6.7 - Formulas and Applications of Rational Equations

We now put our knowledge of rational expressions and their algebraic manipulation to work. We learn to isolate specific variables in formula involving rational expressions and how to solve “word” problems involving rates.

GOAL(S): Be able to solve a formula for a variable. Be able to set-up and solve problems involving rates.

Example (Solving for a Variable) The formula

\[ r = \frac{d}{t} \]

relates the average speed of an object, \( r \), to the total distance travelled by the object, \( d \), over a given period of time \( t \). Solve the formula for both \( d \) and \( t \).

Example (Solving for a Variable) Optometrists use a measurement of a lens, called the focal length, to analyze glasses. When an object is in focus, the formula

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]

relates the focal length, \( f \), of the lens, to the distance from the lens to the object, \( p \), and the distance to your retina to the lens \( q \). Solve the formula for \( p \).

Example (Solving for a Variable) The formula

\[ S = \frac{C}{1 - r} \]

is used to relate a product’s selling price, \( S \), to its cost to the seller, \( C \), and its marketing rate \( r \). Solve for the marketing rate.

A rate is the ratio of two related quantities. When discussing a rate, usually the word "per" is used to separate the units of measurement describing the rate. For example average speed (miles per hour), hourly wage (dollars per hour) and the price of gasoline (dollars per gallon) are all rates.

Example (Solving a Rate Problem Involving Motion and Time) You commute to work a distance of 40 miles and return on the same route. Your average rate on the return trip is 30 miles per hour faster than your average rate on the outgoing trip. If the round trip takes 2 hours what is your average rate on the outgoing trip to work?
Solving a Problem Involving Rates

1. Let the variable $x$ represent one of the unknown quantities.
2. Represent other unknown quantities in terms of $x$.
3. Write an equation, in the unknown $x$, that models the conditions of the statement of the problem. (It may help to first organize the information you have into a table, then use the information to write your equation.)
4. Solve the equation and answer the question (in complete sentences).

Example (Solving a Rate Problem Involving Motion and Time) A passenger train can travel 240 miles in the same amount of time it takes a freight train to travel 160 miles. If the rate of the freight train is 20 miles per hour slower than the rate of the passenger train, find the average rate of each.

Example (Solving a Rate Problem Involving Work) You can design a website in 15 hours. Your friend can design the same website in 10 hours. Assuming you both work just as efficiently together, how long would it take to build the website if you work together?

Example (Solving a Rate Problem Involving Work) An experienced carpenter can build a staircase 3 times faster than an apprentice can. Working together, they can build a staircase in 6 hours. How long would it take each person working alone to do the job?