10.1 - Distance and Midpoint Formulas; Circles

★ We develop the distance formula and the midpoint formula in the plane. The distance formula leads use directly to the standard form of the equation of a circle, which then leads to the general form of the equation of a circle.

GOAL(S): Find the distance between two points in the plane. Find the midpoint of the line segment joining two points in the plane. Determine the center and radius of a circle whose equation is given in standard form. Graph circles in the plane. Convert the general form of a circles equation to standard form.

The Distance Formula

The distance \( d \) between the points \((x_1, y_1)\) and \((x_2, y_2)\) in the cartesian (rectangular) coordinate system is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Example (Using the distance Formula) Find the distance between the points \((-1, -3)\) and \((2, 3)\). Express the answer in simplified radical form and then round to two decimal places.

The Midpoint Formula

The midpoint of the line segment joining the points \((x_1, y_1)\) and \((x_2, y_2)\) in the cartesian (rectangular) coordinate system is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Example (Using the Midpoint Formula) Find the midpoint of the line segment with endpoints \((1, 2)\) and \((7, -3)\)

Circles

A circle is the set of all points in the plane that are equidistant from a fixed point called the center. The distance from the circles center to any point on the circle is called the radius.

★ Lets convert the geometry of the circle to the equation of the circle!

The Standard Form of the Equation of a Circle

The standard form of the equation of a circle with center \((h, k)\) and radius \(r\) is

\[
(x - h)^2 + (y - k)^2 = r^2
\]

Example (The Standard Form) Write the standard form of the equation of a circle with center \((0, 0)\) and radius 4
Example (The Standard Form) Write the standard form of the equation of a circle with center \((5, -6)\) and radius 10

Example (Graphing a Circle from the Standard Form) Find the center and radius of the circle whose equation in standard form is

\[(x + 3)^2 + (y - 1)^2 = 4\]

The General Form of the Equation of a Circle

The general form of the equation of a circle with center \((h, k)\) and radius \(r\) is

\[x^2 + y^2 + Dx + Ey + F = 0\]

Example (Converting from General to Standard Form) Write in standard form:

\[x^2 + y^2 + 4x - 4y - 1 = 0\]

Example (Converting from General to Standard Form) Write in standard form:

\[x^2 + y^2 + 8x - 4y + 16 = 0\]