10.1 - Distance and Midpoint Formulas; Circles

★ We use the pythagorean theorem to develop the distance formula and the midpoint formula in the plane. The distance formula leads us directly to the standard form and general form of the equation of a circle.

GOAL(S): Find the distance between two points in the plane. Find the midpoint of the line segment joining two points in the plane. Determine the center and radius of a circle whose equation is given in standard form. Graph circles in the plane. Convert the general form of a circles equation to standard form.

The Distance Formula

The distance \( d \) between the points \((x_1, y_1)\) and \((x_2, y_2)\) in the cartesian (rectangular) coordinate system is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Example (Using the distance Formula) Find the distance between the points \((-1, -3)\) and \((2, 3)\). Express the answer in simplified radical form and then round to two decimal places.

The Midpoint Formula

The midpoint of the line segment joining the points \((x_1, y_1)\) and \((x_2, y_2)\) in the cartesian (rectangular) coordinate system is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_2 + y_1}{2} \right)
\]

Example (Using the Midpoint Formula) Find the midpoint of the line segment with endpoints \((1, 2)\) and \((7, -3)\)

Circles

A circle is the set of all points in the plane that are equidistant from a fixed point called the center. The distance from the circle’s center to any point on the circle is called the radius.

★ Let’s convert the geometry of the circle to the equation of the circle!

The Standard Form of the Equation of a Circle

The standard form of the equation of a circle with center \((h, k)\) and radius \(r\) is

\[
(x - h)^2 + (y - k)^2 = r^2
\]

Example (The Standard Form) Write the standard form of the equation of a circle with center \((0, 0)\) and radius 4
**Example** (The Standard Form) Write the standard form of the equation of a circle with center \((5, -6)\) and radius 10

**Example** (Graphing a Circle from the Standard Form) Find the center and radius of the circle whose equation in standard form is

\[(x + 3)^2 + (y - 1)^2 = 4\]

**The General Form of the Equation of a Circle**

The general form of the equation of a circle with center \((h, k)\) and radius \(r\) is

\[x^2 + y^2 + Dx + Ey + F = 0\]

**Example** (Converting from General to Standard Form) Write in standard form:

\[x^2 + y^2 + 4x - 4y - 1 = 0\]

**Example** (Converting from General to Standard Form) Write in standard form:

\[x^2 + y^2 + 8x - 4y + 16 = 0\]