* Recall: A function $f$ is a correspondence between two sets $X$ and $Y$ that associates to each element $x$ in $X$ exactly one element $y = f(x)$ in $Y$.

- The set of "inputs" $X$ is called the Domain of $f$.
- The set of "outputs" $Y$ is called the Range of $f$.
**Example (Function)**  This correspondence defines a function:

<table>
<thead>
<tr>
<th>Input</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

- Every input has exactly one output.

Domain = \{1, 2, 3, 4, 5\}  
Range = \{9, 7, 10, 13, 16\}

**Non-Example (Not a function)**  This Correspondence is not a function.

<table>
<thead>
<tr>
<th>Input</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>16</td>
</tr>
</tbody>
</table>

- The input 2 has two outputs, so this correspondence cannot define a function.
Each positive real # has 2 square roots, one positive and one negative.

\[ (-5)^2 = 25 \implies 5^2 = 25 \]

However, each non-negative real # has exactly one principal square root.

\[ \sqrt{25} = 5, \quad \sqrt{16} = 4 \]

So we can define the (principal) square root function:

\[ f(x) = \sqrt{x} \]

The domain of \( f \) is \([0, \infty)\).
<table>
<thead>
<tr>
<th>$X$</th>
<th>$f(x) = \sqrt{x}$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$f(0) = \sqrt{0} = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>1</td>
<td>$f(1) = \sqrt{1} = 1$</td>
<td>$(1, 1)$</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = \sqrt{4} = 2$</td>
<td>$(4, 2)$</td>
</tr>
<tr>
<td>9</td>
<td>$f(9) = \sqrt{9} = 3$</td>
<td>$(9, 3)$</td>
</tr>
<tr>
<td>16</td>
<td>$f(16) = \sqrt{16} = 4$</td>
<td>$(16, 4)$</td>
</tr>
</tbody>
</table>
Example (Square Root Functions) Find the indicated function values and determine the domain of the function:

\[ f(x) = \sqrt{5x - 6}, \quad f(2) \]

\[ f(2) = \sqrt{5(2) - 6} = \sqrt{10 - 6} = \sqrt{4} = 2 \]

Domain of \( f \):

Since we can only take the square root of non-negative numbers, we need

\[ 5x - 6 \geq 0 \]

\[ \Rightarrow 5x \geq 6 \]

\[ \Rightarrow x \geq \frac{6}{5} \]

In interval notation:

\[ \left[ \frac{6}{5}, \infty \right) \]

Domain of \( f(x) = \sqrt{5x - 6} \)
Example (Square Root functions) Find the indicated function value and determine the domain of the function:

\[ g(x) = \sqrt{64 - 8x} \quad g(-3) \]

\[ g(-3) = \sqrt{64 - 8(-3)} = \sqrt{64 + 24} = \sqrt{88} \]

**Domain of \( g \): Need**

\[ 64 - 8x \geq 0 \]

\[ \Rightarrow 64 \geq 8x \]

\[ \Rightarrow \frac{64}{8} \geq x \]

\[ \Rightarrow 8 \geq x \]

In interval notation:

\[ (-\infty, 8] \]

Domain of \( g(x) = \sqrt{64 - 8x} \)
Since each real number has exactly one cube root, we can define the cube root function:

\[ f(x) = \sqrt[3]{x} \]

The domain of the cube root function is \((-\infty, \infty)\).

Let's graph \( f(x) = \sqrt[3]{x} \).
<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = \sqrt[3]{x}$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$f(-2) = \sqrt[3]{-8} = -2$</td>
<td>$(-2, -2)$</td>
</tr>
<tr>
<td>-1</td>
<td>$f(-1) = \sqrt[3]{-1} = -1$</td>
<td>$(-1, -1)$</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = \sqrt[3]{0} = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>1</td>
<td>$f(1) = \sqrt[3]{1} = 1$</td>
<td>$(1, 1)$</td>
</tr>
<tr>
<td>8</td>
<td>$f(8) = \sqrt[3]{8} = 2$</td>
<td>$(8, 2)$</td>
</tr>
</tbody>
</table>

$\text{graph of } f(x) = \sqrt[3]{x}$
Example (Cube Root Functions) Find the indicated function values for each function:

a) \( f(x) = \sqrt[3]{x - 2} \), \( f(127) \)

b) \( g(x) = \sqrt[3]{8x - 8} \), \( g(-7) \)

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a) \( f(127) = \sqrt[3]{127 - 2} = \sqrt[3]{125} = 5 \)

b) \( g(-7) = \sqrt[3]{8(-7) - 8} = \sqrt[3]{-64 - 8} = \sqrt[3]{-64} = -4 \)
Example (Simplifying Radical Expressions)

Let's "simplify" each of the following:

a) \( \sqrt{3^2} \)
   \[ \sqrt{3^2} = \sqrt{9} = 3 = |3| \]

b) \( \sqrt{(-6)^2} \)
   \[ \sqrt{(-6)^2} = \sqrt{36} = 6 = |-6| \]
   \[ \text{Not} -6 \]

c) \( \sqrt{x^2} \)
   \[ \sqrt{x^2} = |x| \]

d) \( 3\sqrt{(-y)^3} \)
   \[ 3\sqrt{(-y)^3} = 3\sqrt{-6y} = -y \]
Simplifying $\frac{n\sqrt{a^n}}{}$

for any real $a$:

1) If $n$ is even, then $\frac{n\sqrt{a^n}}{} = |a|$

2) If $n$ is odd, then $\frac{n\sqrt{a^n}}{} = a$

Example (Simplifying Radical Expressions)

Let's simplify each of the following:

a) $\sqrt{(x+5)^2}$

b) $3\sqrt{x^3}$

c) $\sqrt{25x^6}$

d) $\sqrt{x^2 + 4x + 4}$
0) \[ \sqrt{(x+5)^2} = |x+5| \]

b) \[ \frac{3}{\sqrt[3]{x^3}} = x \]

\[ \text{since } n=2 \text{ is even, need abs. value bars} \]

\[ \text{odd, don't need abs. value bars}. \]

\[ \sqrt{25x^6} = \sqrt{(5x^3)^2} = 15x^3 \]

\[ = |15| \cdot |x^3| \]

\[ = 5|x^3| \]

\[ \sqrt{x^2+xy+y} = \sqrt{(x+2)^2} \]

\[ = |x+2| \]
Example (Simplifying Radical Expressions)

Simplify each of the following:

a) \( \sqrt[5]{(-5)^5} \)

b) \( \sqrt[4]{(x-3)^4} \)

c) \( \sqrt[3]{-64x^3} \)

d) \( \sqrt[7]{(2x+7)^7} \)

\[ a) \sqrt[5]{(-5)^5} \quad \Rightarrow \quad (n=5 \text{ if odd}) \quad 4 \]

\[ = -5 \]

\[ b) \sqrt[4]{(x-3)^4} \quad \Rightarrow \quad (n=4 \text{ if even}) \quad 4 \]

\[ = |x-3| \]

\[ c) \sqrt[3]{-64x^3} = \sqrt[3]{(-64x)^3} = -4x \]

\[ d) \sqrt[7]{(2x+7)^7} = 2x+7 \]