No graphing calculators or notes allowed during the exam. A scientific calculator may be used. You have 1 hour to complete the exam. **Show all of your work.** Good Luck.

**Question 4:** __________________ (20 points)
**Question 5:** __________________ (20 points)
**Question 6:** __________________ (10 points)
Total: __________________ (50 points)

(Do not write in these spaces.)

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**Information/Formulas Which May be Useful:**

\( r \) is a zero of a polynomial \( f \) \( \iff \) \((x - r)\) is a factor of \( f \)

\[ f(f^{-1}(x)) = x \text{ for all } x \text{ in the domain of } f^{-1} \]
\[ f^{-1}(f(x)) = x \text{ for all } x \text{ in the domain of } f \]

\[ a^{\log_a(M)} = M \quad \text{ log }_a(a^r) = r \quad \text{ log }_a(M) = \frac{\log_b(M)}{\log_b(a)} \]

\[ \log_a(M^r) = r \cdot \log_a(M) \]
\[ \log_a \left( \frac{M}{N} \right) = \log_a(M) - \log_a(N) \]

\[ a^r = e^{r \ln(a)} \]
\[ \log_a (M \cdot N) = \log_a(M) + \log_a(N) \]

\[ A = P \left(1 + \frac{r}{n}\right)^{n \cdot t} \]
\[ A = Pe^{r \cdot t} \]

\[ r_e = e^r - 1 \]
\[ r_e = \left(1 + \frac{r}{n}\right)^n - 1 \]
4. Consider the functions
\[ f(x) = \frac{6}{x-3} \quad \text{and} \quad g(x) = \frac{3}{x} \]

(a) (3 points) Find \( f \circ g (2) \).

\[
f \circ g(2) = f(g(2)) = \frac{6}{\left(\frac{3}{2}\right) - 3} = \frac{6}{\left(\frac{3-6x}{2}\right)} = -4
\]

(b) (6 points) Determine the domain of \( f \circ g \).

- **domain of \( f \):** the denominator of \( f \) cannot be zero
  \[ x - 3 \neq 0 \quad \Rightarrow \quad x \neq 3 \]
  So the domain of \( f \) is \((-\infty, 3) \cup (3, \infty)\).

- **domain of \( g \):** the denominator of \( g \) cannot be zero
  \[ x \neq 0 \]
  So the domain of \( g \) is \((-\infty, 0) \cup (0, \infty)\).

The domain of \( f \circ g \) is all \( x \) in the domain of \( g \) such that \( g(x) \) is in the domain of \( f \)
\[ x \neq 0 \quad \text{and} \quad \frac{3}{x} \neq 3 \quad \Rightarrow \quad x \neq 0, 1 \]
So the domain of \( f \circ g \) is
\[ (-\infty, 0) \cup (0, 1) \cup (1, \infty) \]

(c) (6 points) Find a general formula for \( f \circ g (x) \).

\[
f \circ g(x) = f(g(x)) = \frac{6}{\left(\frac{3}{2}\right) - 3} = \frac{6}{\left(\frac{3-6x}{2}\right)} = \frac{6x}{3 - 6x} = \frac{2x}{1 - 2x}
\]

(d) (5 points) Find two functions \( F \) and \( G \) so that \( F \circ G = H \), when \( H(x) = \frac{1}{\sqrt{x - 4}} \)

Two functions that work are: \( F(x) = \frac{1}{x} \) and \( G(x) = \sqrt{x - 4} \).
5. (a) (4 points) Use properties of logarithms to write the following expression as an integer.

\[ 5 \log_5(18) - \log_5(6) \]

\[ 5 \log_5(18) - \log_5(6) = 5 \log_5 \left( \frac{18}{6} \right) \]

\[ = 5 \log_5(3) \]

\[ = 3 \]

(b) (4 points) Use properties of logarithms to write the following expression as a single logarithm.

\[ \frac{1}{2} \log_a (3x - 2) - 3 \log_a ((x + 1)^2) + \log_a (x^4) \]

\[ \frac{1}{2} \log_a (3x - 2) - 3 \log_a ((x + 1)^2) + \log_a (x^4) \]

\[ = \log_a \left( (3x - 2)^{1/2} \right) - \log_a \left( ((x + 1)^2)^3 \right) + \log_a (x^4) \]

\[ = \log_a \left( \sqrt{3x - 2} \right) - \log_a \left( (x + 1)^6 \right) + \log_a (x^4) \]

\[ = \log_a \left( \frac{\sqrt{3x - 2}}{(x + 1)^6} \right) + \log_a (x^4) \]

\[ = \log_a \left( \frac{x^4 \sqrt{3x - 2}}{(x + 1)^6} \right) \]
(c) (6 points) Solve the following exponential equation. (Do not use your calculator to approximate. Give an exact answer.)

\[ 3^{1-2x} - 4^x = 0 \]

\[
\begin{align*}
3^{1-2x} - 4^x &= 0 \\
\Rightarrow 3^{1-2x} &= 4^x \\
\Rightarrow \ln (3^{1-2x}) &= \ln (4^x) \\
\Rightarrow (1 - 2x) \ln(3) &= x \ln(4) \\
\Rightarrow \ln(3) - 2x \ln(3) &= x \ln(4) \\
\Rightarrow \ln(3) &= x \ln(4) + 2x \ln(3) \\
\Rightarrow \ln(3) &= x(\ln(4) + 2 \ln(3)) \\
\Rightarrow \ln(3) &= x \left( \ln(4) + \ln (3^2) \right) \\
\Rightarrow \ln(3) &= x \ln (4 \cdot 3^2) \\
\Rightarrow \ln(3) &= x \ln(36) \\
\Rightarrow x &= \frac{\ln(3)}{\ln(36)}
\end{align*}
\]

(d) (6 points) Solve the following logarithmic equation.

\[ \log_{\frac{1}{3}} (x^2 + x) - \log_{\frac{1}{3}} (x^2 - x) = -1 \]

- First we look for the potential solutions:

\[
\begin{align*}
\log_{\frac{1}{3}} (x^2 + x) - \log_{\frac{1}{3}} (x^2 - x) &= -1 \\
\Rightarrow \log_{\frac{1}{3}} \left( \frac{x^2 + x}{x^2 - x} \right) &= -1 \\
\Rightarrow \frac{x^2 + x}{x^2 - x} &= \left( \frac{1}{3} \right)^{-1} \\
\Rightarrow \frac{x^2 + x}{x^2 - x} &= 3 \\
\Rightarrow x^2 + x &= 3(x^2 - x) \\
\Rightarrow x^2 + x &= 3x^2 - 3x \\
\Rightarrow 2x^2 - 4x &= 0 \\
\Rightarrow 2x(x - 2) &= 0 \\
\Rightarrow 2x = 0 \text{ or } x - 2 = 0 \\
\Rightarrow x = 0 \text{ or } x = 2
\end{align*}
\]

Now we check \( x = 0 \) and \( x = 2 \) in the original equation:

Continued on next page ——>
When \( x = 0 \) on the LHS of the equation

\[
\log_{\frac{1}{3}}(0) - \log_{\frac{1}{3}}(0)
\]

However, \( \log_{\frac{1}{3}}(0) \) is undefined so \( x = 0 \) is not a solution!

When \( x = 2 \) on the LHS of the equation

\[
\log_{\frac{1}{3}}(2^2 + 2) - \log_{\frac{1}{3}}(2^2 - 2) = \log_{\frac{1}{3}}(6) - \log_{\frac{1}{3}}(2)
\]

\[
= \log_{\frac{1}{3}} \left( \frac{6}{2} \right)
\]

\[
= \log_{\frac{1}{3}} (3)
\]

\[
= -1
\]

So \( x = 2 \) is the only solution to the equation.
6. (10 points) Which of the two rates would yield a larger return after one year:

- 5.9% compounded continuously or 6% compounded monthly?

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Effective Interest Rate Calculation</th>
<th>Effective Interest Rate</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.9% compounded continuously</td>
<td>( r_e = e^{0.059} - 1 \approx 0.06077524704... )</td>
<td>0.0608%</td>
<td></td>
</tr>
<tr>
<td>6% compounded monthly</td>
<td>( r_e = \left(1 + \frac{0.06}{12}\right)^{12} - 1 \approx 0.06167781186... )</td>
<td>0.0617%</td>
<td></td>
</tr>
</tbody>
</table>

The rate of 6% compounded monthly will yield a larger return after 1 year.