Announcements  1/28/19

- Quizzes back on Wednesday.
- Today finish 4.5 & 4.6
- Sometime tomorrow I’ll post an exam study guide on the webpage.
- Wednesday Review, then start 4.7.
4.5: Graphs of Sine & Cosine Functions (CONTINUED)

* We finish our discussion on the graphs of Sine and cosine functions of the form

\[ y = A \sin(Bx - C) + D \]

\[ y = A \cos(Bx - C) + D \]

**Last Time**

* Amplitude = \( |A| \) \( \leftarrow \) Vertical Stretch/Shrink

* Period = \( \frac{2\pi}{B} \) \( \leftarrow \) Horizontal Stretch/Shrink (remember \( B > 0 \))

* Phase Shift = \( \frac{C}{B} \) \( \leftarrow \) Horizontal Shift
  \( C > 0 \) shift to right
  \( C < 0 \) shift to left

* One Cycle Interval = \( \left[ \frac{C}{B}, \frac{C}{B} + \frac{2\pi}{B} \right] \) \( \leftarrow \)

Interval over which one cycle of the graph takes place.
* D corresponds to a **Vertical Shift** of the basic sine and cosine curves.

- If D > 0 → Vertical shift is D units up.
- If D < 0 → Vertical shift is D units down.

* Instead of oscillating about the x-axis (the horizontal line y=0), the graphs of the eqn’s.

\[ y = A \sin(Bx - C) + D \]
\[ y = A \cos(Bx - C) + D \]

oscillate about the horizontal line \( y = D \).

**Example (Vertical Shift)**

Determine the amplitude, period, phase shift and vertical shift of

\[ y = -\frac{1}{2} \sin(\pi x + \pi) + 2 \]

and sketch the graph.
Here we have:

\[ A = -\frac{1}{2} \]
\[ B = \pi \]
\[ C = -\pi \]
\[ D = 2 \]

So that:

\[ \text{Amplitude} = \frac{1}{2} \]
\[ \text{Period} = \frac{2\pi}{\pi} = 2 \]
\[ \text{Phase Shift} = -\frac{\pi}{\pi} = -1 \]
\[ \text{Vertical Shift} = 2 \text{ (units up)} \]

And the graph of \( y = -\frac{1}{2} \sin(\pi x + \pi) + 2 \) completes one cycle over the interval:

\[ \left[ \frac{c}{B}, \frac{c}{B} + \frac{2\pi}{B} \right] = [-1, 1] \]
The five key points over the one cycle interval $[-1, 1]$ are

- $x = -1 \rightarrow (-1, 2)$
- $x = -\frac{1}{2} \rightarrow (-\frac{1}{2}, \frac{3}{2})$
- $x = 0 \rightarrow (0, 2)$
- $x = \frac{1}{2} \rightarrow (\frac{1}{2}, \frac{5}{2})$
- $x = 1 \rightarrow (1, 2)$
Key points

\[ y = -\frac{1}{2} \sin(\pi x + \pi) + 2 \]

\((-1, 2)\) ✓

\((-\frac{1}{2}, 3\frac{1}{2})\) ✓

\((0, 2)\) ✓

\((\frac{1}{2}, 3\frac{1}{2})\) ✓

\((1, 2)\)
A How to use your Calculator to graph Trigonometric Functions:

- Make sure your Calculator is in Radian Mode.
- In WINDOW:
  - Set $X_{\text{min}}$ & $X_{\text{max}}$ based on how many cycles you'd like to graph.
  - Set $X_{\text{scale}}$ based on the distance between the $X$-values in your key points.
  - Set $Y_{\text{min}}$ & $Y_{\text{max}}$ based on the amplitude and vertical shift.
  - Set $Y_{\text{scale}}$ based on the Amplitude.
- Enter the equation in $Y = $

Let's see how to do this with our last example.
4.16: Graphs of Other Trig. Functions

Now we look at the graphs of tangent, cotangent, secant and cosecant functions.

Graph of the Tangent Function

\[ y = \tan(x) \]
As $x$ increases from 0 to $\frac{\pi}{2}$, $\tan(x)$ increases from 0 to $\infty$. As $x$ decreases from 0 to $-\frac{\pi}{2}$, $\tan(x)$ decreases from 0 to $-\infty$.

The same behavior occurs at each value of $x$ not in the domain of $y = \tan(x)$.

The graph of $y = \tan(x)$ has a vertical asymptote at each vertical line $x = \frac{\pi}{2} + n\pi$ for $n$ an integer.

Since $\tan(x)$ increases and decreases without bound,

The range of $y = \tan(x)$ is $(-\infty, \infty)$.
Properties of the graph of \( y = \tan(x) \)

- The graph of \( y = \tan(x) \) completes one cycle over the interval \((-\frac{\pi}{2}, \frac{\pi}{2})\).
  
  The period of \( \tan(x) \) is \( \pi \)

- The key points on the graph of \( y = \tan(x) \) over the one cycle interval \((-\frac{\pi}{2}, \frac{\pi}{2})\) are

  \((-\frac{\pi}{4}, -1), (0, 0), (\frac{\pi}{4}, 1)\)

- From the identity \( \tan(x) = \frac{\sin(x)}{\cos(x)} \), we know that \( \tan(x) \) is not defined at values of \( x \) for which \( \cos(x) = 0 \). The domain of \( \tan(x) \) is all real numbers \( x \neq \frac{\pi}{2} + n \pi \) for \( n \) an integer.
A sketching the graph of a tangent function of the form

\[ y = A \tan(Bx - C) \]

is similar to sketching the graph of \( y = A \sin(Bx - C) \).

In the case of tangent, \( A \) still corresponds to a vertical stretch/shrink. However, we do not call \(|A|\) the amplitude since \( \tan(x) \) increases and decreases without bound over every cycle.

\[ \text{Period} = \frac{\pi}{B} \quad \text{if } B > 0 \]

\[ \text{Phase Shift} = \frac{C}{B} \quad \text{if } C > 0 , \text{ to right} \]
\[ \text{if } C < 0 , \text{ to left} \]

\[ \text{One Cycle Interval} = \left( \frac{C}{B} - \frac{\pi}{2B}, \frac{C}{B} + \frac{\pi}{2B} \right) \]

\[ \text{Found by solving the eqn:} \]
\[ Bx - C = -\frac{\pi}{2} \]

\[ \text{or equivalently:} \]
\[ Bx - C = \frac{\pi}{2} \]
Example (Graphing Tangent Functions)

Determine the period, phase shift and one cycle interval of

\[ y = -3 \tan \left( 2x + \frac{\pi}{2} \right) \]

and sketch two cycles of the graph.

Here

\[ A = -3 \]
\[ B = 2 \]
\[ C = -\frac{\pi}{2} \]

So

\[
\begin{align*}
\text{Period} &= \frac{\pi}{B} = \frac{\pi}{2} \\
\text{Phase shift} &= \frac{C}{B} = -\frac{\pi}{4} \\
\text{One cycle interval} &= \left( \frac{C}{B} - \frac{\pi}{2B}, \frac{C}{B} + \frac{\pi}{2B} \right) = \left( -\frac{\pi}{2}, 0 \right)
\end{align*}
\]
Let's determine the three key points of the graph of 
\[ y = -3 \tan(2x + \frac{\pi}{2}) \] over the one cycle interval \((-\frac{\pi}{2}, 0)\).

Again, we break the one cycle interval into 4 equal parts, but we exclude the endpoints:

- To do this, divide the period by 4, then add the result to the left endpoint \(-\frac{\pi}{2}\) three times consecutively.

\[
\frac{\text{Period}}{4} = \frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}
\]

\[
-\frac{\pi}{2} + \frac{\pi}{8} = -\frac{3\pi}{8}
\]

\[
-\frac{3\pi}{8} + \frac{\pi}{8} = -\frac{\pi}{4}
\]

\[
-\frac{\pi}{4} + \frac{\pi}{8} = -\frac{\pi}{8}
\]
So the key points over the arc cycle interval are:

\[ x = -\frac{3\pi}{8} \quad \longrightarrow \quad \left( -\frac{3\pi}{8}, 3 \right) \]

\[ x = -\frac{\pi}{4} \quad \longrightarrow \quad \left( -\frac{\pi}{4}, 0 \right) \]

\[ x = -\frac{\pi}{8} \quad \longrightarrow \quad \left( -\frac{\pi}{8}, -3 \right) \]

\[ y = -3\tan \left( 2x + \frac{\pi}{4} \right) \]

* Now use the key points to sketch the graph of \( y = -3\tan \left( 2x + \frac{\pi}{4} \right) \) over the arc cycle interval \((-\frac{\pi}{2}, 0\)), then sketch more cycles by extending in either direction.
Key Points

\[ y = -3 \tan \left( 2x + \frac{\pi}{2} \right) \]

\[ \left( -\frac{3\pi}{8}, 3 \right) \]

\[ \left( -\frac{\pi}{4}, 0 \right) \]

\[ \left( -\frac{\pi}{6}, -3 \right) \]

\[ y \]

\[ x \]
Example (Graphing a Cosine Function)

Determine the Amplitude, period, phase shift and vertical shift of

\[ y = 3 \cos(x + \pi) - 1 \]

And graph One cycle.

Amplitude = 3

Period = \(2\pi\)

Phase shift = \(-\pi\)

Vertical shift = \(-1\) (1 unit down)

One cycle interval = \([-\pi, \pi]\)

\[ \checkmark \]
Key Points on \((-\pi, \pi]\)

\[
\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2}
\]

- \(x = -\pi \rightarrow (-\pi, 0)
- \(x = -\frac{\pi}{2} \rightarrow (-\frac{\pi}{2}, -1)
- \(x = 0 \rightarrow (0, -1)
- \(x = \frac{\pi}{2} \rightarrow (\frac{\pi}{2}, -1)
- \(x = \pi \rightarrow (\pi, 0)
\]
$y = 3 \cos(x + \pi) - 1$
Properties of the graph
of \( y = \cot(x) \)

- One cycle of the graph of \( y = \cot(x) \) occurs over the interval \((0, \pi)\).
- The period of \( y = \cot(x) \) is \( \pi \).
- The key points of the graph of \( y = \cot(x) \) over one cycle are \((\pi/4, 1), (\pi/2, 0), (3\pi/4, -1)\).
- The domain of \( y = \cot(x) \) is all real numbers except
  \[ x \neq n\pi \quad \text{for } n \text{ an integer} \]
- The vertical asymptotes of the graph of \( y = \cot(x) \) are the vertical lines
  \[ x = n\pi \quad \text{for } n \text{ an integer} \]
- The range of \( y = \cot(x) \) is \((-\infty, \infty)\).
Graph of the Cotangent Function

using the identity \[ \cot(x) = \frac{\cos(x)}{\sin(x)} \]

and similar reasoning as in the case of \( y = \tan(x) \) we find:

\[ y = \cot(x) \]
When graphing a cotangent function

\[ y = A \cot(Bx - C) \]

\[ \rightarrow \text{period} = \frac{\pi}{B} \quad \text{←} \quad B > 0 \]

\[ \rightarrow \text{phase shift} = \frac{C}{B} \quad \text{←} \quad C > 0, \text{ right} \quad C < 0, \text{ left} \]

\[ \rightarrow \text{one cycle interval} = \left( \frac{C}{B}, \frac{C}{B} + \frac{\pi}{B} \right) \]

\[ \text{Found by solving the eqn.} \quad Bx - C = 0 \]

\[ \text{Found by solving the eqn.} \quad Bx - C = \pi \]

**Example (Graphing a Cotangent Function)**

Determine the period, phase shift and one cycle period of

\[ y = 2 \cot\left(\frac{x}{3}\right) \]

and graph two cycles.
Here

\[ A = -2 \]
\[ B = \frac{1}{3} \]
\[ c = 0 \]

So we know

\[
\begin{align*}
\text{period} &= \frac{\pi}{B} = 3\pi \\
\text{phase shift} &= \frac{c}{B} = 0 \\
\text{one cycle interval} &= (0, 3\pi)
\end{align*}
\]

Now find the

To graph \( y = 2 \cot \left( \frac{x}{3} \right) \) find the three key points over the one cycle interval \((0, 3\pi)\)

\[
\begin{align*}
\text{at } x &= \frac{3\pi}{4} \\
\text{at } x &= \frac{3\pi}{2} \\
\text{at } x &= \frac{9\pi}{4}
\end{align*}
\]

\[
\begin{align*}
(3\pi, 2) \\
(3\pi, 0) \\
(9\pi, -2)
\end{align*}
\]
Key points of $y = 2 \cot(3x)$:

- $\left(\frac{3\pi}{4}, 2\right)$
- $\left(\frac{3\pi}{2}, 0\right)$
- $\left(\frac{9\pi}{4}, -2\right)$
The Graphs of Cosecant $\&$ Secant Functions

To sketch the graphs of $y = \csc(x)$ $\&$ $y = \sec(x)$, we can use the graphs of $y = \sin(x)$ $\&$ $y = \cos(x)$.

Because of the reciprocal identities

\[
\csc(x) = \frac{1}{\sin(x)} \quad \text{and} \quad \sec(x) = \frac{1}{\cos(x)}
\]

At a given value of $x$, the corresponding $y$-coordinate on the graph of $\csc(x)$ is the reciprocal of the $y$-coordinate on the graph of $\sin(x)$.

When $\sin(x) = 0$, $\csc(x)$ is undefined. And we get vertical asymptotes.

At a given value of $x$, the corresponding $y$-coordinate on the graph of $\sec(x)$ is the reciprocal of the $y$-coordinate on the graph of $\cos(x)$.

When $\cos(x) = 0$, $\sec(x)$ is undefined. And we get vertical asymptotes.
**y = csc(x)**

**Period:** \(2\pi\)

**Range:** \((-\infty, -1] \cup [1, \infty)\)

**Domain:** All real numbers except \(x \neq n\pi\) for \(n\) an integer.

**Vertical Asymptotes:**
At vertical lines \(x = n\pi\), \(n\) an integer.
\[ y = \sec(x) \]

**Period:** \( 2\pi \)

**Domain:** All real numbers \( x \neq \frac{\pi}{2} + n\pi \), \( n \) an integer

**Range:** \( (-\infty, -1] \cup [1, \infty) \)

**Vertical Asymptotes**

Vertical lines \( x = \frac{\pi}{2} + n\pi \), \( n \) an integer
Example (Graphing Cosecant functions)

Sketch two cycles of the graph of

\[ y = 2 \csc(2x - \pi) \]

* Begin by sketching the graph of

\[ y = 2 \sin(2x - \pi) \]

Here we have:

- \( A = 2 \)
- \( B = 2 \)
- \( C = \pi \)

So we get:

- Amplitude = \(|A| = 2\)
- Period = \( \frac{2\pi}{B} = \pi \)
- Phase Shift = \( \frac{C}{B} = \frac{\pi}{2} \)

One Cycle = \([\frac{\pi}{2}, \frac{3\pi}{2}]\)

Interval
The key points on the graph of $y = 2 \sin(2x - \pi)$ over the one-cycle interval $\left[ \frac{\pi}{4}, \frac{3\pi}{2} \right]$ are:

- $\frac{\pi}{4}$

- $x = \frac{\pi}{2} \rightarrow (\frac{3\pi}{4}, 2)$

- $x = \frac{3\pi}{4} \rightarrow (\frac{3\pi}{4}, 2)$

- $x = \pi \rightarrow (\frac{5\pi}{4}, -2)$

- $x = \frac{5\pi}{4} \rightarrow (\frac{5\pi}{4}, -2)$

- $x = \frac{3\pi}{2} \rightarrow (\frac{3\pi}{2}, 0)$

- $x = \frac{5\pi}{2} \rightarrow (\frac{3\pi}{2}, 0)$

Now sketch two cycles of $y = 2 \sin(2x - \pi)$.

From this we can determine the graph of $y = 2 \csc(2x - \pi)$. 
Key points of $y = 2 \sin(2x - \pi)$

- $\left(\frac{3\pi}{4}, 2\right)$
- $\left(\pi, 0\right)$
- $\left(\frac{5\pi}{4}, -2\right)$
- $\left(\frac{3\pi}{2}, 0\right)$

Asymptotes at zeros of $y = 2 \sin(2x - \pi)$

$y = 2 \csc(2x - \pi)$