Announcements  3/16/19

* Today:  10.7

* Wednesday:  10.8 $ "REVIEW/DISCUSS
FINAL EXAM

* Wednesday  3/27  FINAL EXAM

↓

HMWK 10.7 & 10.6
DUE AT BEGINNING OF
Final.
We know how to represent a point in the plane using the rectangular coordinate system:

- $x$ is the directed distance from the $y$-axis to the point $(x,y)$
- $y$ is the directed distance from the $x$-axis to the point $(x,y)$.
Now we study a different Coordinate System called the Polar Coordinate System.

To form the Polar Coordinate System:
- Fix a point called the pole, (origin)
- From the pole, construct an initial ray called the polar axis.

Each point \( P \) in the plane can be assigned Polar Coordinates \((r, \theta)\) as follows:
- \( r \) = directed distance from the pole to \( P \)
- \( \theta \) = directed angle, measured clockwise from the polar axis to the line segment joining the pole and \( P \).
EXAMPLE

Plot the points in the polar coordinate system

a) \((3, \pi/4)\)

b) \((5, -\pi/3)\)

c) \((-4, 5\pi/3)\)
\((3, \frac{\pi}{4})\)
\( b \left( 5, -\frac{\pi}{3} \right) \)
( -4, \frac{5\pi}{3} )
In rectangular coordinates, each point has a unique representation \((x, y)\).

**THIS IS NOT TRUE IN POLAR COORDINATES**

- \((r, \theta)\) and \((r, \theta + 2\pi)\) represent the same point.

- \((r, \theta)\) and \((-r, \theta + \pi)\) represent the same point \((r\) is a directed distance!\)

- In general:
  \[
  (r, \theta) = (r, \theta + 2\pi n) \quad n \text{ is an integer}
  \]
  \[
  (r, \theta) = (-r, \theta +(2n+1)\pi) \quad n \text{ is an integer}
  \]
Example (You Try)

Plot the point \((4, \frac{5\pi}{6})\) and find three additional polar representations of this point using \(-2\pi < \theta < 2\pi\).
\[
\begin{align*}
\left( \theta, 5\pi/6 \right) &= \left( \theta, -7\pi/6 \right) \\
&= (-\theta, 11\pi/6) \\
&= (-\theta, -\pi/6)
\end{align*}
\]

All represent the same point in polar coordinates.
Coordinate Conversion

We can relate the polar coordinates \((r, \theta)\) to the rectangular coordinates \((x, y)\) of a point in the plane.

To do this:

- Let the pole & the origin coincide
- Let the polar axis & the positive x-axis coincide.

\[
\begin{align*}
\tan(\theta) &= \frac{y}{x} \\
\cos(\theta) &= \frac{x}{r} \\
\sin(\theta) &= \frac{y}{r} \\
1^2 &= x^2 + y^2
\end{align*}
\]

\(r > 0\)

\(*\) still works if \(r < 0\)
Coordinate Conversion

The Polar Coordinates \((r, \theta)\) are related to the rectangular coordinates \((x, y)\):

**Polar-to-Rectangular**

\[
\begin{align*}
x &= r \cos(\theta) \\
y &= r \sin(\theta)
\end{align*}
\]

**Rectangular-to-Polar**

\[
\begin{align*}
\tan(\theta) &= \frac{y}{x} \\
r^2 &= x^2 + y^2
\end{align*}
\]

Example (You Try)

(a) Convert \((\sqrt{3}, \pi/6)\) to rectangular coordinates

(b) Convert \((-1, 1)\) to polar coordinates

**a**  \((\sqrt{3}, \pi/6)\) \rightarrow \text{rectangular}

\[
\begin{align*}
r &= \sqrt{3}, \quad \theta = \pi/6 \\
\Rightarrow x &= r \cos(\theta) = \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2} \\
y &= r \sin(\theta) = \sqrt{3} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}
\end{align*}
\]

\(\rightarrow (\frac{3}{2}, \frac{\sqrt{3}}{2})\) \text{ Rectangular Coordinates}
b) \((-1, 1) \rightarrow \text{Polar Coordinates}\)

\[X = -1, \quad Y = 1 \quad \text{and from formulas}\]

\[
\tan(\theta) = \frac{Y}{X} = \frac{1}{-1} = -1
\]

\[
r^2 = x^2 + y^2 = (-1)^2 + (1)^2 = 2
\]

\[\Rightarrow \tan(\theta) = -1 \quad \text{and} \quad r^2 = 2
\]

\[\text{This equation has two solutions!}
\]

\[\text{use arctangent function (Be Careful)}
\]

\[
tan(\theta) = -1 \rightarrow \theta = \tan^{-1}(-1) = -\frac{\pi}{4}
\]

1. either \(r < 0 \Rightarrow r = -\sqrt{2}
\]

\[\rightarrow \left( -\sqrt{2}, -\frac{\pi}{4} \right)
\]

2. or add \(\pi\) to angle obtained from arctangent \(\Rightarrow \theta = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}
\]

\[\rightarrow \left( \sqrt{2}, \frac{3\pi}{4} \right)
\]