbs} \]

\[ \|\vec{T}_1\| = 146.41 \text{ lbs} \]