Variable Demand Elasticities and Trade Liberalization
Supplemental Appendix

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1 Two-way trade and Free Entry

In this appendix, I detail how allowing for two-way trade greatly complicates the model in terms of characterizing the long-term effects of trade liberalization on entry. To simplify the discussion of this issue, and to relate the model to its predecessor, Melitz and Ottaviano (2008), I will focus on the effects of ad-valorem tariffs, and assume that firms only produce varieties within one product in each industry.

For each foreign country l, the free entry condition is written as:

$$L_H \int_0^{A_l} \frac{(A_l - c)^2}{4\gamma} g_l(c)dc + L_l \int_0^{A_H/t_H} \frac{(A_H - c \cdot t_H)^2}{4\gamma t_H} g_l(c)dc = F_E \tag{1}$$

Under two-way trade, for firms in the home market, the free entry condition is written as,

$$L_H \int_0^{A_H} \frac{(A_H - c)^2}{4\gamma} g_H(c)dc + \sum_{l=1}^{M} L_l \int_0^{A_l/t_l} \frac{(A_l - c \cdot t_l)^2}{4\gamma t_l} g_H(c)dc = F_E \tag{2}$$

where firms in H can now earn profits at home and in M foreign markets. Using Leibniz Rule to differentiate (1) and (2) with respect to t_H, and rearranging, one can derive the effects of t_H on the residual demand level at home A_H,

$$\frac{\partial A_H}{\partial t_H} = \frac{\sum_{l=1}^{M} \frac{\int_0^{\frac{A_H}{t_H}} \frac{\partial}{\partial t_H} \frac{(A_H - c \cdot t_H)^2}{4\gamma} g_l(c)dc \int_0^{\frac{A_l}{t_l}} \frac{\partial}{\partial A_l} \frac{(A_l - c \cdot t_l)^2}{4\gamma t_l} g_H(c)dc}{\int_0^{A_l} \frac{\partial}{\partial A_l} \frac{(A_l - c)^2}{4\gamma} g_l(c)dc \int_0^{A_H} \frac{\partial}{\partial A_H} \frac{(A_H - c)^2}{4\gamma} g_H(c)dc}}{1 - \sum_{l=1}^{M} \frac{\int_0^{\frac{A_H}{t_H}} \frac{\partial}{\partial A_H} \frac{(A_H - c \cdot t_H)^2}{4\gamma t_H} g_l(c)dc \int_0^{\frac{A_l}{t_l}} \frac{\partial}{\partial A_l} \frac{(A_l - c \cdot t_l)^2}{4\gamma t_l} g_H(c)dc}{\int_0^{A_l} \frac{\partial}{\partial A_l} \frac{(A_l - c)^2}{4\gamma} g_l(c)dc \int_0^{A_H} \frac{\partial}{\partial A_H} \frac{(A_H - c)^2}{4\gamma} g_H(c)dc}} \tag{3}$$

where the sign of the numerator is negative, since the derivative of any profit function with respect to the demand intercept is positive, and the derivative of any profit function with respect to the tariff (holding the demand intercept constant) is negative. However, the sign of the denominator is ambiguous. To provide more guidance regarding the sign of the denominator, note that it can be rewritten as:

$$\begin{aligned} D &\equiv 1 - \sum_{l=1}^{M} \frac{\int_0^{\frac{A_H}{t_H}} \frac{\partial}{\partial A_H} \frac{(A_H - c \cdot t_H)^2}{4\gamma t_H} g_l(c)dc \int_0^{\frac{A_l}{t_l}} \frac{\partial}{\partial A_l} \frac{(A_l - c \cdot t_l)^2}{4\gamma t_l} g_H(c)dc}{\int_0^{A_l} \frac{\partial}{\partial A_l} \frac{(A_l - c)^2}{4\gamma} g_l(c)dc \int_0^{A_H} \frac{\partial}{\partial A_H} \frac{(A_H - c)^2}{4\gamma} g_H(c)dc} \\ &= 1 - \sum_{l=1}^{M} \frac{1}{t_l t_H} \frac{Q_l^H Q_H^l}{Q_H^H Q_l^l} \end{aligned} \tag{4}$$

where Q_l^H represents quantity sold from l to country H. To build intuition, I will move forward by assuming that the distribution governing costs in any country j is defined by a Pareto distribution with shape parameter, k_j, defined over the support $(0, \overline{c})$. With this assumption, (4) can be rewritten as:

$$D = 1 - \sum_{l=1}^{M} \rho_l^{k_H+1} \rho_H^{k_l+1} \left(\frac{A_l}{A_H} \right)^{k_H - k_l} \tag{5}$$

where $\rho_j = \frac{1}{t_j} \in [0,1]$. In (5), D can be signed once we know the value of A_H and A_l for all l. These demand intercepts are pinned down by the free entry conditions, which under the Pareto assumption, are written as:

$$E\Pi_H = L_H \frac{A_H^{k_H+2}}{\psi_H} + \sum_{l=1}^{M} L_l \frac{A_l^{k_H+2} \rho_l^{k_H+1}}{\psi_H} \quad = \quad F_E \tag{6}$$

and for all $l \in M$,

$$E\Pi_l = L_l \frac{A_l^{k_l+2}}{\psi_l} + L_H \frac{A_H^{k_l+2} \rho_H^{k_l+1}}{\psi_l} \quad = \quad F_E \tag{7}$$

where $\psi_j = 2\gamma(k_j+1)(k_j+2)\overline{c}^{k_j}$.

In (5) I need to solve for equilibrium values of A_H and A_l for all l to sign the denominator of $\frac{\partial A_H}{\partial t_H}$. However, in (6), and (7) for all $l \in M$, we have an $(M+1)$ by $(M+1)$ system of polynomials in A_H and A_l's, each of arbitrary degree greater than 2. Hence, solving these conditions is intractable. However, we can solve the model under the assumption of symmetry to build intuition about the particular conditions that provide an unambiguous sign on D. Assuming that $k_l = k_H \equiv k$ for all l, it is straightforward to show the following:

$$A_H^{k+2} = \frac{\psi F_E}{L_H} \cdot \frac{1 - \sum_{l=1}^{M} \rho_l^{k+1}}{1 - \sum_{l=1}^{M} \rho_l^{k+1} \rho_H^{k+1}} \tag{8}$$

and,

$$A_l^{k+2} = \frac{\psi F_E}{L_l} \cdot \frac{\overbrace{1 - \rho_H^{k+1}}^{>0}}{1 - \sum_{l=1}^{M} \rho_l^{k+1} \rho_H^{k+1}} \tag{9}$$

where $\psi = 2\gamma(k+1)(k+2)\overline{c}^k$. Since $\sum_{l=1}^{M} \rho_l^{k+1} > \sum_{l=1}^{M} \rho_l^{k+1} \rho_H^{k+1}$, when $A_H > 0$, then it must be the case that $1 - \sum_{l=1}^{M} \rho_l^{k+1} \rho_H^{k+1} > 0$. Hence, in (5), after imposing equality across Pareto parameters,

$A_H > 0$ is a sufficient condition for $\frac{\partial A_H}{\partial t_H} < 0$.

What is the intuition for the sign on $1 - \sum_{l=1}^{M} \rho_l^{k+1} \rho_H^{k+1}$, or the more complex case in (5)? Under the more complex case where Pareto parameters differ, note that I can write $\frac{\partial E\Pi_H}{\partial A_H}$ as:

$$\frac{\partial E\Pi_H}{\partial A_H} = (k_H + 2)\, A_H^{k_H+1} \underbrace{\left(1 - \sum_{l=1}^{M} \rho_l^{k_H+1} \rho_H^{k_l+1} \left(\frac{A_l}{A_H}\right)^{k_H - k_l}\right)}_{D}$$

Hence, if $D > 0$, an increase in the demand level at home, all else equal, increases the average profits of home firms. Under the identical Pareto assumption, the converse is not possible since the elasticity of entering firms (through the Pareto parameters) is identical across countries. However, when the Pareto parameters differ, and in particular when the elasticity of entering firms in the foreign country is more pronounced, it is possible that $\frac{\partial E\Pi_H}{\partial A_H} < 0$, where higher A_H induces a strong entry effect via the foreign exporters.

Since D cannot be signed generally, the question now becomes whether D is greater than zero in the data? Rewriting (5) and substituting aggregate revenues for imports, exports, and domestic sales under the Pareto distribution, we get the following simple representation for D.

$$D = 1 - \sum_{l=1}^{M} \frac{V_l^H V_H^l}{V_H^H V_l^l} \tag{10}$$

where V_l^H represents aggregate revenues in selling from l to H. We can calculate (10) given sufficient information on the value of trade between countries, and domestic sales. Though at a (much) higher level of aggregation than in the manuscript, a number of databases provide such information. For example, the CEPII Trade, Production and Bilateral Protection Database, from Mayer and Zignago (2005), provides domestic GDP along with bilateral exports and imports at the 3-digit ISIC level. Using the CEPII dataset, I calculate (10) for the years (1990-2000). To measure V_l^H, I use reported trade values. To measure V_l^l, I measure value of domestic production in l and subtract the value of total exports from l. Overall, for 98.8% of year-sic observations, $D > 0$. Further, the median value is 0.984, and the mean is 0.736 (heavily skewed by a few very large negative values). Hence, while I cannot unambiguously sign D without imposing strong restrictions on trade costs or by evaluating the model in the neighborhood of symmetry, the data suggest that D is very likely positive for a strong majority of industries in which the US has reciprocal trading relationships.

2 Elasticity Heterogeneity and Welfare Projections

Broadly speaking, heterogeneity in export sources will yield two types of bias related to projecting the welfare effects of tariffs: within-exporters and across-exporters. In the former, a bias arises since assigning a constant elasticity with respect to tariffs ignores the fact that individual firms sell varieties at different elasticities. In the latter, the bias arises from ignoring that exporters may each have a different average productivity of firms that export to the import market (ultimately leading to different average elasticities). As both biases operate in the same manner, I will focus on within-exporter bias in this section. In particular, I will derive explicitly the size and direction of the bias assuming Pareto which will ultimately be a function of an aggregate trade elasticity, and the heterogeneity across firms within an exporter.

Welfare in the model is a function of tariff revenue and consumer surplus. For simplicity, I will assume that there is no domestic sector in the import market, only one exporter l selling varieties of only one product, and that the analysis of tariffs holds the number of entering firms fixed in l. After substituting the budget constraint into the utility function, and substituting the inverse demand curve for each variety (to remove the consumer price in the budget constraint), and rearranging, welfare for the importing market can be written as

$$W = \underbrace{I + \frac{\eta}{2}Q_l^2 + \frac{\gamma}{2}N_l\int_0^{A/t} q(c)^2 g_l(c)dc}_{Consumer\ \ Surplus} + \underbrace{\tau N_l \int_0^{A/t} v(c)g_l(c)dc}_{Tariff\ \ Revenue}$$

where Q_l is total imports, $q(c)$ is firm-level import quantity, and $v(c)$ is firm-level import value. Differentiating with respect to t, defining V_l as total import value, and writing in terms of elasticities, we get:

$$\frac{\partial W}{\partial t} = \frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \frac{1}{t}\gamma N_l \int_0^{A/t} q(c)^2 \epsilon_{q,t} g_l(c)dc + V_l + \frac{1}{t}\tau N_l \int_0^{A/t} v(c)\epsilon_{v,t} g_l(c)dc \qquad (11)$$

The issue of measuring the welfare effects within this setup is apparent in (11), where along with exogenous parameters, one needs estimates of the aggregate elasticities of total imports ($\epsilon_{Q_l,t}$ for quantity and $\epsilon_{V_l,t}$ for value) along with the individual elasticities for each variety in both quantity and value ($\epsilon_{q,t}$ and $\epsilon_{v,t}$, respectively). While the aggregate elasticities can be estimated, the latter two cannot without firm-level data. Hence, I now evaluate how assumptions over the firm-level elasticities $\epsilon_{q,t}$ and $\epsilon_{v,t}$ yield a bias in the effects of tariffs on welfare.

I will evaluate welfare over two cases that involve projecting the effects of tariff cuts. In the first, I will assume that I can properly assess the firm-level response to tariffs. I will label the change in welfare under this setting as $\frac{\partial W}{\partial t}^{True}$. In the second, I will erroneously apply the aggregate import elasticity to all firms, labeling this (biased) welfare calculation as $\frac{\partial W}{\partial t}^{Agg}$. Indeed, this thought experiment is likely relevant in that absent long panel datasets of firm-level trade (which are not available for all countries), aggregate estimates are likely to be used to project the firm-level response to tariffs.

First, I derive the true change in welfare, which requires that firm-level elasticities are perfectly observed. These elasticities can be written as:

$$\epsilon_{q,t} = \frac{(\epsilon_A - 1)\, A}{2\gamma q(c)} + 1$$
$$\epsilon_{v,t} = \frac{(\epsilon_A - 1)\, A^2}{2\gamma t v(c)} + 1$$

Again, ϵ_A is the elasticity of the variety-specific demand intercept with respect to tariffs. Plugging into (11) and simplifying, we get:

$$\begin{aligned} \frac{\partial W}{\partial t}^{True} &= \frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \frac{1}{t}\gamma N_l \int_0^{A/t} \left(\frac{(\epsilon_A - 1)\, A}{2\gamma} q(c) + q(c)^2 \right) g_l(c) dc \\ & \quad + V_l + \tau N_l \int_0^{A/t} \left(\frac{(\epsilon_A - 1)\, A^2}{2\gamma t} + v(c) \right) g_l(c) dc \end{aligned}$$

Noting that $q(c)^2 = \frac{t\pi(c)}{\gamma}$ and total profits can be written as $\Pi_l = N_l \int_0^{A/t} \pi(c) g_l(c) dc$, I can simply $\frac{\partial W}{\partial t}^{True}$ as:

$$\frac{\partial W}{\partial t}^{True} = \frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \frac{1}{t}\gamma \frac{(\epsilon_A - 1)\, A}{2\gamma} Q_l + \Pi_l + V_l + \tau N_l G_l(A/t) \frac{(\epsilon_A - 1)\, A^2}{2\gamma t} + \tau V_l$$

Next, I can use the same process for the case where I erroneously apply the aggregate elasticity to all firms. To begin this derivation, note that the aggregate elasticities are written as:

$$\epsilon_{Q_l,t} = \frac{(\epsilon_A - 1)\, A}{2\gamma \overline{q}_l} + 1$$
$$\epsilon_{V_l,t} = \frac{(\epsilon_A - 1)\, A^2}{2\gamma t \overline{v}_l} + 1$$

Plugging into (11):

$$\frac{\partial W}{\partial t}^{Agg} = \frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \frac{1}{t}\gamma N_l \int_0^{A/t} q(c)^2 \left(\frac{(\epsilon_A - 1)\, A}{2\gamma \overline{q}_l} + 1\right) g_l(c)dc$$
$$+V_l + \tau N_l \int_0^{A/t} v(c) \left(\frac{(\epsilon_A - 1)\, A^2}{2\gamma t \overline{v}_l} + 1\right) g_l(c)dc$$

Simplifying, we get:

$$\frac{\partial W}{\partial t}^{Agg} = \frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \Pi_l \frac{(\epsilon_A - 1)\, A}{2\gamma \overline{q}_l} + \Pi_l + V_l + \tau \frac{(\epsilon_A - 1)\, A^2 V_l}{2\gamma t \overline{v}_l} + \tau V_l$$

Subtracting $\frac{\partial W}{\partial t}^{Agg}$ from $\frac{\partial W}{\partial t}^{True}$,

$$\frac{\partial W}{\partial t}^{True} - \frac{\partial W}{\partial t}^{Agg} = \frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \frac{1}{t}\gamma \frac{(\epsilon_A - 1)\, A}{2\gamma} Q_l + \Pi_l + V_l + \tau N_l G_l(A/t) \frac{(\epsilon_A - 1)\, A^2}{2\gamma t} + \tau V_l$$
$$- \left(\frac{1}{t}\eta Q_l^2 \epsilon_{Q_l,t} + \Pi_l \frac{(\epsilon_A - 1)\, A}{2\gamma \overline{q}_l} + \Pi_l + V_l + \tau \frac{(\epsilon_A - 1)\, A^2 V_l}{2\gamma t \overline{v}_l} + \tau V_l\right)$$

and noting that $N_l G_l(A/t)\overline{v}_l = V_l$, $\frac{\partial W}{\partial t}^{True} - \frac{\partial W}{\partial t}^{Agg}$ is simplified as:

$$\frac{\partial W}{\partial t}^{True} - \frac{\partial W}{\partial t}^{Agg} = \frac{1}{t}\gamma \frac{(\epsilon_A - 1)\, A}{2\gamma} Q_l - \Pi_l \frac{(\epsilon_A - 1)\, A}{2\gamma \overline{q}_l}$$

Of note, the effects of applying the aggregate elasticity have no effect on the measurement of changes to tariff revenue. However, there is a bias in projecting the effects of tariffs on consumer surplus. Defining $\overline{\pi} = \frac{\gamma}{t}\overline{q} \cdot \overline{q}$ as the profits earned by the average-sized exporter, and using the definition of $\epsilon_{Q_l,t}$, the bias in welfare projections (in terms of consumer surplus) are written as:

$$\frac{\partial W}{\partial t}^{True} - \frac{\partial W}{\partial t}^{Agg} = \overbrace{(\epsilon_{Q_l,t} - 1)}^{<0} \left(N_l G_l(A/t)\overline{\pi}_l - \Pi_l\right)$$

Indeed, this within-exporter bias will be downward ($\frac{\partial W}{\partial t}^{Agg} < \frac{\partial W}{\partial t}^{True}$) if profits earned by the average-sized exporter are less than the average export profits earned by firms in exporter l. A right-skewed cost distribution will deliver such a result, which is satisfied by the Pareto distribution.

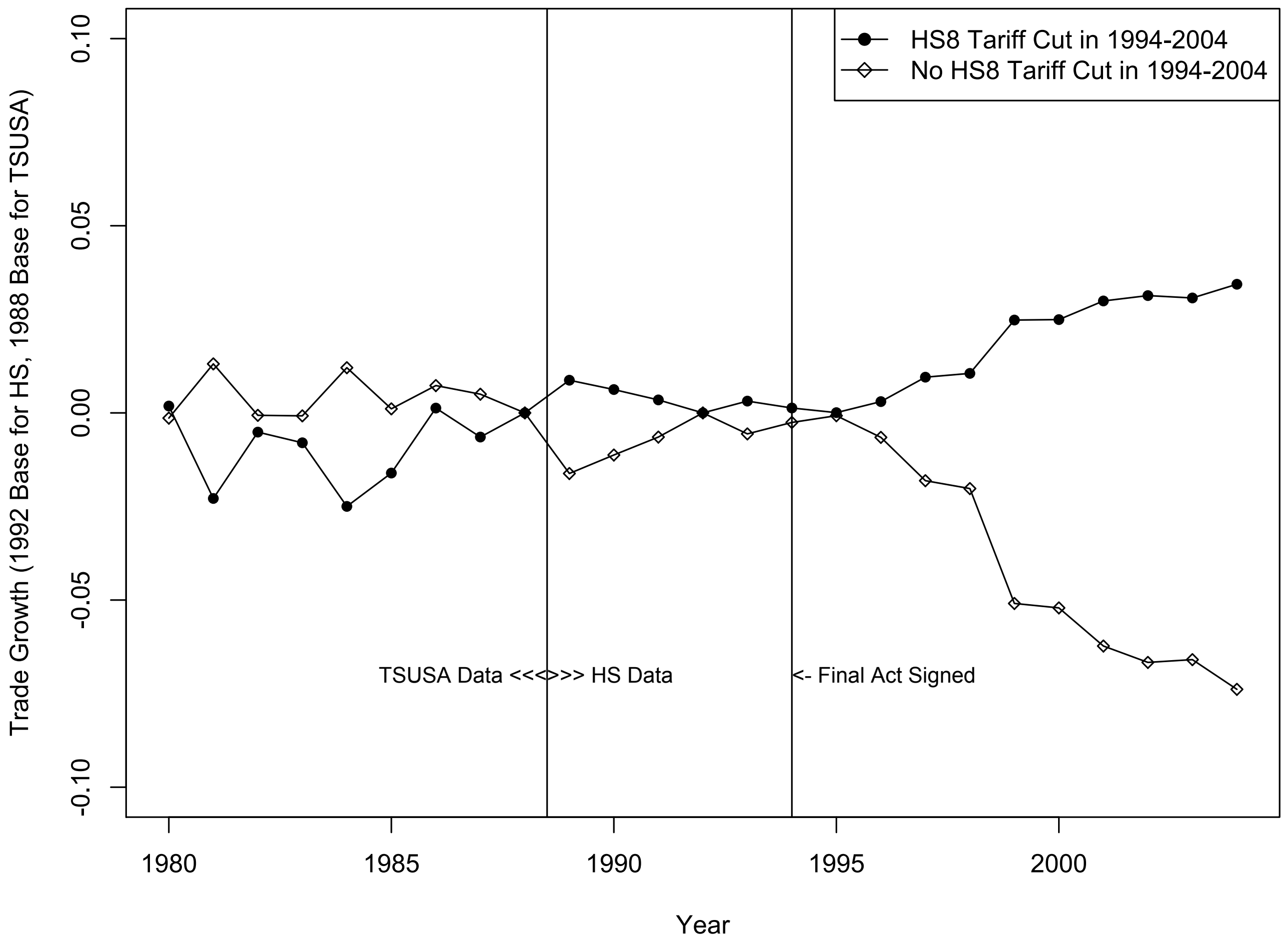
Indeed, the Pareto distribution yields a convenient transformation of $\frac{\partial W}{\partial t}^{True} - \frac{\partial W}{\partial t}^{Agg}$:

$$\begin{aligned}
\frac{\partial W}{\partial t}^{True} - \frac{\partial W}{\partial t}^{Agg} &= -\left(\epsilon_{Q_l,t} - 1\right) \frac{V_l}{2} \frac{k_l}{(k_l+1)(k_l+1)} > 0 \\
\rightarrow \frac{\partial W}{\partial t}^{Agg} &< \frac{\partial W}{\partial t}^{True}
\end{aligned}$$

Using the Pareto distribution, erroneously applying the aggregate elasticity to each firm yields a downward bias on projections of the welfare effects of tariffs. Intuitively, by ignoring the fact that large firms tend to be less-responsive to tariffs, we will overstate the negative impact of tariffs on consumer surplus. This bias, as written in (12), is a function of the aggregate elasticity itself, the total value of imports, and parameters related to the distribution of costs. Indeed, the last term goes to zero as the distribution of firms approaches zero variance ($k_l \rightarrow \infty$).

3 Trade Growth Pre-1989

Figure 1: Bilateral Trade Growth - 1980-2004



In Figure 2 in the manuscript, I provided evidence suggesting that tariffs had a meaningful effect on trade, and that trade growth before tariffs were cut was unrelated to future tariffs cuts. In Figure 1, I extend the illustration of trade growth back to 1980 using TSUSA data from the UC Davis for International Data. The issue with using TSUSA is that there exists no nested concordance with the HS classification system.

To begin, I calculate whether a TSUSA product receives a tariff cut during the Uruguay Round according to the following procedure. Precisely, I adopt the convention that a TSUSA product receives a tariff cut if at least one of the HS8 products that links to it receives a tariff cut. Once I have identified which TSUSA products received a tariff cut, I merge this information with the Exporter-TSUSA bilateral import data over the period 1980-1988.

To measure relative trade growth for observations occurring in 1989 or later, I first calculate the log of the ratio of imports in year t to imports in 1992 for each HS10-Exporter pair. Then, for each year, I de-mean these growth rates across Exporter-SITC4 pairs. SITC4 is used because it has a nested link to both HS8 and TSUSA. Thus, in each year after 1989, average trade growth relative to 1992 is equal to zero. For observations occurring before 1989, I first calculate the log of the ratio of imports in year t to imports in 1988 for each TSUSA-Exporter pair. Then, for each year, I de-mean these growth rates across Exporter-SITC4 pairs. Thus, in each year before 1988, average trade growth relative to 1988 is equal to zero.

In Figure 1, I have decomposed this "zero" into HS8 products that received a tariff cut, and HS8 products that did not. Similar to Figure 2 in the manuscript, we see that there is no obvious difference in pre-Uruguay Round import growth rates between products that received a tariff cut over 1994-2004 and those that didn't.