Chapter 3 - Polynomial and Rational Functions.

3.1 Polynomial Functions and Models

Definition: A polynomial function is is a function of the form

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0, \]

where \( a_n, a_{n-1}, \ldots, a_0 \) are constants and \( n \) is a nonnegative integer.

Notation: coefficients, leading term, leading coefficient, constant term, degree, standard form, smooth function

Definition: A power function is is a function of the form \( f(x) = ax^n \), where \( a \) is a constant and \( n > 0 \) is an integer.

examples: even power functions vs. odd power functions

Definition: The number \( r \) is a zero (or root) of a function \( f \) in case \( f(r) = 0 \). If \( r \) is a zero of \( f \), then the following statements are equivalent:

1. \( r \) is a zero of the function \( f \).
2. \( (r, 0) \) is an \( x \)-intercept of the graph of \( f \).
3. \( x - r \) is a factor of \( f \).
4. \( r \) is a solution to the equation \( f(x) = 0 \).

Definition: If \( (x - r)^m \) is a factor of a polynomial \( f \), and \( (x - r)^{m+1} \) is not a factor of \( f \), then \( r \) is called a zero of multiplicity \( m \) of \( f \).

Graphing aids: If \( r \) is a zero of even multiplicity, then the sign of \( f \) does not change on either side of \( x = r \), and the graph of \( f \) touches the \( x \)-axis at \( r \). If \( r \) is a zero of odd multiplicity, then the sign of \( f \) changes on either side of \( x = r \), and the graph of \( f \) crosses the \( x \)-axis at \( r \).

Turning points: points at which a graph changes direction (yielding local maximum and local minimum points). A polynomial of degree \( n \) has at most \( n - 1 \) turning points.

End behavior: For large values of \( x \) (either positive or negative) the graph of the function \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \) behaves like the graph of \( f(x) = a_n x^n \).