5.5 - Factoring Special Forms

★ In this section we exam some special forms such as the difference of squares, perfect square trinomial and the sum or difference of cubes.

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<th>Difference of Squares</th>
<th>Perfect Square Trinomial</th>
<th>Sum of Cubes</th>
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<td>$x^2 - y^2$</td>
<td>$x^2 + 6x + 9$</td>
<td>$27x^3 + 64$</td>
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GOAL(S): Be able to recognize a difference of squares, a perfect square trinomial and a sum or difference of cubes and correctly apply the factoring formulas associated with each.

The Difference of Squares

If $A$ and $B$ are real numbers, variables of algebraic expressions, then

$$A^2 - B^2 = (A + B)(A - B)$$

Example (Factoring the Difference of Squares)

Factor:

a. $9x^2 - 100$

b. $36y^6 - 49x^4$

Example (Factor Completely: Combining Techniques)

Factor:

a. $81x^4 - 16$

b. $x^3 + 5x^2 - 9x - 45$

Perfect Square Trinomials

If $A$ and $B$ are real numbers, variables of algebraic expressions, then

$$A^2 + 2AB + B^2 = (A + B)^2$$

and

$$A^2 - 2AB + B^2 = (A - B)^2$$

Example (Factoring Perfect Square Trinomials)

Factor:

a. $x^2 + 14x + 49$

b. $4x^2 + 12xy + 9y^2$

c. $9y^4 - 12y^2 + 4$
Example (Factoring: Combining Techniques)

Factor:

a. \( x^2 - 8x + 16 - y^2 \)

b. \( a^2 - b^2 + 10b - 25 \)

Sum or Difference of Cubes

If \( A \) and \( B \) are real numbers, variables of algebraic expressions, then

\[
A^3 + B^3 = (A + B)(A^2 - AB + B^2)
\]

and

\[
A^3 - B^3 = (A - B)(A^2 + AB + B^2)
\]

Example (Factoring the Sum of Cubes)

Factor:

a. \( x^3 + 125 \)

b. \( x^6 + 64y^3 \)

Example (Factoring the Difference of Cubes)

Factor:

a. \( x^3 - 216 \)

b. \( 8 - 125x^3y^3 \)