Firm heterogeneity and acquisition incentives

Alan C. Spearot *
University of California - Santa Cruz
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Abstract

This paper presents a model of acquisitions in which heterogeneous firms acquire cost-lowering assets. Using a variable-elasticity demand system, I show that only mid-productivity firms find an acquisition profitable. In analyzing firm-level data from Compustat, I find evidence for this prediction. In contrast, I also show that as varieties become more substitutable and firms act more like price-takers, high productivity firms are the most likely to acquire. By utilizing Rauch classifications, I find evidence for this prediction within a sample of firms which are most likely to take prices as given when making output decisions.

1 Introduction

Mergers and acquisitions (M&As) are an integral part of industrial reallocation. From the merger of complementary resources to the takeover of inefficient firms, M&As constantly shape and reshape the landscape of domestic and international commerce.

For North America in particular, M&As have served an important role in the reallocation of industry-level resources. For example, Breinlich (2009) shows that acquisitions played

*Email: acespearot@gmail.com. Address: Economics Department, 1156 High Street, Santa Cruz, CA, 95064. Tel.: +1 831 419 2813. I would like to thank Bob Staiger, Charles Engel, John Kennan, Menzie Chinn, Bruce Hansen, Phillip McCalman, Doug Staiger, Mina Kim, Federico Díez and Tor-Erik Bakke for comments. I also thank Jim Anderson for suggesting the use of Rauch classifications. This paper has benefited from presentations at UC-Davis, UC-Santa Cruz, Boston College, Syracuse, Washington State, SCCIE 2007 and EIIT 2006. All remaining errors are my own.
a critical role in the response of firms to the Canada-US free trade agreement. Further, own calculations reveal that M&As are frequently used by a broad cross-section of North American firms. Precisely, over 50% of all agricultural and manufacturing firms engaged in some sort of acquisition behavior over the period 1980-2004, and on average, 21% of all firms acquire in any given year.\(^1\) Overall, firms in North America often view M&As as a viable mode of corporation expansion and reallocation.

Despite the significant amount of reallocation which occurs by M&As, the motivations behind the observed acquisition behavior are broad and much less clear. For example, a popular (and classic) explanation for the observed level of M&A behavior is market power.\(^2\) However, while market power surely plays a role in some cases, the data seem to suggest that other issues may play an equally significant or greater role in acquisition behavior.\(^3\) Indeed, the classic literature has also considered cost-efficiencies alongside market power as a viable motive for acquisitions (Perry and Porter, 1985; Farrell and Shapiro, 1990).

More recently, the literature has added firm-level productivity differences as an important factor affecting acquisition decisions. In particular, Jovanovic and Rousseau (2002) develop a dynamic, closed economy model of capital acquisitions by price-taking firms. In their work, revenues are related one-for-one with installed capital. High productivity firms are willing to produce on a larger scale, and thus have the highest incentive to install additional capital via acquisitions. Within an international context, Nocke and Yeaple (2007) examine how the acquisition and investment behavior of heterogeneous firms respond to various frictions in serving international markets. In their model, firms may wish to acquire the assets of a foreign firm if they are superior to using their own assets in serving a foreign market. In equilibrium, acquisitions are undertaken by the most productive firms as long as capabilities are transferable across borders. If not, greenfield investment is undertaken by the most productive firms, and acquisitions by the least productive.

In this paper, I merge the classic literature on cost-efficiencies with the literature on

\(^1\)Both statistics calculated using the Compustat industrial database. See section three.

\(^2\)See the work of Salant, Switzer and Reynolds (1983) and Deneckre and Davidson (1985) for a classic discussion of the incentives for mergers driven by market power.

\(^3\)If market power were the only reason for acquisitions, we would expect that the ratio of total industry acquisition value to total industry operating income would be fairly large. For example, in a hypothetical merger of two identical cournot competitors into a single firm, the ratio must be at least one half. In contrast, the median of this ratio over SIC4 industries and years is 6.8% (24.2% for the 75th percentile), which is inconsistent with market power being the only motivation for the observed acquisition behavior.
acquisitions by heterogeneous firms. Similar to Nocke and Yeaple (2007), the model involves the trading of assets on a competitive merger market, and within-firm reallocation of production. However, different from Nocke and Yeaple (2007), the main intuition is derived from a purely domestic framework. Within this framework, a number of basic (and unanswered) questions arise. When M&As are a means by which firms can quickly respond uncertain productivity, which firms choose to acquire cost-lowering assets? How do industry level characteristics play into these acquisition decisions? Further, what is the role of cost-reducing acquisitions when acquisitions may occur in multiple locations?

In answering these questions, I make a simple but empirically powerful point. When firms acquire to improve production costs, the characteristics of demand are the critical determinant of acquisition behavior. Specifically, by assuming a demand system with a non-constant elasticity, I show that only mid-productivity firms find an acquisition profitable. By analyzing a 25-year panel of firm-level data from Compustat, I find strong evidence for this prediction. In contrast, as varieties become more substitutable and firms act more like price-takers, I find results similar to Jovanovic and Rousseau (2002) where higher productivity firms are the most likely to acquire. By utilizing a common database (Rauch, 1999), I show that there exists evidence for this prediction within a sample of firms that operate in industries with highly substitutable products.

The setup of the model is fairly simple and one of static acquisition decisions. Firms enter under productivity uncertainty, paying a fixed cost of entry. After paying the fixed cost, firms receive a variety and a "lump" of capital. To embody the stylized fact that it takes longer to build capital than to buy it pre-assembled from somebody else, I assume that no new investment occurs post-entry. Thus, it is assumed that acquisitions are the only way to quickly adjust a firm’s capital stock after productivity is realized. Further, since investment behavior tends to be "lumpy" (Doms and Dunne, 1998), I also assume that firms may either buy the capital of one other firm, or sell the capital of their entire firm. After firms buy and sell within the acquisition market, physical assets are fixed and firms produce varieties for the product market.

The model itself has two distinct features that influence firm-level acquisition behavior.

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4 A more general, dynamic acquisition framework is developed in the appendix.

5 Indeed, "speed is the biggest advantage of M&As over greenfield investment or other entry modes." (OECD, 2001, pg. 36)
By adopting the cost structure of Perry and Porter (1985), acquiring additional capital lowers variable costs, and thus some firms may gain from buying an additional "lump" of capital prior to the product market. Second, as in Melitz and Ottaviano (2005), I assume a utility function which yields a linear demand for each variety. Thus, heterogeneous firms will operate at different demand elasticities, which is an important determinant of the marginal incentive to invest.

By embedding these features into a closed economy model, a sorting result is derived that relates acquisition choices to firm-level productivity. The results of the model show that the least efficient firms sell their capital to higher efficiency firms and exit the market. However, the buying firms are not the most efficient firms. Rather, firms within a mid-range of productivity choose to add capital via acquisitions, with the remaining active firms doing nothing in the acquisition market. The intuition for this result lies at the heart of the assumed demand framework. With linear demand, the highest productivity firms operate on a less-elastic portion of the demand curve, closer to the point where total revenues are maximized. Thus, these firms have insufficient incentives to undertake a cost-lowering acquisition.6

I generalize this result in a number of ways. First, as mentioned above, the degree to which products are substitutable plays a key role in acquisition incentives. As products become more substitutable, firms act more like price-takers. Critically, although the price-taking assumption may imply that firms are small, the flat demand curve functionally provides firms an unbounded market in which to benefit from an acquisition. Thus, high productivity firms, who were previously constrained by bounds on market revenue, now benefit the most from an acquisition.7 Generally, high productivity firms are the most likely to acquire as long as the marginal revenue curve becomes increasingly flat with higher

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6An important corollary of this result is the relationship between a firm’s average $Q$ (ratio of market value to book value) and the marginal incentive to invest, marginal $Q$. The received literature on firm-level investment (Hayashi, 1982) has commonly used the former to proxy for the latter (which is unobservable). Similar to the existing literature, the relationship between productivity and average $Q$ in the present model is monotonic. Thus, the predictions of the model can be restated in terms of average $Q$ rather than productivity. With this relationship, however, it is clear that a firm’s average $Q$ may not be a good proxy for marginal $Q$, where the relationship between the two measures is non-monotonic. In particular, the results of the theoretical model suggest that average $Q$ may be an especially poor proxy for marginal $Q$ for high productivity firms.

7This is also the case with CES demand functions. Specifically, under the CES assumption, the demand curve flattens as price is reduced. Thus, in these models, firms also have a functionally unbounded market for each variety.
quantities (this is derived precisely in the paper). As a second generalization, I show that foreign acquisitions can operate in the same way as domestic acquisitions, where rather than adding to a firm’s domestic capital stock, a firm can acquire foreign capital, thereby diverting export production which would otherwise be produced at a high-cost. Subject to this worldwide cost-reduction, however, the extent to which a firm can increase sales is critically dependent on demand characteristics, as discussed above. Finally, I also discuss marginal capital purchases and a dynamic model of acquisitions, showing that the intuition of the basic model extends to these settings as well.

Empirically, I test the main theoretical prediction of the model using the Compustat North American Industrial database. To do so, I first explicitly derive a relationship between the key productivity parameter and a readily observable measure, sales per worker. Testing the model using non-parametric techniques on a 25-year pooled panel, I show that the main equilibrium prediction is confirmed in the data. That is, within industries and years, firms in a mid-range of sales per worker are the most likely to acquire another firm in the next period, where approximately, the firm at the 75th percentile of sales per worker has the maximum incentive to acquire another firm. At this productivity level, a firm has a probability of acquiring that is 4 percentage points (roughly 20%) above the sample mean (0.21). Further, this same firm has a likelihood of acquiring which is 20 percentage points higher than the firm at the first percentile of sales per worker, and 5 to 10 percentage points higher than firms at the 95-99th percentile.

To test the robustness of the results, and to examine whether alternate theories might explain the empirical findings, I adjust the primary empirical strategy in four ways. First, as acquisition waves and acquisition accounting may occur over multiple periods, I allow for a number of different windows of acquisition activity, rather than the previous approach of using current period sales per worker to predict next period acquisitions. Precisely, using sales per worker estimates from 1990, I show that mid-productivity firms in 1990 are most likely to acquire another firm during the 1991-1994, 2000-2004, and 1991-2004 time frames. The results using the 1995-2004 time frame are slightly less impressive, suggesting that different acquisition incentives, likely for scope (as in Nocke and Yeaple, 2006), were also prevalent during the tech boom in the late 1990’s.

Second, the model suggests that firms operating in industries with highly substitutable products should have acquisition incentives that are increasing in productivity. To test
this hypothesis, I split the sample according to Rauch (1999) to identify industries which are more likely to sell their product on an organized exchange. Within this subsample, I find a positive, monotone, and significant relationship between productivity and acquisition behavior. In contrast, for the remaining sample of firms, which are more likely to operate in highly differentiated industries, acquisition incentives are decidedly non-monotonic, as the theoretical model would suggest.

This particular set of results is important, as no alternate theory can explain the qualitative difference in incentives across industry types. For example, while financing constraints may affect low-productivity firms in any industry, it is unlikely that financing for high-productivity firms would be less available for those in non-homogeneous industries. Thus, the difference in results for high-productivity firms across industry types cannot be explained solely by financing constraints. Further, these results also help rule out market power. More precisely, high-productivity (large) firms might find a merger profitable if they are able to affect industry-level aggregates by their acquisition decisions. However, this assumes a certain degree of market concentration to begin with, and the observed degree of market concentration in homogeneous industries, as measured by a coarse Herfindahl index, is lower than the same measure calculated in non-homogeneous industries. Thus, market power is not a likely explanation of the difference in acquisition incentives between homogeneous and non-homogeneous industries. Overall, the observed acquisition behavior of the highest productivity firms across different industry types is crucial to supporting the intuition presented in the theoretical model.

Third, I examine whether using a pooled and unbalanced panel is biasing the results. On one level, there is concern that pooling observations over time while not estimating a dynamic model will cause a bias in the primary non-parametric estimates (for example, if there is mean reversion in productivity). Further, the theoretical model is a static model within-industries, and thus it is instructive to examine whether time-series variation is driving the empirical results. On a theoretical level, I address the first concern in the technical appendix, where I show that a dynamic model of acquisitions boils down to a static problem under fairly general conditions. Empirically, I address both concerns by estimating the model on year-specific, Rauch-based sub-samples. For both sub-samples, the results are consistent with the pooled results for a majority of years.

Finally, I utilize a number of alternate measures of productivity: sales per worker,
average Q, and TFP, all of which adjusted for the presence of within-firm autocorrelation. In particular, the previous literature has found a positive relationship between average Q and acquisition behavior (Jovanovic and Rousseau, 2002). Regarding TFP, while Compustat does not contain information on value added (and thus TFP is seldom calculated using the Compustat database), it is nevertheless useful as productivity may extend beyond worker productivity. Though slightly less-impressive for TFP, the results using each alternate measure are consistent with the basic predictions of the theoretical model.

The rest of the paper is organized as follows. In section two, I thoroughly develop the theoretical model, highlighting the general acquisition framework and why this framework leads to a new sorting of firms by productivity. In section three, I test the equilibrium predictions of section two. In section four, I conclude. All proofs are available in the technical appendix available at the end of the draft.

2 Model

The basic closed economy model presented in this paper consists of three stages. In stage one, entry decisions are made. Firm-level productivity is uncertain and each potential entrant is ex-ante identical. Firms enter until their expected post-entry profits are equal to the fixed cost of entry. Upon entry, firms receive a fixed lump of capital that may be used in the product market.\footnote{Note that in this model (as in the rest of the literature) the initial capital endowment is not endogenous. I assume this to focus on the ex-post reallocation of capital, and to keep analysis tractable. In the appendix, I develop a simple dynamic model in the spirit of Jovanvic and Rousseau (2002), where under reasonable assumptions, I show that a dynamic model yields a static acquisition decision, similar to the one in this section.}

In stage two, acquisition decisions are made. Post-entry, productivity is realized and firms are allowed to trade industry-specific capital on a perfectly competitive acquisition market. However, capital from the entry stage is indivisible in the acquisition stage; firms may not buy or sell fractions of capital. Additionally, due to unmodeled organizational factors, it is assumed that a firm only has enough resources to acquire one firm in the acquisition stage. Thus, firms are restricted to three options: sell all capital and exit, buy the capital of an exiting firm, or do nothing.

Finally, in stage three, each active firm supplies its individual variety to the product
market. Active firms are monopolists in their own variety, taking other industry variables as given. At this point, any capital accrued during the entry and acquisition stages is fixed, and firms only procure variable factors.

The model is solved by backward induction, and will be introduced in this order.

2.1 Product Market Equilibrium

Consumers

Consumers have preferences over a differentiated industry and a numeraire good, $x_0$. As in Melitz and Ottaviano (2005), quasi-linear preferences of this sort can be written as:

$$U = x_0 + \theta \int_{i \in \Omega} q_i d_i - \frac{1}{2} \eta \left( \int_{i \in \Omega} q_i d_i \right)^2 - \frac{1}{2} \gamma \int_{i \in \Omega} (q_i)^2 d_i$$

(1)

In (1), $\Omega$ represents the measure of varieties, $q_i$ is the consumption of variety $i$, and the parameters $\theta (> 0)$ and $\eta (> 0)$ determine the substitution pattern between the differentiated industry and the numeraire. Finally, $\gamma (> 0)$ represents the degree to which varieties are substitutable. If $\gamma$ were zero, all firms would price at the same level, since products would be homogeneous in the eyes of the consumer.

In an economy with $L$ consumers who each supply one unit of labor at a numeraire wage, the inverse demand function for variety $i$ can be derived as:

$$p_i = \frac{\theta \gamma}{\eta M + \gamma} + \frac{\eta M}{\eta M + \gamma} \bar{p} - \frac{\gamma}{L} q_i = A - bq_i$$

(2)

In (2), $p_i$ is the price of variety $i$, $M$ is the measure of all varieties sold in the product market, and $\bar{p}$ is the average price of these varieties. Naturally, competition will be "tougher" when $M$ is high and/or $\bar{p}$ is low. Thus, the overall level of market "toughness" is captured in $A$, the residual demand level facing each firm. As all firms are small outside their own variety, firms take $A$ as given.
Firms

Capital influences firm decisions through the cost function. Similar to Perry and Porter (1985), the cost function of each firm takes the following form:

\[ C(q_i | \alpha_i, v, K) = \frac{1}{2} \cdot \frac{q_i^2}{\alpha_i K} \]  

(3)

In (3), \( \alpha_i \) is firm-level productivity. Productivity is continuously distributed according to \( G(\alpha) \), defined over \( \alpha \in (0, \infty) \). The variable \( K \) represents capital accumulated during the initial stage and acquisition stage.\(^9\) Firm-level productivity is transferrable across all holdings of capital within the firm.

A firm with productivity draw \( \alpha_i \) and capital level \( K \) faces the following profit maximization problem in stage 3:

\[ \pi(\alpha_i, K) = \max_{q_i} \left\{ (A - b \cdot q_i) \cdot q_i - \frac{1}{2} \frac{q_i^2}{\alpha_i K} \right\} \]  

\[ \text{st} \quad q_i \geq 0 \]  

(4)

Solving (4) and dropping \( i \)'s for notational convenience, profits and prices in the product market are written as:

\[ \pi(\alpha, K) = \frac{A^2 \alpha K}{2(2b\alpha K + 1)} \]  

(5)

\[ p(\alpha, K) = \frac{A b\alpha K + 1}{2b\alpha K + 1} \]  

(6)

In the closed economy, there are two types of firms active in the product market. Firms that "do nothing" (N) in the acquisition stage retain their initial capital level from entry, \( k \), while firms that buy capital (B) in the acquisition stage double their initial capital level, holding \( 2k \). Firms that sell capital (S) in the acquisition stage are not active in the product market.

\(^9\)This cost-structure can be recovered from a Cobb-Douglas production function, given that the level of capital is fixed in the product market stage. In stage three, firms only procure variable factors. The cost function is written as \( C(X_i | v) = \frac{v}{2} \cdot X_i \). With equal intensity of capital and variable factors, the cobb-douglas production function can be written as \( q_i = (\alpha_i X_i)^{\frac{1}{2}} \cdot (K)^{\frac{1}{2}} \). Solving for \( X_i \), and substituting into the above cost function, we get \( C(Q_i | \alpha_i, v, K) = \frac{v}{2} \cdot \frac{v^2}{\alpha_i K} \). Normalizing \( v \) to equal 1 gives the desired result.
Subject to these capital positions, the profits for $N$ and $B$, respectively, are expressed as:

$$
\pi^N(\alpha) = \frac{A^2\alpha k}{(4b\alpha k + 2)} \\
\pi^B(\alpha) = \frac{A^2\alpha k}{(4b\alpha k + 1)}
$$

where,

$$
\pi^B(\alpha) > \pi^N(\alpha) \text{ for } \alpha \in (0, \infty)
$$

Generally, since monopolists operate on the elastic portion of the demand curve, firms have incentive to increase production after a cost-lowering acquisition (an acquisition halves variable costs at every quantity). This is illustrated in Figure 1, where firms of low, middle and high productivity increase production following an acquisition.

However, under the assumption of linear demand, the least efficient and most efficient firms earn minimal returns from a cost-lowering acquisition. In Figure 1, the least efficient firms are limited by a steep marginal cost schedule. Whether or not they acquire, they are still
quite unproductive, and the absolute gains from an acquisition are tiny. The most efficient firms are constrained not by costs, but by the structure of market demand. Specifically, the highest productivity firms operate on a less-elastic portion of the demand curve, which limits the incentive to expand production after a cost-lowering acquisition. Firms in a mid-range of productivity are constrained by neither, and earn relatively high returns from an acquisition. Thus, with linear demand, firms within a mid-range of productivity benefit the most from a cost-lowering acquisition. Indeed, the maximum of $\Delta \Pi = \pi^B (\alpha) - \pi^N (\alpha)$ is at $\alpha = \sqrt{2} / 4b$. Finally, given the cost function in (3), profits exhibit diminishing returns to capital. Thus, $\pi^N (\alpha)$ and $\pi^B (\alpha)$ have the following intuitive ranking.

$$\frac{1}{2} \pi^B (\alpha) < \pi^N (\alpha) < \pi^B (\alpha) \quad (9)$$

This property will be used when characterizing optimal firm-level acquisition decisions as a function of productivity.

### 2.2 Acquisition Stage Equilibrium

**Optimal Acquisition Choice**

Since firms are "small", I assume an acquisition market in which firm-level decisions have no effect on the market-clearing price per firm, $R_a$, or the residual demand level, $A$. First, taking $A$ and $R_a$ as given, I derive optimal firm acquisition decisions as a function of productivity. In the process, I also discuss the polar case in which firms are price-takers. Then, for a given $A$, I show that a unique value of $R_a$ clears the acquisition market. Finally, I prove that there exists a unique value of $A$, subject to firm-level acquisition decisions and the market clearing price per firm, $R_a (A)$.

In the acquisition stage, firms must choose between three options: Sell their firm ($S$), do nothing ($N$), or buy capital ($B$). Respectively, the profits of each option in the acquisition stage are written as:

$$\Pi^S (R_a) = R_a \quad (10)$$

$$\Pi^N (\alpha, A) = \pi^N (\alpha, A) \quad (11)$$

$$\Pi^B (\alpha, A, R_a) = \pi^B (\alpha, A) - R_a \quad (12)$$
Here, the dependence of $\pi^N(\alpha, A)$ and $\pi^B(\alpha, A)$ on $A$ in (7) and (8) is made explicit to emphasize that $A$ is fixed for the moment. In (10), firms sell their capital, collect $R_a$, and exit the market. In (11), firms do nothing in the acquisition market and earn profits given their initial capital endowment, $k$. In (12), firms buy capital, earning $\pi^B(\alpha, A)$ in the product market after paying $R_a$ for an additional lump of capital.

A firm of productivity $\alpha$ chooses the acquisition option which maximizes profits in the acquisition market. Defining $V(\alpha, A, R_a)$ as acquisition market profits given $\alpha$, the acquisition decision of each firm is characterized by the following:

$$V(\alpha, A, R_a) = \max \{ R_a, \pi^N(\alpha, A), \pi^B(\alpha, A) - R_a \}$$

(13)

A convenient transformation of (13) is subtracting $\pi^N(\alpha, A)$ from each option. This gives us the following equivalent representation of the acquisition decision facing each firm:

$$\hat{V}(\alpha, A, R_a) = \max \{ R_a - \pi^N(\alpha, A), 0, \pi^B(\alpha, A) - \pi^N(\alpha, A) - R_a \}$$

This normalizes acquisition market profits relative to the outside option of doing nothing.

Within $\hat{V}(\alpha, A, R_a)$, the function $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ is the benefit of an acquisition. As a function of model parameters, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ is written as:

$$\Delta \Pi(\alpha, A) = \pi^B(\alpha, A) - \pi^N(\alpha, A) = \frac{A^2 \alpha k}{2(2b\alpha k + 1)(4b\alpha k + 1)}$$

(14)

It is straightforward to show that $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ approaches zero for low and high $\alpha$, and reaches its maximum on the interior at $\frac{2\alpha}{\sqrt{2}}$. The optimal acquisition decision derived from $\hat{V}(\alpha, A, R_a)$ is illustrated in Figure 2.

In Figure 2, for $\alpha < \alpha_S$, the profits from selling are greater than profits from doing nothing. Also, the benefit of buying, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$, is less than the acquisition price. Thus, selling is the dominant option for the least efficient firms. There will exist a positive measure of these selling firms for $R_a > 0$.

For "small" $R_a$ ($R_a \leq \frac{(3-2\sqrt{2})A^2}{4b}$), firms with productivity between $\alpha_B$ and $\alpha_B$ find an acquisition profitable. For these firms, the benefit of an acquisition, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$, is greater than the acquisition price. Additionally, for small $R_a$, there exist two disjoint regions of productivity such that doing nothing is optimal. These regions are labeled by $N$ in Figure
2. This follows from the "lumpiness" of assets.\(^{10}\)

For "large" \(R_a\) (\(R_a > \frac{(3-2\sqrt{2})A^2}{4b}\)), no firms find an acquisition profitable. The acquisition price is too large, where \(\pi^B(\alpha, A) - \pi^N(\alpha, A) < R_a\) for all \(\alpha\). Naturally, since there exist selling firms and no buying firms, large \(R_a\) cannot be an acquisition market clearing price. Thus, I henceforth restrict attention to "small" \(R_a\).

The overall shape of Figure 2 follows closely the intuition discussed for firms of low, middle, and high productivity. Precisely, mid-productivity firms have the highest incentive to acquire another firm. They are relatively less constrained by intrinsically high costs, which is the problem for low productivity firms. Further, they have additional room on the revenue side to expand production, which is not the case for the highest productivity firms.

This last point is critically dependent on the slope of the demand curve, \(b\). As illustrated in Figure 2, for any finite level of \(b\), firms in a middle range of productivity have the highest incentive to acquire another firm. But, as \(b\) falls, the level of productivity that maximizes \(\pi^B(\alpha, A) - \pi^N(\alpha, A)\), \(\frac{\sqrt{2}}{4b}\), increases. Indeed, as \(b\) approaches zero, all firms must produce

\(^{10}\)However, it is important to note that the lumpiness of assets is not the reason firms in a middle range of productivity find an acquisition profitable. An alternate acquisition market is developed in the technical appendix where firms may buy any fraction of capital. Under this setup, mid-productivity firms are the only firms that acquire a positive amount of capital.
at the industry average price (see equation 2), and the function $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ is strictly upward sloping. The intuition is that as $b$ falls and products become more substitutable, firms begin to act more like price-takers. Critically, although the price-taking assumption may imply that firms are small, the flat demand curve functionally provides firms with an unbounded market for each variety. In other words, the price taking assumption removes the revenue bounds that are most restrictive for high productivity firms. Thus, as varieties approach perfect substitutability, high productivity firms benefit the most from an acquisition.

In the forthcoming empirical section, I will attempt to control for this issue by using Rauch (1999) classifications to group industries according to whether products are sold on an organized exchange. Indeed, if industries fall into this category, it is more likely that firms within these industries act as price-takers. Thus, along with estimating the overall relationship between productivity and acquisition behavior, I will attempt to isolate the precise relationship between productivity and acquisitions for firms which are most likely to take prices as given when making output decisions.\footnote{Generally, the discussion of product substitutability is suggestive of an additional component affecting acquisition decisions: the ability of firms to maintain a high price with additional output. While this will be discussed at length shortly, note the following examples. When $b = 0$, the demand curve is flat (and of constant elasticity) and high productivity firms have the highest incentive to acquire another firm. This prediction remains when allowing for a downward sloping demand curve with constant elasticity. To see this, consider an identical setup with the exception that inverse demand is $p = Aq^{-\lambda}, 0 < \lambda < 1$. It can be derived that $\pi^B_{\infty}(\alpha) - \pi^N_{\infty}(\alpha) = Z_0 \frac{2^{-\lambda} - 1}{1 + \frac{\lambda}{2}} \left( \frac{1}{1 + \frac{\lambda}{2}} \right)^{\frac{1}{1 + \frac{\lambda}{2}}} k^{\frac{1}{1 + \frac{\lambda}{2}}} > 0$. Clearly, the stage three profits resulting from an acquisition are increasing in productivity.}

**Equilibrium**

Continuing under the assumption that $b > 0$, and once again turning attention to Figure 2, $\alpha_S, \alpha_B$, and $\alpha_B$ represent kinks in $V(\alpha, A, R_a)$. More precisely, these represent firms that are indifferent between acquisition options. Hence, $\alpha_S$ is implicitly defined as:

$$\pi^N(\alpha_S, A) = R_a$$

where,

$$\text{For } \alpha < \alpha_S, S > N$$

\footnote{Continuing under the assumption that $b > 0$, and once again turning attention to Figure 2, $\alpha_S, \alpha_B$, and $\alpha_B$ represent kinks in $V(\alpha, A, R_a)$. More precisely, these represent firms that are indifferent between acquisition options. Hence, $\alpha_S$ is implicitly defined as:

$$\pi^N(\alpha_S, A) = R_a$$

where,

$$\text{For } \alpha < \alpha_S, S > N$$}

14
The preference condition $S > N$ is a straightforward result when observing that stage three
profits are increasing in productivity.

Similarly, $\alpha_B$, and $\bar{\alpha}_B$ can be defined by:

\[
\pi^B (\alpha_B, A) - \pi^N (\alpha_B, A) = R_a \tag{17}
\]
\[
\pi^B (\bar{\alpha}_B, A) - \pi^N (\bar{\alpha}_B, A) = R_a \tag{18}
\]

where,

\[
For \alpha \in (\alpha_B, \bar{\alpha}_B), \ B > N \tag{19}
\]

The condition $B > N$ is immediate from the shape of $\pi^B (\alpha, A) - \pi^N (\alpha, A)$.

Using the indifference conditions in (15), (17), and (18), and the preference conditions
in (16) and (19), the following lemma proves that the features illustrated in Figure 2 are
representative of optimal acquisition choice.

**Lemma 1** In the closed economy, given $A$ and $R_a$, optimal acquisition choice is the fol-
lowing:

For $\alpha \in [0, \alpha_S (A, R_a))$, firms sell
\[
\alpha \in [\alpha_S (A, R_a); \alpha_B (A, R_a)], \text{ firms do nothing}
\]
\[
\alpha \in (\alpha_B (A, R_a); \bar{\alpha}_B (A, R_a)) \text{, firms buy}
\]
\[
\alpha \in [\bar{\alpha}_B (A, R_a); \infty), \text{ firms do nothing}
\]

**Proof.** See Appendix ■

In Lemma 1, the relationship between the equilibrium cutoffs and economy aggregates $A$
and $R_a$ is made explicit. With Lemma 1, given $M_E$ entrants, the demand ($K_D (A, R_a)$) and
supply ($K_S (A, R_a)$) of acquired capital are written as:

\[
K_D (A, R_a) = M_E k (G (\bar{\alpha}_B (A, R_a)) - G (\alpha_B (A, R_a))) \tag{20}
\]
\[
K_S (A, R_a) = M_E k G (\alpha_S (A, R_a)) \tag{21}
\]

The acquisition price, $R_a$, affects $K_D (A, R_a)$ and $K_S (A, R_a)$ through the acquisition cutoffs
$\alpha_S (A, R_a)$, $\alpha_B (A, R_a)$ and $\bar{\alpha}_B (A, R_a)$. Of course, the acquisition market clears if,

\[
K_D (A, R_a) = K_S (A, R_a). \tag{22}
\]
For a given value of $A$, there is a unique $R_a(A)$ that clears the acquisition market. This is proven in the following Lemma:

**Lemma 2**  
*Holding $A$ fixed, there exists a unique $R_a(A)$ that clears the acquisition market.*

**Proof.** See Appendix

The intuition behind Lemma 2 is a simple case of supply and demand. The measure of buying firms is decreasing in the acquisition price, and the measure of selling firms is increasing in the acquisition price. Given that no firms are willing to sell at $R_a = 0$ and no firms are willing to buy at $R_a \geq \frac{(3-2\sqrt{2})A^2}{4b}$, we know that the demand and supply functions cross only once at the equilibrium acquisition price, $R_a(A)$.

With acquisition market clearing in-hand, I now show that there exists a unique equilibrium value of $A$. First, I analyze how the productivity cutoffs summarized in Lemma 1, subject to the acquisition market clearing condition, change with $A$. Conveniently, $\alpha_S$, $\alpha_B$ and $\overline{\alpha}_B$ are all independent of $A$. To see this, note that the equilibrium conditions in (15), (17), and (18), and the market clearing condition in (22), can be combined to yield the following:

$$
\alpha_B(A)k \\
\frac{2}{2(2b\alpha_B(A)k+1)(4b\alpha_B(A)k+1)} = \frac{\alpha_S(A)k}{(4b\alpha_S(A)k+2)}
$$

$$
\frac{\overline{\alpha}_B(A)k}{2(2b\overline{\alpha}_B(A)k+1)(4b\overline{\alpha}_B(A)k+1)} = \frac{\alpha_S(A)k}{(4b\alpha_S(A)k+2)}
$$

$$
G(\overline{\alpha}_B(A)) - G(\alpha_B(A)) = \frac{\alpha_S(A)k}{(4b\alpha_S(A)k+2)}
$$

Above, there exist three equations and three unknowns, $\alpha_S(A)$, $\alpha_B(A)$ and $\overline{\alpha}_B(A)$. Critically, $A$ no longer enters into any equation directly. This feature is a result of profit functions being homogeneous in $A$, along with the acquisition price being the only fixed cost. This immediately yields the following lemma:

**Lemma 3**  
$\frac{\partial \alpha_S(A)}{\partial A} = 0$, $\frac{\partial \alpha_B(A)}{\partial A} = 0$ and $\frac{\partial \overline{\alpha}_B(A)}{\partial A} = 0$

**Proof.** Immediate.

Using Lemma 3, the uniqueness of $A$ is now trivial. Using the inverse demand function for each variety in (2), the unique value of $A$ is defined for any level of entry, $M_E$, by
\[ A = \frac{\theta \gamma}{\gamma + \eta M_E (1 - G(\alpha_S) - \Phi(\alpha_S, \alpha_B, \alpha_B))} \]  

(23)

where,\(^{12}\)

\[ \Phi(\alpha_S, \alpha_B, \alpha_B) = \int_{\alpha_S}^{\alpha_B} \frac{b\alpha k + 1}{2b\alpha k + 1} dG(\alpha) + \int_{2\alpha}^{\alpha_B} \frac{2b\alpha k + 1}{4b\alpha k + 1} dG(\alpha) + \int_{-\alpha}^{\infty} \frac{2b\alpha k + 1}{2b\alpha k + 1} dG(\alpha) \]

and where (23) is written in terms of \( M_E \) using the fact that,

\[ M = M_E (1 - G(\alpha_S)) \]  

(24)

Since \( M_E \) varieties enter and \( M_E G(\alpha_S) \) varieties sell and exit (from Lemma 1), \( M \) varieties are sold in the product market, as defined by (24). Since \( 1 - G(\alpha_S) > \Phi(\alpha_S, \alpha_B, \alpha_B) \), the unique value of \( \hat{A} \) is positive. The uniqueness of \( A \) is summarized in the following lemma.

**Lemma 4** In the closed economy, there exists a unique solution \( \hat{A} > 0 \), as written in (23).

With Lemmas 1, 2, and 4, the following Proposition summarizes the acquisition stage equilibrium in the closed economy.

**Proposition 1** Given \( M_E \) entering firms, the closed economy acquisition equilibrium con-
sists of a unique $A, R_a, \alpha_S, \underline{\alpha}_B, \text{ and } \overline{\alpha}_B$ such that:

For $\alpha \in [0, \alpha_S)$, firms sell

$\alpha \in [\alpha_S, \underline{\alpha}_B)$, firms do nothing

$\alpha \in (\underline{\alpha}_B, \alpha_B)$, firms buy

$\alpha \in [\overline{\alpha}_B, \infty)$, firms do nothing

**Proof.** Follows directly from Lemmas 1, 2, and 4.

The highlight of Proposition 1 is that the highest productivity firms acquire nothing. These firms operate on a less-elastic portion of the demand curve, which limits the incentive to expand production after a cost-lowering acquisition. In contrast, the highest productivity firms *would* acquire if we assumed perfect substitutability between varieties ($b = 0$). Thus, when acquisitions lower production costs, the structure of competition and demand are important components of the equilibrium acquisition decisions of heterogeneous firms.

To close the model, I now present the free entry condition. In stage one, $M_E$ firms enter until their expected post-entry profits equal the fixed cost of entry. Imposing the acquisition market clearing condition (22), the free entry condition is written as:

$$
\int_{\alpha_S}^{\underline{\alpha}_B} \pi^N (\alpha) \, dG (\alpha) + \int_{\underline{\alpha}_B}^{\alpha_B} \pi^R (\alpha) \, dG (\alpha) + \int_{\alpha_B}^{\infty} \pi^N (\alpha) \, dG (\alpha) = F_E
$$

Since profits are increasing in $A$, and $A$ is decreasing in $M_E$ (by 23), additional entry lowers the expected profits of all entrants. Thus, provided that the fixed cost of entry is not prohibitive, there exists a unique, positive measure of entering firms.

**Generalizations**

**Demand**

The primary result of the model is that mid-productivity firms have the highest incentive to acquire another firm. In deriving this result, the assumed demand function makes analysis particularly clean and instructive. However, this begs the following question: what are

13 High productivity firms also acquire within the framework developed by Jovanovic and Rousseau (2002). In their work, revenues are related one-for-one with installed capital. High productivity firms are willing to produce on a larger scale, and thus have the highest incentive to install additional capital.
the general characteristics of demand required to deliver this particular result? To address this question, I will consider a monopolist producing subject to a general inverse demand function \( P(q) \), and the same cost function as listed in (3).

Note that the crucial feature of the model is how the marginal value of additional capital changes with productivity. While \( \frac{\partial \Pi}{\partial k} > 0 \) for every firm, and thus all firms earn some returns from an acquisition, \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} \) may change sign depending on a firm’s productivity level. In the appendix, I derive the following properties of \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} \).

**Proposition 2** \( \frac{\partial \Pi}{\partial k} > 0 \) for all \( \alpha \). Assuming that \( \frac{\partial MR(q)}{\partial q} < 0, \frac{\partial^2 \Pi}{\partial k \partial \alpha} > 0 \) if \( 1 + \alpha k \frac{\partial MR(q)}{\partial q} > 0 \), and \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} < 0 \) otherwise.

**Proof.** See Appendix. ■

For any demand function with a finite slope, as \( \alpha \) approaches zero, \( 1 + \alpha k \frac{\partial MR(q)}{\partial q} > 0 \) and thus \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} > 0 \). The critical question is how \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} \) is valued for higher productivity firms. As a polar case, consider firms that are price takers, where \( \frac{\partial MR(q)}{\partial q} = 0 \). Here, \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} = 1 > 0 \) for all \( \alpha \). Thus, the incentive to acquire another firm is increasing in productivity. In contrast, for linear demand \( (P = A - bq) \), \( \frac{\partial MR(q)}{\partial q} = \frac{\partial}{\partial q}(A - 2bq) = -2b \). Thus, \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} > 0 \) for \( \alpha < \frac{1}{2bk} \), and \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} < 0 \) otherwise. Generally, a mid-productivity firm will have the highest marginal value of capital as long as \( \alpha k \frac{\partial MR(q)}{\partial q} \) is decreasing over all \( \alpha \), and does not asymptote to a value above negative one. On the other hand, if the demand function is sufficiently convex, and \( \alpha k \frac{\partial MR(q)}{\partial q} \) is everywhere greater than negative one, then acquisition incentives will be increasing in productivity.

**More Generalizations**

There are a number of other ways that the intuition from this section can be generalized. In this subsection, three will be discussed, with additional theory presented in the appendix for interested readers.

The first extension examines how acquisition decisions change when allowing for marginal capital purchases. As detailed in the preceding discussion of general incentives, with linear demand, the marginal value of capital will be positive for all firms, but highest for firms in a mid-range of productivity. When including a per-unit price of acquired capital, I find that low-productivity and high-productivity firms choose to sell some of their assets, with some
low-productivity firms liquidating all assets and exiting the market. Thus, the qualitative difference with the lumpy-asset model is that while high-productivity firms remain in the market, they choose to hold very little capital. The intuition is that, to operate given superior productivity, these firms only need a small amount of capital to produce the desired level of output.

The second extension involves dynamics. Precisely, is there a coherent dynamic model which yields predictions close to the static model presented above? On a basic level, the answer to this question is yes, where in the appendix a framework is developed allowing for marginal capital purchases via acquisitions and new investment. The main result is that while new investment requires conjecturing over the expected value of future returns under uncertainty, the incentives governing acquisitions reduce to a static problem (similar to the marginal purchases model described above). The key assumption is that the only difference between buying capital via acquisitions and doing so via new investment is timing, where acquisitions are assumed to be operable immediately and new investment takes one period to become operable. Precisely, any effects of acquisitions beyond the initial period are identical to those of new investment after accounting for a discount rate and depreciation. Thus, the long-run effects of acquisitions are embodied in the price of new capital, which itself determines the optimal amount of new investment.

Finally, the model can be easily extended to allow for foreign acquisitions. While a complete treatment of foreign acquisitions and the role of trade costs is presented in a companion paper (Spearot, 2008), the appendix contains a version of the model assuming costless trade and two identical countries. The main result is that foreign acquisitions function identically to domestic acquisitions, where firms in a middle range of productivity choose to acquire. The intuition is quite simple. If firms purchase capital domestically, they add to their overall domestic capital stock. This makes variable factors more efficient, and facilitates revenue gains via increased sales. In contrast, with foreign acquisitions, domestic factors become more efficient by diverting export production which would otherwise be produced at home. Given the fixed nature of capital, a lower production level at home yields a lower marginal cost for the last unit produced at home. Thus, firms sell more to the integrated world market at a lower average cost. In both cases, acquiring additional capital improves efficiency, and through these efficiency gains, firms can increase sales. However, the extent to which a firm can increase sales is critically dependent on demand characteristics,
as discussed above.

3 Empirics

This section tests the primary prediction of the theoretical model in section two, which is that mid-productivity firms are most likely to acquire another firm. In doing so, I will derive an explicit link between the theory and an observable firm-level measure, sales per worker. Further, I also test for any differential relationship within a sample of firms that are more likely to act as price-takers. Finally, I will use a number of alternate measures of productivity to provide additional evidence for the predictions of the theoretical model.

3.1 A link to the theory

In section two, acquisition decisions are derived as a function of endowed firm-level productivity, $\alpha$. However, as is clear from (14), the important factor specific to each firm is not the productivity level, but the capital adjusted productivity level, $\alpha_k$. Conveniently, this can be directly related to a readily observable measure of productivity - sales per worker.

To see this, first note from above that optimal pre-acquisition quantity and price are written as:

$$q(\alpha_k) = \frac{A\alpha_k}{2b\alpha_k + 1}$$
$$p(\alpha_k) = A \frac{\alpha_k + 1}{2b\alpha_k + 1}$$

Thus, sales of each variety can be written as:

$$Sales(\alpha_k) = A^2\alpha_k \frac{(b\alpha_k + 1)}{(2b\alpha_k + 1)^2}$$

Next, note that from the assumed cost function in (3), variable input requirements are equal to $\frac{q^2}{\alpha_k}$. Assuming that labor is the primary variable input, optimal labor procurement as a function of capital adjusted productivity is written as:

$$L(\alpha_k) = \frac{A^2\alpha_k}{(2b\alpha_k + 1)^2}$$

21
Thus, the \textit{pre-acquisition} level of sales per worker is written as:

\[
\frac{Sales}{L}(\alpha k) = (b\alpha k + 1)
\]

Clearly, within a given industry (in which the parameter \(b\) is common across firms), sales per worker is linearly related to the critical term of interest, \(\alpha k\). Defining \(\tilde{S} \equiv \frac{Sales}{L}(\alpha k) - 1 = b\alpha k > 0\) as an \textit{adjusted} sales per worker, I can thus estimate \(\alpha k\) by controlling for any industry-year specific effects embodied in \(b\). Then, I can estimate equation (14) using non-parametric techniques. This is precisely the approach that I will use later in this section.

Along with sales per worker, I will also utilize two other proxies for productivity, average \(Q\) and TFP, which will be described in detail later in this section. With this in mind, I now turn to describing the data, sample, and estimation.

\subsection*{3.2 Data and Sample}

The sample of active firms is constructed using the \textit{Compustat} North American Industrial database. Within the mergers literature, this database has also been used by Jovanovic and Rousseau (2002) and Breinlich (2009). The time period of analysis is 1980-2004. The primary sample is constructed using firms from industries with two-digit SIC codes less than 40. These are primarily agricultural, commodity, and manufacturing firms. This will yield a sample totaling 60510 observations.

To identify firms that acquire, I construct the following binary measure of acquisition behavior:

\[
DAcq_{i,t} = 1 (Value_{i,t} > 0)
\]  

In (26), \(Value_{i,t}\) represents a positive outflow of cash or funds towards acquisitions (\textit{Compustat} Item 129, in Millions of US$) for firm \(i\) in year \(t\).\footnote{A binary measure treats large and small acquisitions as equal, which may bias the results. However, the results using a log-adjusted \(Value_{i,t}\) rather than the binary measure are qualitatively identical.}

To measure sales per worker, I first construct a naive measure by dividing yearly net sales of firm \(i\) in year \(t\) (\(Sales_{i,t}\)) in millions of dollars (\textit{Compustat} item 12) by yearly employment in millions of workers (\textit{Compustat} item 29 divided by 1000). However, two additional steps are taken to yield a measure which is both easily interpretable, and closely applied to the
theory. First, I take the natural log of sales per worker to control for outliers which will distort the illustrations required for a non-parametric analysis. Second, the model detailed above describes merger activity in a static model within a given industry. Thus, log sales per worker is demeaned within industry-year pairs, yielding the final measure, $SalesEmp_{i,t}$.

### 3.3 Specification and Results

"Short" Run Acquisitions

I will use a simple nonparametric specification to estimate the relationship between productivity and acquisition activity. The procedure I will use is an "additive model", which allows for joint estimation of both parametric and nonparametric components of an empirical specification. Following the procedure in Wood (2007), using the MGCV package for R, I will estimate the following linear probability model:

$$DAcq_{i,t+1} = s(SalesEmp_{i,t}) + \beta Other_{i,t} + \epsilon_{i,t}$$

(27)

Equation (27) estimates the relationship between the right-hand side variables and acquisition behavior in the next period. Using this approach prevents an obvious endogeneity problem between acquisition behavior and covariates from the same period. Thus, acquisition behavior covers the period 1981-2004, and firm and industry covariates cover 1980-2003.

I will test the robustness of this approach by utilizing different acquisition windows later in this section.

In (27), $s(SalesEmp_{i,t})$ represents a smooth function in $SalesEmp_{i,t}$.

Further, the term $Other_{i,t}$ includes Year, two-digit SIC sector, and a foreign incorporation dummy variable (which is a dummy variable identifying firms incorporated in Canada). To facilitate relatively quick estimation, $s()$ will be estimated using a penalized cubic-spline regression. This allows for a generally specified smooth fit, along with a penalty in the likelihood function for too much "wiggliness". The optimal degree of smoothing is chosen by a generalized cross-validation procedure.

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15For identification purposes, $E\{s()\} = 0$. Thus, $s()$ measures the relative effect of its argument.

16In previous versions of this paper, I have used locally linear models and other types of smoothing splines. The results are comparable. In addition, I have constructed bootstrap percentile intervals in a previous draft, though inference is largely unaffected by this alternate approach.
Figure 3: Baseline Nonparametric Results

Each panel illustrates the relative probability of acquisition activity as a function of SalesEmp, for all North American firms, 1980-2004. The left panel allows for SIC2 and Year fixed effects, while right panel interacts SIC2 and Year fixed effects. The expected value of $s(SalesEmp)$ is normalized to zero for identification. 90% Bayesian confidence intervals are provided. In addition, the vertical dashed lines represent (from left to right) the 1st, 5th, 10th, 25th, 50th, 75th, 90th, 95th and 99th percentiles of SalesEmp. The hashed marks on the x-axis represent data points.

Figure 3 presents the results from estimating (27), where the left panel estimates a model with industry and year fixed effects, and the right panel uses a within estimator to estimate a model with their interaction. The vertical axis measures the relative probability of acquisition activity, and the horizontal axis measures the relative value of sales per worker within industry-year pairs.

Clearly, the results in both panels of Figure 3 support the basic incentives described in section two. That is, mid-productivity firms are the most likely to acquire another firm, where approximately, the firm at the 75th percentile of sales per worker has the maximum incentive to acquire another firm. At this productivity level, a firm has a probability of acquiring that is 4 percentage points (roughly 20%) above the sample mean (0.21). Further, this same firm has a likelihood of acquiring which is 20 percentage points higher than the firm at the first percentile of sales per worker, and 5 to 10 percentage points higher than firms at the 95-99th percentile.

The results presented in Figure 3 are clearly consistent with the theoretical model. However, other explanations for the observed acquisition behavior must be considered. Since
acquisitions usually require a significant amount of financing, it is possible that firms less-likely to acquire are more constrained by financing issues. For high productivity firms, who tend to be large in my sample, this is explanation is unlikely. Further, for these same firms, I will provide additional evidence against a financing story by examining the incentives for acquisitions across different industry types. For low productivity firms, it is not possible to ascertain whether the observed acquisition activity is due to fundamentals or financing issues.

Finally, the above results show that, within industries and years, firms with sales per worker in a middle range are the most likely to acquire in the next period. Some might find this to be a bit coarse, as some acquisitions are negotiated a year or more prior to accounting for them on financial statements. Further, some might also view acquisitions as part of a process longer than one year, where in a given year productivity might warrant an acquisition, but financing constraints and other issues delay such an acquisition until later years. Finally, some firms may simply account for a given acquisition over a number of years rather than in one period. The above results do not address these specific issues, and thus before looking at results within different types of industries, I will examine an adjusted dataset which allows for different windows of acquisition activity.

"Long" Run

I will modify the specification in (27) by using a number of different dependent variables (instead of $DAcq_{i,t+1}$) based on windows of acquisition behavior. Precisely, I will look at acquisition behavior within the 1991-1994, 1995-2004, 2000-2004, and 1991-2004 time frames. The dependent variable will remain discrete, taking the value of one if acquisitions occur during the given time frame, and zero otherwise. Further, I will use 1990 as the baseline period for productivity (sales per worker). Thus, these particular regressions will evaluate the acquisition incentives, medium and long run, of North American firms operating in 1990.

These medium-long run results are presented in Figure 4. Clearly, firms in a middle-range of productivity have the highest incentive to acquire relative to low and high productivity firms. Although the precision is not as sharp relative to Figure 3, this is expected as it is no longer a panel (there are 2203 firms in this particular sample rather that 60510 firm-year combinations in the previous sample). Interestingly, the estimates for the full acquisition
Each panel illustrates the relative probability of acquisition activity over different acquisition windows as a function of SalesEmp in 1990 for North American firms. The expected value of \( s(\text{SalesEmp}) \) is normalized to zero for identification. 90% Bayesian confidence intervals are provided. In addition, the vertical dashed lines represent (from left to right) the 1st, 5th, 10th, 25th, 50th, 75th, 90th, 95th and 99th percentiles of SalesEmp. The hashed marks on the x-axis represent data points.
window 1991-2004 (lower-right panel) and 1995-2004 (upper-right panel) exhibit a strong hump shape, along with a mild upward sloping characteristic in productivity. This might be explained by a certain degree of mergers for scope during the high-tech boom of the 90’s. Theoretically, this would be consistent with Nocke and Yeaple (2006), where higher productivity firms tend to acquire for scope. However, there is a fundamental relationship between sales per worker and acquisition incentives that is consistent with the theoretical model, and persistent over long and distant windows of acquisition activity.

**Homogeneous industries**

As mentioned earlier, the strongest empirical support will come from estimating incentives across different industry types, and showing that any difference in results is consistent with the theory. As detailed in section two, industries in which products are more substitutable are more likely to exhibit acquisition incentives which are increasing in productivity. Otherwise, the incentives are non-monotone, as presented above. To split the sample into industries which are likely and not likely to sell highly substitutable products, I will harness a widely used classification system. Rauch (1999) presents a framework for classifying industries as differentiated, reference priced, or sold on an organized exchange. Specifically, industries in the last category will be defined as "homogeneous", and according to the theory, I hypothesize that acquisition incentives in these industries will be monotone and increasing in productivity.

To test this hypothesis, Rauch classifications are collected, and industries are grouped according to their conservative Rauch classifications. Then, each industry is mapped by hand to a *Compustat* SIC code. Precisely, an industry is defined as homogeneous if it is identified as such in the Rauch classifications, and there exists a clear mapping into a *Compustat* SIC code. The industries defined as homogeneous are listed in Table 1.

In Table 1, a few points are worth noting. First, as the matching is accomplished at the four-digit SIC level, the two-digit codes are made available only for reference. Second, most products fall into agriculture or mining industries. A few exceptions involve those products in SIC 20-22, which are processed agricultural products. Finally, a few products may seem oddly placed on this list. For example, Cigarettes and Cigars are obviously differentiated to some degree. However, the results below hold with and without the inclusion of Cigarettes and Cigars. Further, in most industries there will be some level of differentiation (for example, different grades of salmon or tuna). Thankfully, the theory only predicts that as
substitutability increases, acquisition incentives will be skewed toward higher productivity firms. Despite being labeled as homogeneous industries (for the sake of simplicity), perfect substitutability is not required to measure a qualitative difference between industries.

Moving forward, (27) is estimated using each sub-sample, where the group of homogeneous industries includes 5627 observations, and the non-homogeneous group includes 54883 observations. The results are presented in Figure 5. Clearly, acquisition incentives are different for firms which operate in industries selling highly substitutable products. In the left panel of Figure 5, we see that the optimal non-parametric fit, as determined by a generalized cross-validation procedure, is essentially linear. Further, the 90% confidence intervals are fairly tight and the overall fit is increasing in sales per worker. This is consistent with predictions from models such as Jovanovic and Rousseau (2002), and the intuition presented in section two for industries with a high substitutability between varieties. In contrast, the results using a sample restricted to non-homogeneous firms (right panel of Figure 5) predict that firms in a middle-range of sales per worker are the most likely to acquire. Overall, the incentives governing acquisitions are different for firms selling homogeneous goods in a way consistent with the theoretical model.

This dichotomy helps differentiate between alternate theories of acquisition behavior. For example, one possible explanation of the base results in Figure 3 is that high productivity firms require particularly scarce assets. More precisely, high productivity firms are large
because they hold "good" assets, whether tangible or intangible. Thus, if they are to acquire new assets, the only assets which are useful are those which are even more efficient. But, these target assets are more likely to be held by other large and efficient firms, who may be less likely to sell. Thus, in equilibrium, high productivity firms may be less likely to acquire simply because the assets which they desire are in low supply. Critically, this intuition is not specific to any industry. Thus, if this was causing a noticeable bias in the empirical results, we would expect to see non-monotone acquisition incentives within homogenous industries along with non-homogeneous industries. Indeed, Figure 5 suggests that something else is driving the results.

One additional issue to consider is market power. Precisely, firms may acquire for market power they have sufficient "weight" to push around industry-level aggregates in a favorable direction via mergers and acquisitions. High productivity firms, who earn rather paltry returns on acquisitions for cost-reduction, are the most likely to exert such power via the acquisition market. Is this story consistent with the evidence presented in Figure 5? This would be a consistent explanation for the results using the sample of homogeneous industries.
if these industries exhibited a greater degree of market concentration than non-homogeneous industries. Using *Compustat*, the average and median Herfindahl indices (calculated over SIC4-Year pairs) within homogeneous and non-homogeneous product categories are 0.396 and 0.317 for the former group, and 0.42 and 0.35 for the latter. Thus, while both measures are fairly large (firms not listed in Compustat are not included in the calculations), the homogeneous sector exhibits less concentration, and thus market power is not likely to be a sufficient explanation for the dichotomy present in Figure 5 when comparing homogeneous and non-homogenous industries.

**Yearly Estimation**

Next, I test the robustness of the results as they relate to using a pooled and unbalanced panel. On one level, there is concern that pooling observations over time while not estimating a dynamic model will cause a bias in the primary non-parametric estimates (for example, if there is mean reversion in productivity). Further, the theoretical model is a static model within-industries, and thus it is instructive to examine whether time-series variation is driving the empirical results.

On a theoretical level, I address the first concern in the technical appendix, where I show that a dynamic model of acquisitions boils down to a static problem under fairly general conditions. Empirically, I address both concerns by estimating the Rauch-based samples by year. The results from estimating equation (27) for each Rauch-based sample for each year, 1992-2003, are presented in Figures 6 and 7. In Figures 6 and 7, the labeled year represents the year in which the right-hand side variables are measured. As described above, acquisitions are measured a year after productivity measures to prevent any direct endogeneity issues. Clearly, the general features present in both panels of Figure 5 are present when estimating by year. In Figure 6, with the exception of 1992 and 2003, all non-parametric estimates summarize a monotone and increasing relationship between relative sales per worker and the probability of acquisition behavior. Although the lack of observations often yields a fit which is fairly imprecise, the results suggest that the characteristics in the left panel of Figure 5 are fairly consistent over years.

Next, in Figure 6, we see that the results for non-homogeneous industries are also consistent with those in the right panel of Figure 5. That is, with the exception of 2001 and 2003,

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17 Results for all other years are available upon request.
Each panel illustrates the relative probability of acquisition activity as a function of SalesEmp for North American firms in each year, 1992-2003. Further, the sample is restricted to firms which operate in homogeneous industries (see Rauch, 1999). The expected value of $s(SalesEmp)$ is normalized to zero for identification. 90% Bayesian confidence intervals are provided. In addition, the vertical dashed lines represent (from left to right) the 1st, 5th, 10th, 25th, 50th, 75th, 90th, 95th and 99th percentiles of SalesEmp. The hashed marks on the x-axis represent data points.
Each panel illustrates the relative probability of acquisition activity as a function of SalesEmp for North American firms in each year, 1992-2003. Further, the sample is restricted to firms which do not explicitly operate in homogeneous industries (see Rauch, 1999). The expected value of $s$(SalesEmp) is normalized to zero for identification. 90% Bayesian confidence intervals are provided. In addition, the vertical dashed lines represent (from left to right) the 1st, 5th, 10th, 25th, 50th, 75th, 90th, 95th and 99th percentiles of SalesEmp. The hashed marks on the x-axis represent data points.
the non-parametric results in Figure 6 suggest that acquisition incentives are highest for firms between the 50th and 90th percentile of sales per worker. Again, estimating the model on year-specific samples does not seem to alter the basic relationship between productivity and acquisition behavior within each industry type.

3.4 Alternate Measures of Productivity

As a final test of the theory, I will utilize alternate measures of productivity to test the predictions from section two and the overall robustness of the results presented thus far. The first measure is constructed by simply allowing for within-firm autocorrelation while estimating sales per worker. The second and third require a more detailed explanation.

The second involves testing the model as a Q-Theory of Investment. As Jovanovic and Rousseau (2002) find that high-Q firms tend to acquire, testing the model similar to a Q-Theory of Investment is of high interest. On a basic level, $Q$ and productivity should be positively related, as $Q$ measures the ratio of the expected future stream of profits (market value) to the replacement value of assets (book value). If a firm has intangible attributes that differentiate it (positively) from the rest of the sample (higher productivity, for example), this firm’s value of $Q$ should be higher. Within the context of the relationship developed in section two, $Q$ will be positively related to productivity so long as any estimation controls for the level of asset holdings.\footnote{To see this, define $Q$ for firm $j$ as $\frac{V^j(\alpha, K_j)}{r_a K_j}$, where $V^j(\alpha, K_j)$ and $K_j$ are the value function prior to acquisition decisions and asset holdings of firm $j$, respectively. In addition, $r_a$ is the replacement cost per-unit of capital. Clearly, $Q$ is increasing in $\alpha$, conditional on $K_j$. In terms of $K_j$, $V^j(\alpha, K_j)$ will exhibit diminishing returns to capital, and $r_a K_j$ will exhibit constant returns. Thus, comparing firms of equal $\alpha$ which differ only by $K_j$, the firm with the higher $K_j$ will have the lower $Q$.}

With regard to the existing literature, $Q$ has been shown to be positively related to the level of firm-specific profits (Villalonga, 2004). In addition, Wernerfelt and Montgomery (1988) use $Q$ as a measure of firm performance. In contrast, Nocke and Yeaple (2006) identify a potential problem with using $Q$ as a measure of firm performance. They report that the empirical relationship between $Q$ and firm size (sales) is actually negative.\footnote{To resolve this paradox, they develop a model in which firms merge to expand the scope of the firm to additional varieties.} Empirically, this is problematic since according to my model, firm size should be increasing in $Q$. I will provide a resolution to this issue below.
In constructing $Q$, I will modify the existing literature in the following way. I assume that the observed value $\hat{Q}_{i,t}$ is a function of productivity, capital, and fluctuations specific to industries, years, and the country of ownership. Specifically, I adopt the following functional form

$$\hat{Q}_{i,t} = f(\alpha_{i,t}) \cdot SIC_j \cdot Year_t \cdot Corp_c \cdot Cap^{\beta_Q}_{i,t}$$ (28)

In (28), $Cap_{i,t}$ is the value of property, plants and equipment (Compustat item 8). Controlling for $Cap$ is necessary as productivity and $Q$ have a positive monotone relationship only after controlling for capital holdings. The observed value, $\hat{Q}_{i,t}$, will be constructed identically to Jovanovic and Rousseau (2002). For firm $i$ in year $t$, the market value is defined as the sum of the market value of common equity (stock) at current share prices ($Compustat$ item 24 multiplied by $Compustat$ item 25), book value of preferred stock ($Compustat$ item 130), and the book values of short and long-term debt ($Compustat$ items 34 and 9). The book value of firm $i$ in year $t$ is computed in a similar fashion to the market value, with the exception of replacing the market value of common equity with the book value of common equity ($Compustat$ item 60).

The key in (28) is the estimation of $f(\alpha_{i,t})$, which motivated by the model, is assumed to be an increasing function in its argument. To recover this function in relative terms, I take logs of (28) and define $\xi_{i,t} = \log(f(\alpha_{i,t}))$. This yields the following estimating equation:

$$\log(\hat{Q}_{i,t}) = \beta_Q^{Cap} \log(Cap)_{i,t} + \beta_Q^{SIC} SIC + \beta_Q^{Year} Year + \beta_Q^{Can} FINC + \xi_{i,t}$$ (29)

In (29), I now have a vector of industry controls and coefficients, $\beta_Q^{SIC} SIC$, a vector of year controls and coefficients, $\beta_Q^{Year} Year$, and a dummy variable identifying firms which are incorporated in Canada, $FINC$. In estimating (29), I will also allow for within-firm autocorrelation. As information may be slowly revealed to the market over time, it is possible that firm-level fundamentals from previous periods, while obsolete, are influencing market values in current periods. Indeed, correcting for autocorrelation yields a positive correlation between sales and $Q$, contrary to the existing literature, and consistent with the model.20

Finally, the third proxy for productivity will involve a simple TFP calculation. Precisely,
TFP will be defined as the residual from the following regression:

\[
\log(\text{NetSales}_{i,t}) = \beta_{\text{cap}}^{\text{TFP}} \log(\text{Cap}_{i,t}) + \beta_{\text{Emp}}^{\text{TFP}} \log(\text{Emp}_{i,t}) + \\
\beta_{\text{sic}}^{\text{TFP}} \text{SIC} + \beta_{\text{year}}^{\text{TFP}} \text{Year} + \beta_{\text{can}}^{\text{TFP}} \text{FINC} + \epsilon_{i,t}
\]  

(30)

In (30), NetSales_{i,t} represents yearly sales in millions of dollars (Compustat item 12), and Emp_{i,t} represents employment in millions of workers. I will utilize a variant of this regression which allows for within-firm autocorrelation, and one which does not.

The results from using four alternate measures of productivity are illustrated in Figure 8. Focusing on the top-left panel, we see that correcting sales per worker for within-firm, one-year autocorrelation does nothing to the qualitative nature of the results. Further, in the top-right panel, using a measure of average Q to proxy for productivity also yields similar results. There exists one outlier (on the negative side) which distorts the results somewhat, but the basic story remains over a majority of data points: mid-productivity firms have the highest incentive to acquire another firm.

The results using simple calculations of TFP are somewhat different, though still fairly supportive. That is, in the lower panels of Figure 8, it appears that correcting for autocorrelation makes a difference. Specifically, in the lower-left panel, we see that between the 1st and 99th percentiles of TFP, firms in a middle range have the highest incentive to acquire. In the lower-right panel, where TFP is estimated correcting for autocorrelation, we see that within these bounds the incentives to acquire are more skewed toward high-productivity firms, where the maximum incentive to acquire occurs between the 90th and 95th percentile of TFP. However, it is important to note that above the 95th percentile the probability of acquisitions drops sharply, with both confidence bands following suit. Thus, while acquisition incentives seem to be more skewed toward higher productivity firms, the results still support the basic features of the theoretical model presented in section two.

4 Conclusion

This paper presented an industry-equilibrium model of acquisition behavior that endogenizes the reallocation of capital after productivity has been realized. The incentives for acquisition, driven by firm heterogeneity in productivity, result from cost-lowering acquisitions and a
Figure 8: Acquisition Incentives - Alternate Productivity Measures

Each panel illustrates the relative probability of acquisition activity as a function of productivity for North American firms, 1980-2004. SalesEmp, Q, and TFP measures are used as described in the text, with the former two controlling for serial correlation at the firm level, and TFP with and without this correction. The expected value of $s(\text{Productivity})$ is normalized to zero for identification. 90% Bayesian confidence intervals are provided. In addition, the vertical dashed lines represent (from left to right) the 1st, 5th, 10th, 25th, 50th, 75th, 90th, 95th and 99th percentiles of productivity. The hashed marks on the x-axis represent data points.
linear demand framework. The main results of the model show that mid-productivity firms are the most likely to acquire another firm. This prediction is empirically supported using a panel of North American firms. Further, the degree to which firms may set price is shown to be a crucial theoretical and empirical determinant of acquisition decisions. Overall, the paper supports the notion that the incentives to engage in cost-lowering investments are complex, but fundamentally influenced by the characteristics of demand facing each firm.

Future work is bound to focus on two areas: matching in the acquisition market, and foreign acquisitions. With regard to the former, part of the simplicity of the model is based on the assumption of a common price of capital within each industry. In reality, acquisition prices are bargained after potential matches are identified, and there is likely a tremendous amount of uncertainty within each potential acquisition. With regard to the latter, a large share of FDI involves the transfer of ownership across borders. While a companion paper (Spearot, 2008) extends the above model to a setting with multiple countries and trade costs, more work is needed to precisely address the welfare effects of foreign acquisitions relative to other forms of market-entry.

References


A Technical Appendix

A.1 Proof of Lemma 1

To prove Lemma 1 for small $R_a$, I need to establish that $\alpha_S < \alpha_B < \bar{\alpha}_B$. Once this ranking is established, Lemma 1 is immediate via the preference conditions in (16) and (19). To show $\alpha_S < \alpha_B$, first note that from (15) and (17) it must be the case that:

$$\pi^B(\alpha_B) - \pi^N(\alpha_B) = \pi^N(\alpha_S)$$

Rearranging,

$$\frac{1}{2}\pi^B(\alpha_B) - \pi^N(\alpha_B) = \pi^N(\alpha_S) - \frac{1}{2}\pi^B(\alpha_B)$$

Since $\frac{1}{2}\pi^B(\alpha_B) < \pi^N(\alpha_B)$, the RHS must also be negative in equilibrium. This is only possible if $\alpha_S < \alpha_B$. By definition, $\alpha_B < \bar{\alpha}_B$. Using this result and $\alpha_S < \alpha_B$, this completes the proof that $\alpha_S < \alpha_B < \bar{\alpha}_B$.

A.2 Proof of Lemma 2

Holding $A$ fixed, differentiating the acquisition profit functions (15), (17), and (18) with respect to $R_a$, and using the properties of $\frac{\pi^B(\alpha)}{\partial \alpha} - \frac{\pi^N(\alpha)}{\partial \alpha}$, I get:

$$\frac{\pi^N(\alpha_S(R_a(A),A))}{\partial \alpha} \frac{\partial \alpha}{\partial R_a(A)} \overset{>0}{=} 1$$

$$\left(\frac{\pi^B(\alpha_B(R_a(A),A))}{\partial \alpha} - \frac{\pi^N(\alpha_B(R_a(A),A))}{\partial \alpha}\right) \frac{\partial \alpha_B}{\partial R_a(A)} \overset{>0}{=} 1$$

$$\left(\frac{\partial \pi^B(\alpha_B(R_a(A),A))}{\partial \alpha} - \frac{\pi^N(\alpha_B(R_a(A),A))}{\partial \alpha}\right) \frac{\partial \alpha}{\partial R_a(A)} \overset{<0}{=} 1$$

40
These derivatives clearly yield \( \frac{\partial \alpha_S(R_a(A), A)}{\partial R_a(A)} > 0 \), \( \frac{\partial \alpha_B(R_a(A), A)}{\partial R_a(A)} > 0 \), and \( \frac{\partial \sigma_B(R_a(A), A)}{\partial R_a(A)} < 0 \). Differentiating (20) and (21) with respect to \( R_a(A) \), and imposing \( \frac{\partial \alpha_S(R_a(A), A)}{\partial R_a(A)} > 0 \), \( \frac{\partial \alpha_B(R_a(A), A)}{\partial R_a(A)} > 0 \), and \( \frac{\partial \sigma_B(R_a(A), A)}{\partial R_a(A)} < 0 \),

\[
\frac{\partial K_D(R_a(A))}{\partial R_a} = M_E k g(\alpha_B(R_a(A), A)) \frac{\partial \alpha_B(R_a(A), A)}{\partial R_a} - M_E k g(\alpha_B(R_a(A), A)) \frac{\partial \sigma_B(R_a(A), A)}{\partial R_a} < 0
\]

(31)

\[
\frac{\partial K_S(R_a(A))}{\partial R_a} = M_E k \left( g(\alpha_S(R_a(A), A)) \frac{\partial \alpha_S(R_a(A), A)}{\partial R_a} \right) > 0
\]

(32)

Naturally, \( K_D(R_a(A)) \) is decreasing and \( K_S(R_a(A)) \) is increasing in the acquisition price. Thus, if \( K_D(R_a) \) is larger (smaller) than \( K_S(R_a) \) at low (high) values of \( R_a \), there exists a unique \( R_a \) that clears the acquisition market. As \( R_a \to 0 \), using (10), (11) and (12),

\[
\alpha_S(R_a(A), A) \to 0, \quad \alpha_B(R_a(A), A) \to 0, \quad \sigma_B(R_a(A), A) \to \infty
\]

This yields,

\[
K_D(R_a(A)) \to M_E k, \quad K_S(R_a(A)) \to 0
\]

(33)

Similarly, as \( R_a \to \frac{(3-2\sqrt{2})A^2}{4b} \):

\[
\alpha_S(R_a(A), A) > 0, \quad \alpha_B(R_a(A), A) \to \frac{\sqrt{2}}{4bk}, \quad \sigma_B(R_a(A), A) \to \frac{\sqrt{2}}{4bk}
\]
which yields:

\[ K_D (R_a (A)) \to 0 \quad (34) \]
\[ K_S (R_a (A)) > 0 \]

Thus, using (33) and (34), there exists a unique value \( \hat{R}_a (A) \) that clears the acquisition market.

**A.3 Demand**

As in standard monopoly models, optimal output is determined by:

\[ P' \cdot q + P = \frac{q}{\alpha k} \quad (35) \]

By the envelope theorem, the marginal value of additional capital can be written as:

\[
\frac{\partial \Pi}{\partial k} = \left( P' \cdot q + P - \frac{q}{\alpha k} \right) \frac{\partial q}{\partial k} + \frac{q^2}{\alpha k^2}
\]
\[
= \frac{q^2}{\alpha k^2} > 0 \quad (36)
\]

Note that the marginal value of additional capital (net of capital costs) is always positive. Using (35), equation (36) can be written as:

\[
\frac{\partial \Pi}{\partial k} = \alpha (P' \cdot q + P)^2 = \alpha (MR(q))^2
\]

Differentiating with respect to \( \alpha \), we get:

\[
\frac{\partial^2 \Pi}{\partial k \partial \alpha} = (MR(q))^2 + 2\alpha (MR(q)) \frac{\partial MR(q)}{\partial q} \frac{\partial q}{\partial \alpha}
\]
As the final intermediate step, note that using (35) \( \frac{\partial q}{\partial \alpha} \) is written as,

\[
\frac{\partial q}{\partial \alpha} = \frac{kMR(q)}{1 - \alpha k \frac{\partial MR(q)}{\partial q}}
\]

and thus \( \frac{\partial^2 \Pi}{\partial k \partial \alpha} \) is simplified as:

\[
\frac{\partial^2 \Pi}{\partial k \partial \alpha} = \begin{pmatrix}
\begin{array}{c}
\partial MR(q) \\
\partial q
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
1 \\
1 - \alpha k \frac{\partial MR(q)}{\partial q}
\end{array}
\end{pmatrix}
\]

\( (37) \)

A.4 Alternative Acquisition Market

To show that the non-monotonic behavior of capital acquisitions is not a function of "lumpy" capital purchases, consider the following maximization problem of a firm purchasing capital on the margin.

\[
\max_s \left\{ \frac{A^2 \alpha_s (k+s)}{2(2b\alpha_s (k+s) + 1)} - r * s \right\}
\]

such that: \( s \geq -k \)

Here, \( s \) is the level of acquisition by the firm, and \( r \) is the acquisition price. The unconstrained solution to this problem is:

\[
s^* = -k - \frac{1}{2\alpha b} + \frac{\sqrt{2A}}{4b\sqrt{r\alpha}}
\]

Clearly, \( \lim_{\alpha \to 0} s^* = -\infty \) and \( \lim_{\alpha \to \infty} s^* = -k \). Taken with the constraint on initial capital holdings, this is suggestive that high and low productivity firms are likely to sell (note that \( s^* = -k \) at \( \alpha = \frac{2r}{A^2} \)). Further, note that the maximum of \( s^* \) occurs at \( \alpha = \frac{8r}{A^2} \), where \( \frac{\partial s^*}{\partial \alpha} > 0 \) for \( \alpha < \frac{8r}{A^2} \) and \( \frac{\partial s^*}{\partial \alpha} < 0 \) for \( \alpha > \frac{8r}{A^2} \). At this maximum, profits equal \( \frac{A^2}{16r^2} - k \). Since \( s^* \) has its limits below zero and a maximum that must be above zero for an acquisition
equilibrium to exist (low enough $r$), the intermediate value theorem guarantees that there exists a $\alpha$ and $\bar{\alpha}$ such that for $\alpha \in (\alpha, \bar{\alpha})$ firms will purchase capital on the margin. Thus, acquisition behavior is qualitatively similar to the case with lumpy capital purchases.

A.5 A Dynamic Extension

In this appendix subsection, I show how the intuition from the simple static model extends to a dynamic setting with capital and productivity heterogeneity, and the option of new investment. In doing so, I will extend the marginal investment model from the previous appendix. In terms of analysis, I will closely follow the model of Jovanovic and Rousseau (2002)

Motivated earlier, I assume that the only difference between acquisitions and new investment is the time required to make the investment operational. In the static model, all new investment took place "behind the scenes", before productivity was realized. In the dynamic setup, I assume that acquisitions are immediately operational, and new investment is operational only in the next period. Further, I assume that productivity varies from period to period, subject to a stochastic process $G(\alpha' | \alpha)$. Thus, the entire payoff to new capital investment will be subject to productivity uncertainty, while part of the payoff from acquisitions will be subject to no productivity uncertainty. Formally, the optimization problem for a firm with productivity $\alpha$ and capital holdings $k$ is written as:

$$V(\alpha, k) = \max_{I_A \geq -k, I_N \geq 0} \left\{ \max \left\{ \begin{array}{c} r_a k, \pi(\alpha, k + I_A) - r_a I_A - r_I I_N \\ + \beta \int V(\alpha', (k + I_A) + I_N) dG(\alpha' | \alpha) \end{array} \right\} \right\}$$

(38)

In (38), $I_A$ represents the level of acquisition activity, $I_N$ represents the level of new investment, $r_I$ is the price of this new investment per unit, and $r_a$ is the price of an acquisition per unit. Also, $\beta$ is the discount factor, and $\delta$ is the rate of depreciation.

Focusing on firms that do not sell and exit ($I_A > -k$), an interior solution to maximizing
\( V(\alpha, k) \) is characterized by the following first order conditions:

\[
\frac{\partial \pi (\alpha, k + I_A)}{\partial K} + \delta \beta \int \frac{\partial V (\alpha, (k + I_A)\delta + I_N)}{\partial K} dG(\alpha' | \alpha) = r_a
\]

(39)

\[
\beta \int \frac{\partial V (\alpha, (k + I_A)\delta + I_N)}{\partial K} dG(\alpha' | \alpha) = r_I
\]

(40)

As mentioned earlier, the objective of this section is to examine how acquisition incentives might change in a dynamic model with additional investment options. In short, these incentives change very little. Substituting (40) into (39), we get the following implicit determination of the acquisition level, \( I_A \):

\[
\frac{\partial \pi (\alpha, k + I_A)}{\partial K} = r_a - \delta r_I
\]

(41)

Clearly, the solution for optimal acquisition behavior is identical to that in the previous section, with the exception that \( r_a - \delta r_I \) replaces the simple marginal acquisition price. By exploiting the time-sensitivity of new investment and acquisitions, the effect of acquisitions on the continuation value is identical to the effect of new investment (after accounting for depreciation). Thus, any effects of new investment on acquisition decisions will be a function of the cost of new investment, \( r_I \). To the extent \( r_I \) is embodied by industry and year controls, identification of acquisition incentives only requires a proxy for \( \alpha, k, \) and other controls to account for changes aggregate measures (\( A, r_a, r_I, b \)).

A.6 Free Trade

In this section, the closed economy model is extended to a two-country reciprocal markets model of trade. Each country is identical in every dimension, yielding a symmetric equilibrium. For simplicity, and to prove the main point of the section, I assume that trade is costless. I will focus on firms in the home country as I lay out the framework of the open
The consumer’s problem is virtually identical to the closed economy, with the exception that \( M \) (measure of active firms) and \( p \) (average price of varieties) now include foreign varieties. There exist two potential sources of production, one located in each country. Given the two potential sources of production, the product market maximization problem for an active firm in stage three is the following:

\[
\pi\left(\alpha_i, K^H, K^F\right) = \max_{q_i, q_i^x, q_i^F} \left\{ \begin{array}{c}
(A - b \cdot q_i) \cdot q_i + \left( A - b \cdot \left( q_i^x + q_i^F \right) \right) \cdot \left( q_i^x + q_i^F \right) \\
- \frac{1}{2} \cdot \frac{(q_i^x + q_i)^2}{\alpha_i K^H} - \frac{1}{2} \cdot \frac{(q_i^F)^2}{\alpha_i K^F}
\end{array} \right\}
\]

such that: \( q_i \geq 0, q_i^F \geq 0 \), and \( q_i^x \geq 0 \)

The top line in equation (42) represents revenues in the home and foreign market, and the second line represents the costs of production in each market. In (42), \( q_i \) is home production for the domestic market, \( q_i^x \) is home production for exports, and \( q_i^F \) is foreign production for the foreign market. Finally, \( K^H \) and \( K^F \) represent home and foreign capital holdings, respectively.\(^{21}\)

In the open economy, there are three types of firms active in the product market. Firms that "do nothing" \((N)\) in the acquisition market retain their initial capital level at home, \(k\). Firms that buy domestic capital \((B)\) in the acquisition market double their home capital level, holding \(2k\). Firms that buy foreign capital \((B^*)\) in the acquisition market retain \(k\) at home and acquire \(k\) units abroad. Again, firms that sell and exit \((S)\) in the acquisition stage are not active in the product market.

\(^{21}\)Throughout this section, it is assumed that \(K_H \geq K_F\), which implies that production costs abroad are never less than costs at home. Thus, firms never find it optimal to serve the home market via imports from production facilities abroad.
Solving (42) for all active firms in the product market and dropping $i'$s, profits can be written as:

**No Acquisition (N)**

\[
\pi^{N,FT}(\alpha) = \frac{A^2\alpha k}{2(b\alpha k + 1)} \tag{43}
\]

**Domestic Acquisition (B)**

\[
\pi^{B,FT}(\alpha) = \frac{A^2\alpha k}{2b\alpha k + 1} \tag{44}
\]

**Foreign Acquisition (B*)**

\[
\pi^{B*,FT}(\alpha) = \frac{A^2\alpha k}{2b\alpha k + 1} \tag{45}
\]

Note that if $b$ is replaced with $2b$ in (43) and (44), the profit functions are identical to the closed economy. This is intuitive as free trade represents an integrated world market twice the size of the closed economy.

Further, it is not coincidental that $\pi^{B*,FT}(\alpha) = \pi^{B,FT}(\alpha)$. Both foreign and domestic acquisitions provide a firm with two lumps of capital (plants) in an integrated world market. Via profit maximization, firms equalize marginal costs across their entire holding of plants. Given that trade is costless on the margin, the stage three profits of holding an additional plant abroad and an additional plant at home are identical.

To push this point further, I now move back to stage two and derive optimal acquisition decisions. Assuming that physical capital cannot be moved across borders, there exist two segmented capital acquisition markets, one in each country. Within each market, firms that purchase capital do so at a price $R_a$ per lump.\footnote{The assumption of symmetry will ensure that $R_a$ is identical in each country.} The benefits of buying a domestic firm
and foreign firm, respectively, are written as:

\[
\begin{align*}
\Delta \Pi_{FT}(\alpha) &= \pi^{B,FT}(\alpha) - \pi^{N,FT}(\alpha) = \frac{A^2\alpha k}{2(bak + 1)(2bak + 1)} \quad (46) \\
\Delta \Pi_{FT}^*(\alpha) &= \pi^{B^*,FT}(\alpha) - \pi^{N,FT}(\alpha) = \frac{A^2\alpha k}{2(bak + 1)(2bak + 1)} \quad (47)
\end{align*}
\]

Clearly, these incentives are identical. As stated above, the profits in the product market from \(B\) and \(B^*\) are identical, since firms simply equalize marginal costs across their entire capital holdings, and produce for an integrated world market. However, the effect of acquisitions on production costs, which yields this equivalence, is more subtle. If firms purchase capital domestically, they add to their overall domestic capital stock. This makes variable factors more efficient, and facilitates revenue gains via increased sales. In contrast, with foreign acquisitions, domestic factors become more efficient by diverting export production which would otherwise be produced at home. Given the fixed nature of capital, a lower production level at home yields a lower marginal cost for the last unit produced at home. Thus, firms sell more to the integrated world market at a lower average cost.

In both cases, acquiring additional capital improves efficiency at home. Through these efficiency gains, the firm can increase sales. However, the extent to which a firm can increase sales is critically dependent on their endowed productivity level. As argued in the closed economy model, the effects of a cost reduction depend on the shape of the demand curve and the nature of competition. Under the assumption of linear demand, high productivity firms are precisely the firms which operate on a less elastic portion of the demand curve. This is precisely where the incentives to invest are low. Thus, mid-productivity firms have the highest incentive to acquire, whether at home or abroad.

Focusing on foreign acquisitions, these incentives are different from the existing literature on foreign investment and firm heterogeneity. In the received literature, the incentives to invest abroad are almost always based on market access. For example, in Helpman, Melitz,
and Yeaple (2004), only high productivity firms can profitably invest abroad. The intuition is that high productivity firms benefit the most from avoiding trade costs, and thus are the only firms which can afford the fixed cost of foreign investment. These incentives are similar in Nocke and Yeaple (2007), where if firm-specific productivity is transferrable across borders, a result is derived in which the highest productivity firms either acquire or invest in new facilities abroad. Head and Reis (2003) is most closely related to the present result, where they extend the Helpman, Melitz, and Yeaple (2004) framework to countries with asymmetric costs of production. In their work, parameter values exists such that low productivity firms are the only firms that choose to invest abroad. The intuition is that if the host country has relatively low wages, high-cost firms will invest in order to offset their low endowed productivity.

Fundamentally, the incentives in (46) and (47) are a result of a different mechanism, which is the effect of domestic and foreign investment on domestic efficiency. Practically, these efficiency gains can be realized in a number of ways. For example, after the purchase of another plant, an existing plant may be able to reduce shifts which are either produced using overtime labor, or less-skilled workers. Generally, mergers that reduce the combined production of the merged firms will result in either the least efficient assets, or those being operated inefficiently, to be retired.

Moving forward, I now characterize the equilibrium of the free trade model. Given that (46) and (47) are identical to the closed economy if $b$ is replaced with $2b$, the equilibrium of the model is trivially similar to the closed economy. The only difference is that the location of acquisitions is indeterminate, given the equivalence of (46) and (47). Denoting $\alpha_{SFT}$, as the firm which is marginally indifferent between selling and doing nothing, and $\alpha_{FT}^{FT}$

---

23 In Nocke and Yeaple (2007), if firm-specific capabilities not transferrable across borders, then the least productive firms acquire abroad and the most productive firms invest greenfield.

24 Another example would be airline mergers, where the number of flights of the merged firm are often reduced following a merger. In this case, the merged firm may retire older planes which are either less efficient or incur higher maintenance costs. For a good review of airline mergers, see Kim and Singal (1993).
and $\bar{\pi}^{FT}$ as the firms indifferent between buying at home or abroad and doing nothing, the equilibrium of the model is summarized in the following proposition.

**Proposition 3** The free trade acquisition equilibrium consists of a unique $A, R_o, \alpha_s^{FT}, \alpha^{FT}, \bar{\alpha}^{FT}$, and $\bar{\pi}^{FT}$ such that:

For $\alpha \in [0, \alpha^{FT}_S)$, firms sell

$\alpha \in [\alpha^{FT}_S, \alpha^{FT}_B]$, firms do nothing

$\alpha \in (\alpha^{FT}_B, \bar{\alpha}^{FT})$, firms buy capital (location indeterminate)

$\alpha \in [\bar{\pi}^{FT}_B, \infty)$, firms do nothing

**Proof.** Identical to the closed economy.

With the exception of export behavior and the possibility of foreign acquisitions, there are no qualitative differences between the closed economy and free trade. Mid-productivity firms find an acquisition profitable, as they are not constrained by either low productivity, or bounds on market revenues. The location of acquisitions is indeterminate, since domestic and foreign acquisitions offer the same benefits subject to the same cost. However, this indeterminacy is broken under a couple of alternative scenarios. These are summarized in the following corollary.

**Corollary 1** If there exists a infinitesimal fixed cost of exporting, firms with $\alpha \in (\alpha^{FT}_B, \bar{\alpha}^{FT})$ acquire abroad. In contrast, if there is an infinitesimal fixed cost of foreign investment, firms with $\alpha \in (\alpha^{FT}_B, \bar{\alpha}^{FT})$ acquire domestically.

**Proof.** Straightforward when comparing (46) and (47)

If exporting incurs a tiny cost, but foreign investment does not, it is clear that acquiring firms will only do so abroad. In (46), since both types of firms incur the fixed cost, the equation does not change. However, in (47), the firm benefits from avoiding the exporting.
costs, and thus (47) increases. In contrast, if the tiny fixed cost is only occurred with foreign investment, acquiring firms will only do so domestically, since foreign acquisitions are now less profitable for all $\alpha$.

In summary, the present model shows that moving from a closed economy to an open economy will not necessarily change the incentives to acquire another firm. However, one incentive that is absent from this setup is the incentive to avoid trade costs. It is this incentive that is central to most simple foreign investment models, such as Helpman, Melitz and Yeaple (2004). Generally, the incentive to avoid trade costs is increasing in productivity. Since countries are able to manipulate trade costs via trade agreements or unilateral actions, deriving the precise effects of trade costs will help understand the broad effects of trade agreements on both trade flows and investment patterns. Not surprisingly, the full treatment of trading costs and foreign investment costs is extremely complex. Allowing for a trade cost yields a model which pins market access incentives, ala Helpman, Melitz and Yeaple (2004), against cost incentives as described above.$^{25}$

$^{25}$This is an area of current work.