7.5 - Multiplying with More Than One Term and Rationalizing Denominators

★ We continue to sharpen our algebraic skills for working with radical expressions. We practice distributing multiplication over radical expressions and develop a very useful technique known as **rationalizing denominators**.

**GOAL(S):** Be able to distribute multiplication over radical expressions. Be able to rationalize denominators.

**Example** (Distribute Multiplication over Radicals) Multiply:

(a) \( \sqrt{6}(x + \sqrt{10}) \)

(b) \( \frac{3}{\sqrt{2}} \left( \frac{3}{\sqrt{2}^2} - \frac{3}{\sqrt{7}} \right) \)

(c) \( (6\sqrt{5} + 3\sqrt{2})(2\sqrt{5} - 4\sqrt{2}) \)

**Example** (Distribute Multiplication over Radicals) Multiply:

(a) \( (\sqrt{5} + \sqrt{6})^2 \)

(b) \( (\sqrt{5} + \sqrt{6})(\sqrt{5} - \sqrt{6}) \)

(c) \( (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) \)

★ When given an expression containing radicals in the denominator it sometimes useful to rewrite the expression as an equivalent expression with no radicals in the denominator. This is known as **rationalizing the denominator**.

**Example** (Rationalizing Denominators) Rationalize the denominators:

(a) \( \frac{\sqrt{3}}{\sqrt{7}} \)

(b) \( \frac{\sqrt{2}}{\sqrt{9}} \)

(c) \( \frac{\sqrt{7}}{\sqrt{25}} \)

**Example** (Rationalizing Denominators) Rationalize the denominators. Assume all variables represent positive real numbers.

(a) \( \frac{\sqrt{2x}}{\sqrt{7y}} \)

(b) \( \frac{\sqrt[3]{x}}{\sqrt[3]{9y}} \)

(c) \( \frac{5\sqrt[3]{6x}}{\sqrt[3]{8x^2y^4}} \)

★ To rationalize a denominator containing two terms we multiply the numerator and denominator by the conjugate of the denominator.

**Example** (Rationalizing Denominators w/ Two Terms) Rationalize each denominator.

(a) \( \frac{8}{3\sqrt{2} + \sqrt{4}} \)

(b) \( \frac{3 + \sqrt{7}}{\sqrt{5} + \sqrt{4}} \)

(c) \( \frac{2 + \sqrt{5}}{\sqrt{6} - \sqrt{2}} \)

**Example** (Rationalizing the Numerator) Rationalize the numerator. Assume all variable represent non-negative real numbers:

\( \frac{\sqrt{x} + 3 - \sqrt{x}}{3} \)