10/1/18  Announcements:

* EXAM  Next week

Wed OCT 10
Covers:
1.6, 1.7, 4.2, 4.3, 5.3, 5.4, 5.5

Homework Due too!

More Info on Exam to Cover:

Outline / Study guide
Try & Post before Friday.
Recall: The absolute value of a real number $a$ is the distance from 0 to $a$ on the number line.

$$\begin{align*}
131 &= 3 \\
|\text{-3}| &= 3
\end{align*}$$

Distance = 3
Today we look at absolute value equations and absolute value inequalities.

Examples of Absolute Value Equations

\[ |x| = 2 \]
\[ |3x - 11| = |x + 5| \]

Examples of Absolute Value Inequalities

\[ |x - 4| < 3 \]
\[ |2x + 3| > 5 \]

Goal(s): Be able to solve absolute value equations and inequalities

How? Rewrite without absolute value bars, then solve.
Example (Solving an Absolute Value Equation)

Solve:  
\[ \text{a) } |x| = 2 \quad \text{b) } |x| = -3 \]

\[ \text{a) To solve } |x| = 2, \text{ means to find all values of } x \text{ whose distance from 0 on the number line is 2.} \]

\[ \text{Exactly two values for } x: \]
\[ x = 2, -2 \]

\[ \Rightarrow \text{ The solutions to } |x| = 2 \text{ are} \]
\[ x = 2, -2 \]
b) To solve \( |x| = -3 \) is to find all real numbers whose distance from 0 on the number line is -3.

Absolute value is always non-negative (positive or zero). So \( |x| = -3 \) has no solutions.

Rewriting an Absolute Value Equation Without Absolute Value Bars

If \( c \) is a positive real number and \( u \) is an algebraic expression, then

\[ |u| = c \quad \text{is equivalent to} \quad u = c \quad \text{or} \quad u = -c \]
If equations are equivalent, then they have the same solutions.

Example (solving an Absolute Value Equation)

\[ |2x - 3| = 11 \]

Think \( n = 2x - 3 \) and \( c = 11 \)

\[ |2x - 3| = 11 \iff \begin{cases} 2x - 3 = 11 \\ 2x - 3 = -11 \end{cases} \]

\( \Rightarrow \) Solve: \( 2x - 3 = 11 \)
\( \Rightarrow \) Solve: \( 2x - 3 = -11 \)
\[ 2x = 11 + 3 \]
\[ \Rightarrow 2x = 14 \]
\[ \Rightarrow x = \frac{14}{2} \]
\[ \Rightarrow x = 7 \]

\[ 2x = -11 + 3 \]
\[ \Rightarrow 2x = -8 \]
\[ \Rightarrow x = \frac{-8}{2} \]
\[ \Rightarrow x = -4 \]

Solutions to \(|2x-11|=3\) are
\[ x = -4 \text{ or } x = 7 \]

What about \(|x-2|=0\)?

\[ x = 2 = 0 \]

When \(c = 0\), same, but since \(0 = -0\)

\[ x - 2 = 0 \text{ or } x - 2 = -0 \]

Same, no need to repeat. Solve one.
Example (Solving an Absolute Value Equation)

Solve: \[ |3x - 11| = |x + 5| \]

* Two abs. Value expressions are equal exactly when the expressions inside the abs. Value bars are equal or opposite.

\[ \begin{align*}
|3x - 11| &= |x + 5| \\
3x - 11 &= x + 5 \quad \text{or} \\
3x - 11 &= -(x + 5) 
\end{align*} \]

\( x = 2 \)

Solution to \( |x - 2| = 0 \) is \( x = 2 \)
\[ \text{Solve: } 3x - 11 = x + 5 \]

\[ \Rightarrow 3x - 11 - x = 5 \]

\[ \Rightarrow 2x = 5 + 11 \]

\[ \Rightarrow 2x = 16 \]

\[ \Rightarrow x = \frac{16}{2} \]

\[ \Rightarrow x = 8 \]

\[ \text{Solutions to } |3x - 11| = |x + 5| \text{ are} \]

\[ x = 8 \text{ or } x = \frac{3}{2} \]

Rewriting an Absolute Value Equation w/o two absolute values w/o Absolute Value bars

If \(|u| = |v|\), then \(u = v\) or \(u = -v\)

To solve \(|u| = |v|\),

Need to solve both

\(u = v\) or \(u = -v\)
What about absolute value inequalities?

Example (Solving an Absolute Value Inequality)

Solve: a) $|x| < 2$  
   b) $|x| < -3$

To solve $|x| < 2$, we need to find all values of $x$ whose distance from 0 on the # line is less than 2.

[Graphically, we get:]

$$-2 < x < 2$$

In interval notation: \((-2, 2)\)

As an inequality without absolute values:

\(-2 < x < 2\)  
No abs. value bars!
b) To solve $|x| < -3$, we need to find all values of $x$ whose distance from $0$ on the number line is less than $-3$.

Absolute value is always non-negative, so $|x| < -3$ has **no solutions**.

Rewriting $|u| < c$ without absolute values:

If $c$ is a positive real number and $u$ represents an algebraic expression, then

$|u| < c$ is equivalent to $-c < u < c$

(also valid if $<$ replaced by $\leq$)
Example (Solving an Absolute Value Inequality)

Solve: \(|x - y| < 3\)

Here \(u = x - y\) and \(c = 3\)

\(|x - y| < 3\)

\(-3 < x - y < 3\)

\(-3 + y < x < 3 + y\)

\(1 < x < 7\)

In interval notation: \((1, 7)\)
Example (Solving an Absolute Value Inequality)

Solve:  
\[ a) \quad |x| > 2 \quad b) \quad |x| < -3 \]

a) To solve \( |x| > 2 \), we need to find all values of \( x \) whose distance from 0 on the number line is greater than 2.

Graphically, we get:

\[ (-\infty, -2) \cup (2, \infty) \]

in interval notation:

inequality without absolute values:

\[ x < -2 \text{ or } x > 2 \]

No Abs. value bars!
b) To solve $|x| > 3$, we need to find all values of $x$ whose distance from 0 on the # line is greater than 3.

Absolute value is always non-negative. So the solutions to $|x| > 3$ are all real #s. In interval notation:

$$(-\infty, \infty)$$

**Rewriting $|u| > c$ without Absolute Values**

If $c$ is a positive real # and $u$ represents an algebraic expression, then

$|u| > c$ is equivalent to $u < -c$ or $u > c$

(Valid if we replace ">" by "\geq")
Example (Solving an Absolute Value Inequality)

Solve: \[ |2x + 3| \geq 5 \]

* Here \( u = 2x + 3 \) \& \( c = 5 \)

\[ |2x + 3| \geq 5 \]

\[ \iff \begin{cases} 2x + 3 \leq -5 \\ 2x + 3 \geq 5 \end{cases} \]

Solve: \[ 2x + 3 \leq -5 \]

\[ \implies 2x \leq -8 \]

\[ \implies x \leq -4 \]

Solve: \[ 2x + 3 \geq 5 \]

\[ \implies 2x \geq 2 \]

\[ \implies x \geq 1 \]

\[ \implies x \leq \frac{5}{2} \]
Solutions to $|2x + 3|^{7/5}$ are:

$x \leq -4$

or

$x \geq 1$

In interval notation:

$(-\infty, -4] \cup [1, \infty)$
5.3: Greatest Common Factors $\$$

Factoring by Grouping

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Recall: A polynomial is a single term or the sum of two or more terms containing variables with whole number exponents.

**Examples of Polynomials**

- Single Variable Polynomial: $-49x^3 + 85x^2 - 2503$

- Two Variable Polynomial: $xy^3 + 12x^2y - 9x + 8y$

* The numbers in front of the variables are called coefficients. In this class, the coefficients will always be integers.*
A polynomial is **simplified** when it contains no like terms.

\[ 2x^3 + 3x^2 - x^2 + y = 2x^3 + 2x^2 + y \]

- Not Simplified
- Simplified

* A simplified polynomial:

\[ \begin{cases} 
\text{- with one term is called a} & \text{monomial} \\
\text{- with two terms is called a} & \text{binomial} \\
\text{- with three terms is called a} & \text{trinomial} 
\end{cases} \]

\[ \begin{array}{c|c|c}
5x^2y & 3x + 4 & -2xy + 7x - y \\
\text{Monomial} & \text{Binomial} & \text{Trinomial} \\
\end{array} \]
To multiply polynomials, we use the distributive property.

\[ 3x^2(2x+1) = 6x^3 + 3x^2 \]

\[ (x+3)(x^2+x) = x^3 + x^2 + 3x^2 + 3x \]
\[ = x^3 + 4x^2 + 3x \]

\[ (x^2+1)(x^3+x+2) = x^5 + x^3 + 2x^2 + x^3 + x + 2 \]
\[ = x^5 + 2x^3 + 2x^2 + x + 2 \]
To factor a polynomial means to find an equivalent expression that is a product.

\[ 21x^2 + 28x = 7x(3x+4) \]

\[ x^3 - 5x^2 + 3x - 15 = (x-5)(x^2 + 3) \]

Think of factoring a polynomial as "reversing" polynomial multiplication.
Goal(s): Be able to find the greatest common factor of a polynomial & be able to factor a polynomial by grouping.

* When factoring a polynomial, the first step is to find the "greatest common factor". The greatest common factor (or GCF) of a polynomial is a monomial expression with greatest coefficient and highest degree which divides each term of the polynomial.
Example (Factoring out the GCF)

Factor out:

\[
\text{GCF}
\]

a) \(21x^2 + 28x\)

b) \(x^3 - 5x^2 + 3x - 15\)

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a) (Step 1) Find the GCF of all terms of \(21x^2 + 28x\)

- Look for largest power of \(x\) which divides all terms:

  \[\Rightarrow x\]

- Look for GCF of the coefficients:

  \[\Rightarrow 7\]
The GCF of the polynomial $21x^2 + 28x$ is: $7x$

**Step 2** Write each term of $21x^2 + 28x$ as a product of $7x$ and another factor:

$21x^2 = (7x)(3x)$

$28x = (7x)(4)$

**Step 3** "Reverse" the distributive property of multiplication to factor out $7x$ from $21x^2 + 28x$:

$21x^2 + 28x = 7x(3x + 4)$

GCF factored out
b) (Step 1) Find the GCF of all terms of \( x^3 - 5x^2 + 3x - 15 \):

- Look for largest power of \( x \) which divides all terms:

  \[ \Rightarrow x^0 = 1 \]

- Look for GCF of all coefficients:

  \[ \Rightarrow 1 \]

\[ \Rightarrow \text{In this case the GCF of the polynomial is } 1 \]

\( \text{factoring out 1 is pointless here} \)

\*[When the GCF of a polynomial is 1, more techniques are needed to begin factoring.]
Factoring Out the GCF

1. Determine the GCF of all terms in the polynomial.

2. Express each term as the product of the GCF and another factor.

3. Use the distributive property to factor out the GCF.
Example (Factor out the GCF)

a) \(9x^5 + 15x^3\)

b) \(16x^2y^3 - 24x^3y^4\)

c) \(12x^5y^4 - 4x^4y^3 + 2x^3y^2\)

d) \(-3x^3 + 12x^2 - 15x\)

\(\text{(Step 1)}\) Find GCF of all terms:

- Largest power of \(x\) dividing all terms: \(x^3\)
- GCF of the coefficients: \(3\)

\(\Rightarrow\) so the GCF is \(3x^3\)

\(\text{(Step 2)}\) Write each term as a product of GCF and another factor:

\(\Rightarrow 9x^5 = (3x^3)(3x^2)\) \(,\) \(15x^3 = (3x^3)(5)\)
(Step 3) "Reverse" the distributive property to factor out GCF:

\[ 9x^5 + 15x^3 = 3x^3(3x^2 + 5) \]

b) \[ 11x^2y^3 - 24x^3y^4 \]

(Step 4) Find GCF of all terms:

\[
\begin{align*}
&\text{Largest power of } x \text{ dividing all terms: } \quad \Rightarrow x^2 \\
&\text{Largest power of } y \text{ dividing all terms: } \quad \Rightarrow y^3 \\
&\text{GCF of the coefficients: } \quad \Rightarrow 1
\end{align*}
\]
So the GCF of the polynomial is \(8x^2y^3\).

**Step 2** Wrote each term as a product of GCF and another factor:

\[16x^2y^3 - 24x^3y^4 = (8x^2y^3)(2)\]

\[-24x^3y^4 = (8x^2y^3)(-3xy)\]

**Step 3** Using the "reverse" distributive property to factor out the GCF:

\[16x^2y^3 - 24x^3y^4 = 8x^2y^3(2 - 3xy)\]
c) \[ 12x^5y^4 - 4x^4y^3 + 2x^3y^2 \]

(Step 1) Find GCF of all terms:

\[ \Rightarrow 2x^3y^2 \]

(Step 2) Write each term as a product of GCF and another factor:

\[ 12x^5y^4 = (2x^3y^2)(6x^2y^2), \quad -4x^4y^3 = (2x^3y^2)(-2xy), \quad 2x^3y^2 = (2x^3y^2)(1) \]

(Step 3) Use distributive property to factor out GCF:

\[ 12x^5y^4 - 4x^4y^3 + 2x^3y^2 = 2x^3y^2(6x^2y^2 - 2xy + 1) \]
(Step 1) Find GCF all terms:

$\Rightarrow 3x$, But we take $-3x$

(Step 2) Work each term as product of GCF and another factor:

$\Rightarrow -3x^3 = (-3x)(x^2)$

$12x^2 = (-3x)(-4x)$

$-15x = (-3x)(5)$

(Step 3) Use distributive property to factor out GCF:

$-3x^3 + 12x^2 - 15x = -3x(x^2 - 4x + 5)$
When the GCF of a polynomial is 1, it still may be possible to factor the polynomial by grouping together terms which have a common factor. This technique is known as factoring by grouping.

Example (Factoring by Grouping)

Factor by Grouping:

\[ x^3 - 5x^2 + 3x - 15 \]

Step 1) Group together terms which have a common (monomial factor):

\[ x^3 - 5x^2 + 3x - 15 = (x^3 - 5x^2) + (3x - 15) \]
Step 2) Factor out the common factor from each "group":

\[ x^3 - 5x^2 + 3x - 15 = (x^3 - 5x^2) + (3x - 15) \]

\[ = x^2(x - 5) + 3(x - 5) \]

(Step 3) Use distributive property to factor out the common binomial factor:

\[ = x^2(x - 5) + 3(x - 5) \]

\[ = (x^2 + 3)(x - 5) \]

\[ \text{factored} \]
Factoring by Grouping

1. Group together terms with common monomial factor.

2. Factor out the common monomial factor from each group.

3. Use the distributive property to factor out the common binomial factor.

* Usually, we form two groups.
* Rearranging terms may be necessary.

* Even so, a common binomial factor may not exist. \( \Rightarrow \) To factor, in this case, we need more techniques.
Example (Factor by Grouping)

\[ 3x^2 + 12x - 2xy - 8y \]

* Carry out same procedure!

\[ 3x^2 + 12x - 2xy - 8y = (3x^2 + 12x) + (-2xy - 8y) \]

\[ = 3x(x + 4) - 2y(x + 4) \]

\[ = (3x - 2y)(x + 4) \]