7.4 - Adding, Subtracting and Dividing Radical Expressions

In this section we continue to develop the algebra of radical expressions, building off of the work of previous sections.

GOAL(S): Be able to add and subtract radical expressions. Use the quotient rule (for radical expressions) to divide and simplify radical expressions.

Example (Adding/Subtracting Radical Expressions) Simplify by combining like radicals:

(a) \(7\sqrt{2} + 8\sqrt{2}\)  
(b) \(9\sqrt{7} - 6x\sqrt{7} + 12\sqrt{7}\)  
(c) \(8\sqrt{5x} - 5\sqrt{5x} + 4\sqrt{5x}\)

It may be necessary to simplify one or more radicals before we notice that we have like radicals. So always simplify radicals before attempting to add or subtract.

Example (Adding/Subtracting Radical Expressions) If possible, simplify by combining like radicals:

(a) \(3\sqrt{20} + 5\sqrt{45}\)  
(b) \(7\sqrt{18} + 5\sqrt{8}\)  
(c) \(4\sqrt{27x} - 8\sqrt{12x}\)  
(d) \(8\sqrt{5} - 6\sqrt{2}\)

Example (Adding/Subtracting Radical Expressions) If possible, simplify by combining like terms.

(a) \(3\frac{3}{\sqrt{24}} - 5\frac{3}{\sqrt{81}}\)  
(b) \(5\frac{3}{\sqrt{x^2y}} + \frac{3}{\sqrt{27x^3y^4}}\)

The Quotient Rule for Radicals

If \(\sqrt{a}\) and \(\sqrt{b}\) are real numbers and \(b \neq 0\), then

\[\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}}\]

Example (Using the Quotient Rule) Simplify by using the quotient rule. Assume all variables represent positive real numbers.

(a) \(\frac{\sqrt{24}}{\sqrt{125}}\)  
(b) \(\frac{\sqrt{9x^3y}}{\sqrt{y^{10}}}\)  
(c) \(\frac{\sqrt{8y^2}}{\sqrt{x^{12}}}\)

Example (Using the Quotient Rule) Simplify, if possible, using the quotient rule. Assume all variables represent positive real numbers.

(a) \(\frac{\sqrt{40x^5}}{\sqrt{2x}}\)  
(b) \(\frac{\sqrt{50xy}}{2\sqrt{2}}\)  
(c) \(\frac{\sqrt{48y^2}}{\sqrt{6xy^{-2}}}\)