6.6 - Rational Equations

Recall that an equation that is true for all values in the domain of the variable(s) is called an identity (or usually, just an equation since the two expressions are actually equal). For example

\[2x(x^2 - 1) = 2x^3 - 2x\]

is an identity since it is true for all values in the domain of the variable \(x\). (The domain of the variable, in this case, is all real numbers).

On the other hand, an equation that is true for just some (or maybe none) of the real numbers in the domain of the variable is called a conditional equation. These are the types of equations we solve. For example

\[x^2 - 9 = 0\]

is a conditional equation since it is only true for certain values in the domain of the variable \(x\) (only true for \(x = -3, 3\) while the domain of the variable is all real numbers).

**GOAL(S):** Be able to solve (conditional) rational equations. Be able to solve problems involving rational functions that model applied situations.

**Example (Solving Rational Equations)**

Solve:

\[\frac{x + 4}{2x} + \frac{x + 20}{3x} = 3\]

**Solving (Conditional) Rational Equations**

1. List the restrictions on the variable. Avoid any values of the variable which make the denominator zero.

2. Clear all denominators from the equation by multiplying both sides by the LCD of all rational expressions occurring in the equation.

3. Solve the resulting equation.

4. Discard any proposed solution that is in the list of restrictions on the variable. Check the other proposed solutions in the original equation.

**Example (Solving Rational Equations)**

Solve:

\[\frac{x + 1}{x + 10} = \frac{x - 2}{x + 4}\]

**Example (Solving Rational Equations)**

Solve:

\[\frac{x}{x - 3} = \frac{3}{x - 3} + 9\]
Example (Solving Rational Equations)

Solve: \[ \frac{2x}{x-3} + \frac{6}{x+3} = -\frac{28}{x^2-9} \]

Example (Solving Rational Equations: Applications)

Recall that a city uses the function

\[ f(x) = \frac{120x}{100-x} \]

to model the cost, \( f(x) \), in thousands of dollars, to remove \( x \)% of a lakes pollutants.

If voters commit $80 thousand to a project to clean up the lake, what percentage of the pollutants can they expect to be removed?