Productivity and the Role of the Global Acquisition Market

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Abstract

This paper presents a model of domestic and foreign acquisitions with heterogeneous firms. The model shows that acquisitions positively affect aggregate productivity by transferring capital from the least efficient firms to higher efficiency firms. However, contrary to the existing literature, these acquiring firms are in a mid-range of productivity. The results of my model show that the most productive firms *do not* find domestic acquisitions profitable in either a closed or open economy, and *do not* find foreign acquisitions profitable in relatively integrated open economies. This is a result of a demand-cost framework in which mark-ups are variable and, conditional on total capital holdings, concentration of production in one location reduces firm-level efficiency. With this new framework, I show that if policy actions reduce the amount of acquisitions that occur, aggregate productivity falls. For example, foreign investment restrictions always reduce aggregate productivity. Further, reciprocal trade liberalization, which has a strong negative effect on acquisition demand from abroad, may also reduce aggregate productivity.

JEL Classifications: F12, F23, G34, L2
Keywords: Firm Heterogeneity, Mergers and Acquisitions, Proximity-Concentration, Trade Liberalization, Aggregate Productivity

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1 Introduction

Without question, cross-border mergers and acquisitions (M&A) are one of the fastest growing aspects of globalization. In absolute terms, according to the OECD (2001), the value of cross-border M&A increased five-fold over the period 1990-1999. Relatively, the growth of cross-border M&A has also been substantial, where the share of North American firms that acquired cross-border rather than domestically increased 133% between 1985 and 2004.[1] Further, as noted by Navaretti and Venables (2006), cross-border M&A make-up a majority of foreign direct investment (FDI) between developed countries, and are increasing as a share of FDI to developing countries and transition economies.

In contrast with the steady growth of cross-border M&A, the policy treatment of foreign acquisitions has varied greatly by country and sector. In the United States, despite a fairly open foreign investment policy, all large inward foreign acquisitions are subject to review by the "Committee on Foreign Investment in the United States" (CFIUS). In India, prior to 1991, ownership by foreigners was restricted to only minority shares (Chhibber and Majumdar, 1999). In China, acquisition review varies greatly by industrial sector and province, and often involves multiple government agencies with different agendas.[2] Complicating matters further is apparent substitution between exports and FDI in response to policy choices, and the likelihood that governments discount this substitution when choosing policies (see Blonigen, 2002). Overall, given the complex incentives behind trade and investment, it is hardly shocking that governments have chosen such divergent paths in regulating foreign acquisitions. This point is not lost on the United Nations Conference on Trade and Development; "Indeed, perhaps to a greater extent than many other aspects of globalization, cross-border M&As and the expanding global market for firm ownership and control in which they occur — raise questions about the balance of their benefits and costs for host countries" (UNCTAD, 2000).

However, a fact often lost in this entire discussion is that while the prevalence of foreign acquisitions has increased over time, on average (and at the median), foreign acquisitions tend to be less common.[3] Thus, along with examining the causes and effects of cross-border M&A, it is equally (if not more) important to examine how these causes and effects relate to domestic M&A. Therefore, alongside the role of acquisitions within FDI, three equally appropriate questions arise: (1) why do firms acquire cross-border rather than acquire domestically, (2) how are these decisions affected by policy parameters, and (3) what are the implications for measures of efficiency and welfare?

Unfortunately, in most acquisition models with heterogeneous firms, the location of acquisitions is restricted for analytical simplicity. For example, in Jovanovic and Rousseau (2002), and Breinlich (2008), all acquisitions occur in one market. Nocke and Yeaple (2007), building on the framework in Helpman, Melitz and Yeaple (2004), present a model which is primarily concerned with the choice of foreign investment; greenfield or acquisition. In their work, domestic acquisitions are included. However, a simplifying feature is that domestic acquisitions are a function of necessity, where firms only acquire domestically if endowed with capabilities that are insufficient for any production.

Overall, the existing literature fails to model an active decision between domestic and foreign

[1] Author's calculation using the *Thomson SDC Platinum* database.

[2] For a concise review of Chinese acquisition policy, see http://www.hg.org/articles/article_443.html

[3] For example, using the *Thomson SDC Platinum* database, the yearly mean and median foreign share of acquisitions over target SIC2-country pairs for worldwide mergers over the period 1980-2006 is uniformly less than 0.5.

acquisitions. In this paper, I present a rich model of investment via acquisitions in which firms choose between domestic and foreign acquisitions. Broadly, my theoretical contributions are two-fold. First, I show that a trade-off between cost reduction and market access determines who acquires, and in which location the acquisition takes place. Second, specific to a model with both domestic and foreign acquisitions of industry specific factors, I articulate a new channel through which changing costs of trade affect aggregate productivity. Precisely, I show that the degree to which liberalization improves aggregate productivity depends crucially on whether the liberalization is focused on trade or investment flows, and whether foreign acquisitions are an important component of the acquisition market.

The model is setup in three stages. In the first stage, firms enter under productivity uncertainty, paying a fixed cost of entry. As a part of the entry process, firms receive an initial allocation of capital. To embody the stylized fact that it takes longer to build capital than to buy it pre-assembled from somebody else, I assume that no new investment occurs post-entry. Thus, it is assumed that acquisitions are the only way to quickly adjust a firm's capital stock after productivity is realized.[4] After firms buy and sell within the acquisition market, physical assets are fixed and firms procure variable factors to produce varieties for the product market. Given that capital is fixed at the time of production, firms face an upward sloping marginal cost schedule. The degree to which marginal costs are increasing in quantity is inversely proportional to both a firm's endowed productivity level, and the amount of capital within the firm.

Broadly, acquiring additional capital may increase profits via two channels of cost reduction. First, similar to classic models of FDI, a firm that acquires capital abroad increases market access to the foreign market by avoiding the traditional costs of trade (tariffs, for example). Since high-productivity firms tend to be the largest potential exporters, high-productivity firms tend to have the largest incentives to avoid the costs of trade.

Second, an acquisition may also reduce marginal production costs. Indeed, this type of cost-reduction is the vehicle which delivers the novel results of this paper. One can best understand this mechanism by focusing on the various ways in which firms can reallocate production post-acquisition. Generally, if production by the merged firm is less than the combined production of the old firms (a common prediction in most acquisition models), the acquisition allows the acquiring firm to reallocate production amongst the best assets of both firms. This might include reducing the amount of overtime shifts, firing the least productive workers, retiring the oldest machinery, or reducing output at inefficient plants and increasing output at more efficient plants.[5] Analytically, I adopt a simple characterization of this mechanism which is similar to neoclassical investment models, such as Hayashi (1982), where investment in new capital increases the marginal productivity of other factors. If acquiring domestically, a cost-reduction occurs with certainty, where the newly acquired plant always diverts a portion of production from the existing plant, allowing both to operate on more efficient portions of their marginal cost curves. If acquiring abroad, a cost-reduction occurs for the same reason if the acquisition diverts export production that would have otherwise occurred at the existing domestic plant (at a higher marginal production

[4] Indeed, "speed is the biggest advantage of M&As over greenfiend investment or other entry modes." (OECD, 2001, pg. 36)

[5] Indeed, on an anecdotal level, this last type reallocation seems to have occured immediately after the Daimler-Benz merger with Chrysler in 1997. Specifically, there is a positive correlation between pre-merger labor productivity (96-97) and post-merger growth rates of capacity utilization (99-00 relative to 96-97) within Chrysler plants. Contact the author for details of this calculation, which uses data from the *The Harbour Report*, 1997-2001.

cost).

Whether domestic or foreign, the degree to which a reduction in marginal costs via capital acquisitions translates into increased profits depends crucially on demand characteristics. To make this point, I assume a utility function which yields a linear demand for each variety (ala Melitz and Ottaviano, 2008), thus allowing for heterogenous firms to operate at different demand elasticities. For this particular demand system, I find that mid productivity firms benefit the most from a reduction in marginal costs via acquisitions. On one end, high productivity firms operate near the point of unit-elasticity, and thus are constrained by bounds on market revenues. On the other end, low productivity firms are constrained by their poor endowed productivity - a firm-level characteristic they never fully overcome. Relatively, mid productivity firms are constrained by neither, and earn the highest returns from a reduction in marginal costs. Generally, I find that this result holds as long as demand isn't "too convex". Of note, the CES demand system, which is widely used in the literature, is an example of a demand system which is "too convex".

It is the resolution of these two incentives, market access and cost reduction, which determines the set of firms that acquire and in what location. In a closed economy equilibrium, in which no market access considerations exist, I show that low productivity firms sell capital to mid productivity firms, with the remaining active firms doing nothing in the acquisition market. In contrast, in an open economy model with large trade costs, market access considerations dominate, and high-productivity firms are the most likely to find an acquisition profitable.

Further, I rigorously analyze the case of two symmetric and relatively integrated markets. Specifically, I show that if trade costs are relatively small, there exists a unique industry equilibrium in which low productivity firms sell, and mid-productivity firms acquire. Among those firms that acquire, the most productive acquire abroad. The intuition for this result can be explained as follows. While it is true that high-productivity firms have the highest incentive to avoid trade costs, the relative size of trade costs tempers this incentive. Whether doing so at home or abroad, mid-productivity firms have higher incentives to acquire another firm, primarily driven by cost-reduction. In equilibrium, mid-productivity firms bid up the acquisition price, making an acquisition for market access unprofitable for the most productive firms.

The acquisition equilibrium itself provides a rich setting in which to assess the impact of trade and investment parameters on aggregate productivity. The results as they pertain to trade liberalization are provocative. In equilibrium, if foreign acquisitions play a significant role in the acquisition market, trade liberalization will reduce aggregate productivity. What defines a significant role? I find that this result holds unless foreign acquisitions do not occur, or the mass around the most productive acquiring firms (foreign) is small relative to the least productive acquiring firms (domestic).

The intuition for this result is fairly simple. Since the acquisition market dictates a transfer of capital from inefficient firms to more efficient firms, changes to the acquisition market have an effect on aggregate productivity. When all acquisitions are foreign, aggregate productivity worsens with trade liberalization, as foreign firms have a greater incentive to export, and thus contract their demand for assets used by inefficient domestic firms. In contrast, when all acquisitions are domestic, aggregate productivity improves with trade liberalization via an expansion of export-driven domestic acquisitions. In between, the effect of trade liberalization on aggregate productivity is analytically ambiguous. However, more productive firms, who are larger potential exporters and thus more affected by trade costs, are precisely the firms that are more likely to acquire abroad rather than domestically. Overall, the effects of trade costs on these firms tend to dominate, and trade liberalization reduces aggregate productivity.

The case of investment liberalization is more straightforward. By removing foreign investment restrictions, additional demand for domestic assets by foreign firms pushes up the acquisition price, which leads to more inefficient firms selling and exiting. Thus, with a more liberal policy regarding foreign investment, aggregate productivity improves.

Related literature

Broadly, this paper contributes to a number of different areas related to trade, FDI, firm heterogeneity, and M&A. Generally, it adds to the growing literature examining the role of firm heterogeneity in trade and investment decisions. The critical innovation in the paper is that "concentration", whereby firms locate production only in one location, may affect variable efficiency at the margin. Without this feature, the model would be based solely on issues discussed in Brainard (1997), and deliver predictions similar to Helpman, Melitz, and Yeaple (2004). However, given that concentration of production may affect variable efficiency at the margin, the model delivers a rich set of predictions regarding the sorting of firms into investment and export choices, and the relationship between trade costs and productivity. Overall, the model suggests that industry specific factors related to demand and costs have a strong influence on equilibrium firm behavior. This is not unlike Nocke and Yeaple (2007), where the degree to which capabilities are transferrable across borders - and industry level attribute - has a critical effect on firm-level incentives for greenfield investment vis-a-vis M&A. Head and Reis (2003) is also similar, where there exist industry parameters such that low-productivity firms optimally invest abroad, hiring low-wage workers to offset their poor labor productivity.

Tangentially, the paper is related to the classic literature on mergers and acquisitions, in which the predominant explanation for M&A behavior is market power.[6] However, while market power surely plays a role in some cases, the data seem to suggest that other issues may play an equally significant or greater role in acquisition behavior.[7] Indeed, the classic literature has also considered cost-efficiencies alongside market power as a viable motive for acquisitions (Perry and Porter, 1985; Farrell and Shapiro, 1990). Analytically, I use the same cost structure as in Perry and Porter (1985) to tractably model acquisitions that may affect the efficiency of variable factors. While I do not add to the literature discussing market power, my paper does identify the set of firms that can use cost-reductions as a viable merger defense. That is, unless acquisitions are for scope or market access, high-productivity firms cannot use variable cost efficiencies as a merger defense.

Related to scope, a number of authors have modeled mergers and acquisitions as a form of expansion to additional varieties. A recent and notable example of this is Nocke and Yeaple (2006), where firms only acquire for scope, and only domestically. My paper makes the strong assumption that acquisitions occur within the industry in which the firm currently operates. Empirically, both within-industry and extra-industry incentives are likely important, where roughly 50% of

[6] See the work of Salant, Switzer and Reynolds (1983) and Deneckre and Davidson (1985) for a classic discussion of the incentives for mergers driven by market power.

[7] If market power were the only reason for acquisitions, we would expect that the ratio of total industry acquisition value to total industry operating income would be fairly large. For example, in a hypothetical merger of two identical cournot competitors into a single firm, the ratio must be at least one half. In contrast, the median of this ratio over SIC4 industries and years is 6.8% (24.2% for the 75th percentile), which is inconsistent with market power being the only motivation for the observed acquisition behavior. Both statistics calculated using the *Compustat* database.

acquisitions are outside of a firm's primary SIC2 classification[8]. However, unless firms acquire capital which is utilized in industries not substitutable with their primary industry, the incentives summarized above, and detailed below, will be relevant for acquisition decisions.

Outline

The rest of the paper is organized as follows. In section two, I thoroughly develop the closed economy model, highlighting the general acquisition framework and why this framework leads to a new sorting of firms by productivity. In section three, I extend the intuition from the closed economy to an open economy model with two identical countries. In section four, I evaluate the effects of trade and investment costs on the equilibrium sorting of firms into acquisition decisions, and on aggregate productivity. In section five, I briefly conclude.

2 Model

The basic closed economy model presented in this paper consists of three stages. In stage one, entry decisions are made. Firm-level productivity is uncertain and each potential entrant is ex-ante identical. Firms enter until their expected post-entry profits are equal to the fixed cost of entry. Upon entry, firms receive a fixed "lump" of capital that may be used in the product market.[9]

In stage two, acquisition decisions are made. Post-entry, productivity is realized and firms are allowed to trade industry-specific capital on a perfectly competitive acquisition market. However, since investment behavior tends to be "lumpy" (Doms and Dunne, 1998), I assume that capital from the entry stage is indivisible in the acquisition stage; firms may not buy or sell fractions of capital. Additionally, due to unmodeled organizational factors, it is assumed that a firm only has enough resources to acquire one firm in the acquisition stage. Thus, firms are restricted to three options: sell all capital and exit, buy the capital of an exiting firm, or do nothing. Very little generality is lost in this restriction, and is an issue discussed later in the draft.

Finally, in stage three, each active firm supplies its individual variety to the product market. Active firms are monopolists in their own variety, taking other industry variables as given. At this point, any capital accrued during the entry and acquisition stages is fixed, and firms only procure variable factors.

The model is solved by backward induction, and will be introduced in this order.

[8] See Breinlich (2008)

[9] Note that in this model (as in the rest of the literature) the initial capital endowment is not endogenous. I assume this to focus on the ex-post reallocation of capital, and to keep analysis tractable. In Spearot (2009), I develop a simple dynamic model in the spirit of Jovanvic and Rousseau (2002), where under reasonable assumptions, I show that a dynamic model reduces to a static acquisition decision, similar to the one in this section.

2.1 Product Market Equilibrium

Consumers

Consumers have quasi-linear preferences over a differentiated industry and a numeraire good, x_0. Similar to Melitz and Ottaviano (2008), preferences of this sort can be written as:

$$U = x_0 + \theta \int_{i\in\Omega} q_i di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i di \right)^2 - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i)^2 \, di \tag{1}$$

In (1), Ω represents the measure of varieties, q_i is the consumption of variety i, and the parameters θ (> 0) and η (> 0) determine the substitution pattern between the differentiated industry and the numeraire. Finally, γ (> 0) represents the degree to which varieties are substitutable. If γ were zero, all firms would price at the same level, since products would be homogeneous in the eyes of the consumer.

In an economy with L consumers who each supply one unit of labor at a numeraire wage, the inverse demand function for variety i can be derived as:

$$p_i = \underbrace{\frac{\theta\gamma}{\eta M + \gamma} + \frac{\eta M}{\eta M + \gamma}\overline{p}}_{A} - \underbrace{\frac{\gamma}{L}}_{b} q_i = A - bq_i \tag{2}$$

In (2), p_i is the price of variety i, M is the measure of all varieties sold in the product market, and $\overline{p}$ is the average price of these varieties. Naturally, competition will be "tougher" when M is high and/or $\overline{p}$ is low. Thus, the overall level of market "toughness" is captured in A, the residual demand level facing each firm. As all firms are small outside their own variety, firms take A as given.

Firms

Capital influences firm decisions through the cost function. Similar to Perry and Porter (1985), the cost function of each firm takes the following form:

$$C\left(q_i|\alpha_i, K_i\right) = \frac{1}{2} \cdot \frac{q_i^2}{\alpha_i K_i} \tag{3}$$

In (3), α_i is firm-level productivity. Productivity is continuously distributed according to $G(\alpha)$, defined over $\alpha \in (0, \infty)$. The variable K_i represents capital accumulated during the initial stage and acquisition stage for firm i.[10] Firm-level productivity is transferrable across all holdings of capital within the firm.

A firm with productivity draw α_i and capital level K_i faces the following profit maximization

[10] This cost-structure can be recovered from a Cobb-Douglas production function, given that the level of capital is fixed in the product market stage. In stage three, firms only procure variable factors at a price v per unit. The cost function is written as $C\left(X_i|v\right) = v \cdot X_i$. With equal intensity of capital and variable factors, the Cobb-Douglas production function can be written as $q_i = \left(2\alpha_i X_i\right)^{\frac{1}{2}} K_i^{\frac{1}{2}}$. Solving for X_i, and substituting into the above cost function, we get $C\left(Q_i|\alpha_i, v, K_i\right) = \frac{v}{2} \cdot \frac{q_i^2}{\alpha_i K_i}$. Normalizing v to equal 1 gives the desired result, $C\left(Q_i|\alpha_i, K_i\right) = \frac{1}{2} \cdot \frac{q_i^2}{\alpha_i K_i}$

problem in stage 3:

$$\begin{aligned} \pi\left(\alpha_i, K_i\right) &= \max_{q_i}\left\{\left(A - b\cdot q_i\right)\cdot q_i - \frac{1}{2}\frac{q_i^2}{\alpha_i K_i}\right\} \\ st &: \quad q_i \geq 0 \end{aligned} \tag{4}$$

Solving (4) and dropping $i's$ for notational convenience, profits and prices in the product market are written as:

$$\pi\left(\alpha, K\right) = \frac{A^2 \alpha K}{2\left(2b\alpha K + 1\right)} \tag{5}$$

$$p\left(\alpha, K\right) = A\frac{b\alpha K + 1}{2b\alpha K + 1} \tag{6}$$

In the closed economy, there are two types of firms active in the product market. Firms that "do nothing" (N) in the acquisition stage retain their initial capital level from entry, k, while firms that buy capital (B) in the acquisition stage double their initial capital level, holding $2k$. Firms that sell capital (S) in the acquisition stage are not active in the product market. Subject to these capital positions, the profits for N and B, respectively, are expressed as:

$$\pi^N\left(\alpha\right) = \frac{A^2 \alpha k}{\left(4b\alpha k + 2\right)} \tag{7}$$

$$\pi^B\left(\alpha\right) = \frac{A^2 \alpha k}{\left(4b\alpha k + 1\right)} \tag{8}$$

where,

$$\pi^B\left(\alpha\right) > \pi^N\left(\alpha\right) \text{ for } \alpha \in \left(0, \infty\right)$$

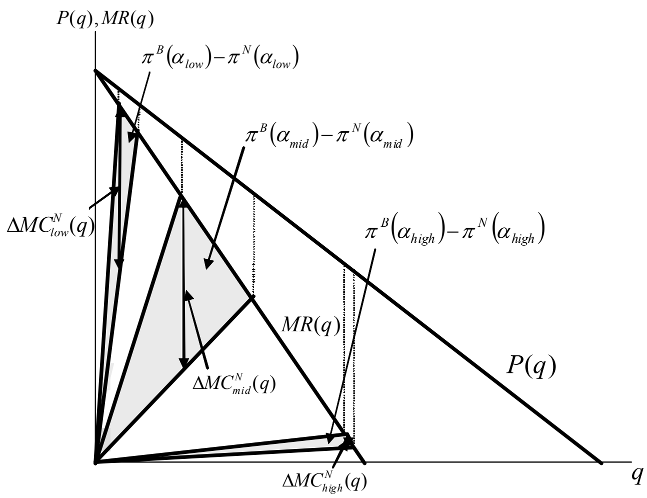
Generally, since monopolists operate on the elastic portion of the demand curve, firms have incentive to increase production after a cost-lowering acquisition (an acquisition halves variable costs at every quantity). This is illustrated in Figure 1, where firms of low, middle and high productivity increase production following an acquisition.

However, under the assumption of linear demand, the least efficient and most efficient firms earn minimal returns from a cost-lowering acquisition. In Figure 1, the least efficient firms are limited by a steep marginal cost schedule. Whether or not they acquire, they are still quite unproductive, and the absolute gains from an acquisition are tiny. The most efficient firms are constrained not by costs, but by the structure of market demand. Specifically, the highest productivity firms operate on a less-elastic portion of the demand curve, which limits the incentive to expand production after a cost-lowering acquisition. Firms in a mid-range of productivity are constrained by neither, and earn relatively high returns from an acquisition. Thus, with linear demand, firms within a mid-range of productivity benefit the most from a cost-lowering acquisition. Indeed, the maximum of $\Delta\Pi = \pi^B\left(\alpha\right) - \pi^N\left(\alpha\right)$ is at $\alpha = \frac{\sqrt{2}}{4bk}$.

Finally, given the cost function in (3), profits exhibit diminishing returns to capital. Thus, $\pi^N\left(\alpha\right)$ and $\pi^B\left(\alpha\right)$ have the following intuitive ranking:

$$\frac{1}{2}\pi^B\left(\alpha\right) < \pi^N\left(\alpha\right) < \pi^B\left(\alpha\right) \tag{9}$$

Figure 1: Effect of acquisitions on profits



This property will be used when characterizing optimal firm-level acquisition decisions as a function of productivity.

2.2 Acquisition Stage Equilibrium

Optimal Acquisition Choice

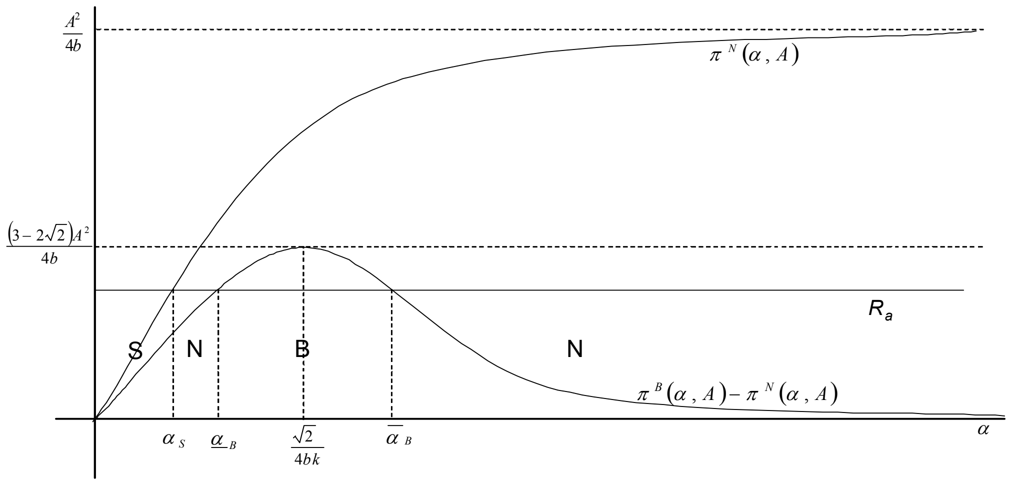
Since firms are "small", I assume an acquisition market in which firm-level decisions have no effect on the market-clearing price per firm, R_a, or the residual demand level, A. First, taking A and R_a as given, I derive optimal firm acquisition decisions as a function of productivity. Then, for a given A, I show that a unique value of R_a clears the acquisition market. Finally, I prove that there exists a unique value of A, subject to firm-level acquisition decisions and the market clearing price per firm, $R_a(A)$.

In the acquisition stage, firms must choose between three options: Sell their firm (S), do nothing (N), or buy capital (B). Respectively, the profits of each option in the acquisition stage are written as:

$$\Pi^S(R_a) = R_a \tag{10}$$
$$\Pi^N(\alpha, A) = \pi^N(\alpha, A) \tag{11}$$
$$\Pi^B(\alpha, A, R_a) = \pi^B(\alpha, A) - R_a \tag{12}$$

Here, the dependence of $\pi^N(\alpha, A)$ and $\pi^B(\alpha, A)$ on A in (7) and (8) is made explicit to emphasize that A is fixed for the moment. In (10), firms sell their capital, collect R_a, and exit the market. In (11), firms do nothing in the acquisition market and earn profits given their initial capital

Figure 2: Optimal Acquisition Choice



endowment, k. In (12), firms buy capital, earning $\pi^B(\alpha, A)$ in the product market after paying R_a for an additional lump of capital.

A firm of productivity α chooses the acquisition option which maximizes profits in the acquisition market. Defining $V(\alpha, A, R_a)$ as acquisition market profits given α, the acquisition decision of each firm is characterized by the following:

$$V(\alpha, A, R_a) = \max\left\{R_a, \pi^N(\alpha, A), \pi^B(\alpha, A) - R_a\right\} \tag{13}$$

A convenient transformation of (13) is subtracting $\pi^N(\alpha, A)$ from each option. This gives us the following equivalent representation of the acquisition decision facing each firm:

$$\widehat{V}(\alpha, A, R_a) = \max\left\{R_a - \pi^N(\alpha, A), 0, \pi^B(\alpha, A) - \pi^N(\alpha, A) - R_a\right\}$$

This normalizes acquisition market profits relative to the outside option of doing nothing.

Within $\widehat{V}(\alpha, A, R_a)$, the function $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ is the benefit of an acquisition. As a function of model parameters, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ is written as:

$$\pi^B(\alpha, A) - \pi^N(\alpha, A) = \frac{A^2\alpha k}{2(2b\alpha k + 1)(4b\alpha k + 1)} \tag{14}$$

It is straightforward to show that $\pi^B(\alpha, A) - \pi^N(\alpha, A)$ approaches zero for low and high α, and reaches its maximum on the interior at $\frac{\sqrt{2}}{4bk}$. The optimal acquisition decision derived from $\widehat{V}(\alpha, A, R_a)$ is illustrated in Figure 2.

In Figure 2, for $\alpha < \alpha_S$, the profits from selling are greater than profits from doing nothing. Also, the benefit of buying, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$, is less than the acquisition price. Thus, selling is the dominant option for the least efficient firms. There will exist a positive measure of these selling firms for $R_a > 0$.

For "small" R_a ($R_a \leq \frac{(3-2\sqrt{2})A^2}{4b}$), firms with productivity between $\underline{\alpha}_B$ and $\overline{\alpha}_B$ find an acquisition profitable. For these firms, the benefit of an acquisition, $\pi^B(\alpha, A) - \pi^N(\alpha, A)$, is greater than the acquisition price. Additionally, for small R_a, there exist two disjoint regions of productivity such that doing nothing is optimal. These regions are labeled by N in Figure 2. This follows from the "lumpiness" of assets. For "large" R_a ($R_a > \frac{(3-2\sqrt{2})A^2}{4b}$), no firms find an acquisition profitable. The acquisition price is too large, where $\pi^B(\alpha, A) - \pi^N(\alpha, A) < R_a$ for all α. Naturally, since there exist selling firms and no buying firms, large R_a cannot be an acquisition market clearing price. Thus, I henceforth restrict attention to "small" R_a.

The overall shape of Figure 2 follows closely the intuition discussed for firms of low, middle, and high productivity. Precisely, mid-productivity firms have the highest incentive to acquire another firm. They are relatively less constrained by intrinsically high costs, which is the problem for low productivity firms. Further, they have additional room on the revenue side to expand production, which is not the case for the highest productivity firms.

This last point is critically dependent on the shape of the demand curve, and not the restriction that acquisition behavior is lumpy. Generally, mid-productivity firms optimally hold the most capital only when relative pricing power falls sufficiently with output. This is formalized in the appendix, where I derive the optimal level of capital holdings for a monopolist producing subject to an inverse demand function $P(q)$ and the cost function used in (3). In particular, I show that the marginal value of added capital is decreasing in productivity if $\alpha k > \frac{1}{-\frac{\partial MR(q(\alpha k))}{\partial q}}$. This condition is never satisfied, for example, when firms are price takers ($\frac{\partial MR(q(\alpha k))}{\partial q} = 0$), or when adopting a demand system of constant elasticity.[11] In both cases, $\frac{\partial MR(q(\alpha k))}{\partial q}$ is either zero or falls (in absolute terms) as productivity increases. On the other hand, if $\frac{\partial MR(q(\alpha k))}{\partial q}$ is constant (as in linear demand), rises (in absolute terms) as productivity increases, or simply does not fall (in absolute terms) as quickly as αk rises, the incentives to invest will fall with higher productivity. Overall, when investment reduces variable costs, the structure of market demand is a critical component of investment incentives.

Equilibrium

Once again turning attention to Figure 2, α_S, $\underline{\alpha}_B$, and $\overline{\alpha}_B$ represent kinks in $V(\alpha, A, R_a)$. More precisely, these represent firms that are indifferent between acquisition options. Hence, α_S is implicitly defined as:

$$\pi^N(\alpha_S, A) = R_a \tag{15}$$

where,

$$For\ \alpha < \alpha_S\ ,\ S \succ N \tag{16}$$

The preference condition $S \succ N$ is a straightforward result when observing that stage three profits are increasing in productivity.

[11] This can also be seen by deriving acquisition incentives subject to CES demand. To see this, consider an identical setup as above with the exception that inverse demand is $p = Aq^{-\lambda}$, $0 < \lambda < 1$. It can be derived that $\pi^B_{ces}(\alpha) - \pi^N_{ces}(\alpha) = Z\alpha^{\frac{1+\lambda}{1-\lambda}}$, where $Z = A^{\frac{2}{1-\lambda}}(1-\lambda)\left(2^{\frac{1+\lambda}{1-\lambda}} - 1\right)\left(\frac{1}{1+\lambda}\right)^{\frac{1+\lambda}{1-\lambda}} k^{\frac{1+\lambda}{1-\lambda}} > 0$. Clearly, the stage three profits resulting from an acquisition are increasing in productivity.

Similarly, $\underline{\alpha}_B$, and $\overline{\alpha}_B$ can be defined by:

$$\begin{aligned}\pi^B\left(\underline{\alpha}_B, A\right) - \pi^N\left(\underline{\alpha}_B, A\right) &= R_a \qquad (17)\\ \pi^B\left(\overline{\alpha}_B, A\right) - \pi^N\left(\overline{\alpha}_B, A\right) &= R_a \qquad (18)\end{aligned}$$

where,

$$For\ \alpha \in \left(\underline{\alpha}_B, \overline{\alpha}_B\right),\ B \succ N \qquad (19)$$

The condition $B \succ N$ is immediate from the shape of $\pi^B(\alpha, A) - \pi^N(\alpha, A)$.

Using the indifference conditions in (15), (17), and (18), and the preference conditions in (16) and (19), the following lemma proves that the features illustrated in Figure 2 are representative of optimal acquisition choice.

Lemma 1 *In the closed economy, given A and R_a, optimal acquisition choice is the following:*

$$\begin{aligned}For \quad & \alpha \in [0, \alpha_S(A, R_a)),\ \textit{firms sell}\\ & \alpha \in [\alpha_S(A, R_a), \underline{\alpha}_B(A, R_a)],\ \textit{firms do nothing}\\ & \alpha \in (\underline{\alpha}_B(A, R_a), \overline{\alpha}_B(A, R_a)),\ \textit{firms buy}\\ & \alpha \in [\overline{\alpha}_B(A, R_a), \infty),\ \textit{firms do nothing}\end{aligned}$$

Proof. See Appendix ■

In Lemma 1, the relationship between the equilibrium cutoffs and economy aggregates A and R_a is made explicit. With Lemma 1, given M_E entrants, the demand $(K_D(A, R_a))$ and supply $(K_S(A, R_a))$ of acquired capital are written as:

$$\begin{aligned}K_D(A, R_a) &= M_E k\left(G\left(\overline{\alpha}_B(A, R_a)\right) - G\left(\underline{\alpha}_B(A, R_a)\right)\right) \qquad (20)\\ K_S(A, R_a) &= M_E k G\left(\alpha_S(A, R_a)\right) \qquad (21)\end{aligned}$$

The acquisition price, R_a, affects $K_D(A, R_a)$ and $K_S(A, R_a)$ through the acquisition cutoffs $\alpha_S(A, R_a)$, $\underline{\alpha}_B(A, R_a)$ and $\overline{\alpha}_B(A, R_a)$. Of course, the acquisition market clears if,

$$K_D(A, R_a) = K_S(A, R_a). \qquad (22)$$

For a given value of A, there is a unique $R_a(A)$ that clears the acquisition market. This is proven in the following Lemma:

Lemma 2 *Holding A fixed, there exists a unique $R_a(A)$ that clears the acquisition market.*

Proof. See Appendix ■

The intuition behind Lemma 2 is a simple case of supply and demand. The measure of buying firms is decreasing in the acquisition price, and the measure of selling firms is increasing in the acquisition price. Given that no firms are willing to sell at $R_a = 0$ and no firms are willing to buy at $R_a \geq \frac{(3-2\sqrt{2})A^2}{4b}$, the demand and supply functions cross only once at the equilibrium acquisition price, $R_a(A)$.

With acquisition market clearing in-hand, I now show that there exists a unique equilibrium value of A. First, I analyze how the productivity cutoffs summarized in Lemma 1, subject to the acquisition market clearing condition, change with A. Conveniently, α_S, $\underline{\alpha}_B$ and $\overline{\alpha}_B$ are all independent of A. To see this, note that the equilibrium conditions in (15), (17), and (18), and the market clearing condition in (22), can be combined to yield the following:

$$\begin{aligned}
\frac{\underline{\alpha}_B(A)k}{2\left(2b\underline{\alpha}_B(A)k+1\right)\left(4b\underline{\alpha}_B(A)k+1\right)} &= \frac{\alpha_S(A)k}{\left(4b\alpha_S(A)k+2\right)} \\
\frac{\overline{\alpha}_B(A)k}{2\left(2b\overline{\alpha}_B(A)k+1\right)\left(4b\overline{\alpha}_B(A)k+1\right)} &= \frac{\alpha_S(A)k}{\left(4b\alpha_S(A)k+2\right)} \\
G\left(\overline{\alpha}_B(A)\right) - G\left(\underline{\alpha}_B(A)\right) &= G\left(\alpha_S(A)\right)
\end{aligned}$$

Above, there exist three equations and three unknowns, $\alpha_S(A)$, $\underline{\alpha}_B(A)$ and $\overline{\alpha}_B(A)$. Critically, A no longer enters into any equation directly. This feature is a result of profit functions being homogeneous in A, along with the acquisition price being the only fixed cost. This immediately yields the following lemma:

Lemma 3 $\frac{\partial \alpha_S(A)}{\partial A} = 0$, $\frac{\partial \underline{\alpha}_B(A)}{\partial A} = 0$ *and* $\frac{\partial \overline{\alpha}_B(A)}{\partial A} = 0$

Proof. Immediate. ■

Using Lemma 3, the uniqueness of A is now trivial. Using the inverse demand function for each variety in (2), the unique value of A is defined for any level of entry, M_E, by

$$\widehat{A} = \frac{\theta\gamma}{\gamma + \eta M_E\left(1 - G\left(\alpha_S\right) - \Phi\left(\alpha_S, \underline{\alpha}_B, \overline{\alpha}_B\right)\right)} \tag{23}$$

where,[12]

$$\Phi\left(\alpha_S, \underline{\alpha}_B, \overline{\alpha}_B\right) = \int_{\alpha_S}^{\underline{\alpha}_B} \frac{b\alpha k+1}{2b\alpha k+1} dG\left(\alpha\right) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{2b\alpha k+1}{4b\alpha k+1} dG\left(\alpha\right) + \int_{\overline{\alpha}_B}^{\infty} \frac{b\alpha k+1}{2b\alpha k+1} dG\left(\alpha\right)$$

and where (23) is written in terms of M_E using the fact that,

$$M = M_E\left(1 - G\left(\alpha_S\right)\right). \tag{24}$$

Since M_E varieties enter and $M_E G\left(\alpha_S\right)$ varieties sell and exit (from Lemma 1), M varieties are sold in the product market, as defined by (24). Since $1 - G\left(\alpha_S\right) > \Phi\left(\alpha_S, \underline{\alpha}_B, \overline{\alpha}_B\right)$, the unique value of $\widehat{A}$ is positive. The uniqueness of A is summarized in the following lemma.

Lemma 4 *In the closed economy, there exists a unique solution $\widehat{A} > 0$, as written in (23).*

[12] This is derived from the equation for the average price. Using (6), individual prices are:

$$\begin{aligned}
p^N(\alpha) &= A\frac{b\alpha k+1}{2b\alpha k+1} \\
p^B(\alpha) &= A\frac{2b\alpha k+1}{4b\alpha k+1}
\end{aligned}$$

Given Lemma 1, I can write the equation for the average price as:

$$\overline{p} = \frac{A}{1 - G\left(\alpha_S\right)}\left(\int_{\alpha_S}^{\underline{\alpha}_B} \frac{b\alpha k+1}{2b\alpha k+1}\partial G + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \frac{2b\alpha k+1}{4b\alpha k+1}\partial G + \int_{\overline{\alpha}_B}^{\infty} \frac{b\alpha k+1}{2b\alpha k+1}\partial G\right)$$

With Lemmas 1, 2, and 4, the following Proposition summarizes the acquisition stage equilibrium in the closed economy.

Proposition 1 *Given M_E entering firms, the closed economy acquisition equilibrium consists of a unique A, R_a, α_S, $\underline{\alpha}_B$, and $\overline{\alpha}_B$ such that:*

$$\begin{array}{ll} For & \alpha \in [0, \alpha_S),\ \textit{firms sell} \\ & \alpha \in [\alpha_S, \underline{\alpha}_B],\ \textit{firms do nothing} \\ & \alpha \in (\underline{\alpha}_B, \overline{\alpha}_B),\ \textit{firms buy} \\ & \alpha \in [\overline{\alpha}_B, \infty),\ \textit{firms do nothing} \end{array}$$

Proof. Follows directly from Lemmas 1, 2, and 4. ■

The highlight of Proposition 1 is that the highest productivity firms acquire nothing. These firms operate on a less-elastic portion of the demand curve, which limits the incentive to expand production after a cost-lowering acquisition. In contrast, as discussed earlier, the highest productivity firms *would* acquire if I assumed CES demand or a setting in which firms were price takers.[13] Thus, when acquisitions lower production costs, the structure of competition and demand are important components of the equilibrium acquisition decisions of heterogeneous firms.

To close the model, I present the free entry condition. In stage one, M_E firms enter until their expected post-entry profits equal the fixed cost of entry. Imposing the acquisition market clearing condition (22), the free entry condition is written as:

$$\int_{\alpha_S}^{\underline{\alpha}_B} \pi^N(\alpha)\, dG(\alpha) + \int_{\underline{\alpha}_B}^{\overline{\alpha}_B} \pi^B(\alpha)\, dG(\alpha) + \int_{\overline{\alpha}_B}^{\infty} \pi^N(\alpha)\, dG(\alpha) = F_E \tag{25}$$

Since profits are increasing in A, and A is decreasing in M_E (by 23), additional entry lowers the expected profits of all entrants. Thus, provided that the fixed cost of entry is not prohibitive, there exists a unique, positive measure of entering firms.

3 Open Economy

In the previous section, acquisition incentives are derived for firms that procure capital and sell products in the same market. In an open economy, firms may also buy capital as well as sell products in distant markets. In this section, I detail how the incentives to acquire another firm are affected by the location of capital and product markets. In particular, I focus on how the costs of trade and investment affect acquisition decisions, and hence aggregate productivity.

To capture the standard features of the trade and investment literature as simply as possible, I assume that there are two countries which are identical in every dimension, each with segmented markets for products (varieties) and assets. Further, to reduce clutter, and to focus on the key ideas, I adopt two restrictions. First, I hold A constant for all varieties in both product markets. Under this restriction, there are no aggregate effects of entry, or the acquisition market, on prices. Second, as entry only effects firms through A, I do not include a free entry condition, instead focusing on the product market and acquisition stages. In a supplementary technical appendix, I

[13] High productivity firms also acquire within the framework developed by Jovanovic and Rousseau (2002). In their work, revenues are related one-for-one with installed capital. High productivity firms are willing to produce on a larger scale, and thus have the highest incentive to install additional capital.

relax these restrictions and solve the model with an endogenous value of A, pinned down by a free entry condition.[14] I also discuss conditions such that this value is uniquely determined. Overall, the main results as they pertain to trade liberalization remain unchanged.

Firms have two product markets in which to sell their variety, and two locations at which production may take place. The basic cost structure, at a given location, will remain identical to the closed economy model. In each product market, the inverse demand function for each variety is labeled as $p_i = A - b \cdot q_i$. A trade cost, t, is incurred per unit of exports. For any value of trade cost, each firm solves the following profit maximization problem:

$$\pi\left(\alpha_i, K_i^H, K_i^F\right) = \max_{q_i, q_i^x, q_i^F} \left\{ \begin{array}{c} \underbrace{\underbrace{(A - b \cdot q_i) \cdot q_i}_{\text{home}} + \underbrace{\left(A - b \cdot \left(q_i^x + q_i^F\right)\right) \cdot \left(q_i^x + q_i^F\right)}_{\text{foreign}}}_{\text{Revenues}} \\ \underbrace{-\underbrace{\frac{1}{2}\frac{1}{\alpha_i} \cdot \frac{\left(q_i^x + q_i\right)^2}{K_i^H}}_{\text{home}} - \underbrace{\frac{1}{2}\frac{1}{\alpha_i} \cdot \frac{\left(q_i^F\right)^2}{K_i^F}}_{\text{foreign}}}_{\text{production costs}} - \underbrace{t \cdot q_i^x}_{\text{trade costs}} \end{array} \right\} \tag{26}$$

$$\textit{such that} \quad : \quad q_i \geq 0 \ q_i^F \geq 0 \ \textit{and} \ q_i^x \geq 0$$

In (26), q_i is the home production of variety i for home consumption, q_i^x is home production of variety i for sale in the foreign country, and q_i^F is foreign production of variety i for the foreign market. These production levels are dictated by costs in each country, which are fundamentally dictated by the capital holdings of firm i in each country, K_i^H at home and K_i^F abroad. These capital holdings will be determined in the acquisition stage. Lastly, note that export production at home increases the costs of non-export production at home, and vice versa. This will be a particularly important feature when discussing foreign acquisitions.

Prior to acquisition decisions, all firms are endowed with k units of capital at home, and zero units of capital abroad. For the sake of tractability, I will assume that firms choose one of the following options in the acquisition market: (S) Sell all domestic capital and exit, (N) do nothing, (B) buy an additional "lump" of domestic capital, or (B^*) buy an additional "lump" of foreign capital.

Generally, if firms remain "domestic", in the sense that they purchase no foreign capital in the acquisition market ($K_i^F = 0$), profits are written as:

$$\pi\left(\alpha, K^H, 0\right) = \left\{ \begin{array}{ll} \frac{A^2 \alpha K^H}{4b\alpha K^H + 2} & \alpha \leq \frac{t}{2bK^H(A-t)} \\ \frac{A^2 \alpha K^H}{2b\alpha K^H + 2} - \frac{t(4bA\alpha K^H - 2b\alpha K^H t - t)}{4b(2b\alpha K^H + 2)} & \alpha > \frac{t}{2bK^H(A-t)} \end{array} \right\} \tag{27}$$

In (27), where $i's$ have been dropped for notational convenience, when $\alpha \leq \frac{t}{2bK^H(A-t)}$, firms do not export. If a firm's productivity is too low, the maximum trade-cost adjusted marginal revenue of serving the foreign market is lower than the equilibrium marginal cost of only serving the domestic market. If $\alpha > \frac{t}{2bkK^H(A-t)}$, the opposite is the case, where firms have low enough production costs such that additional domestic production intended for exports is optimal. Note that this cutoff decreases as the level of domestic capital holdings, K^H, increases. Conditional on productivity,

[14] The supplementary technical appendix is available at http://people.ucsc.edu/~aspearot

exporting is more likely when firms hold more capital at home.

Product market profits of options (N) and (B) are defined as:

$$\begin{aligned}\pi^N(\alpha) &\equiv \pi(\alpha, k, 0) \\ \pi^B(\alpha) &\equiv \pi(\alpha, 2k, 0)\end{aligned}$$

As in the closed economy, $\pi^B(\alpha) > \pi^N(\alpha)$, since variable costs are lower after an acquisition. Of course, the difference in $\pi^B(\alpha)$ and $\pi^N(\alpha)$ will be weighed against the acquisition price, and the added option of foreign acquisitions.

Finally, firms that purchase a foreign firm (B^*) hold k units of capital at home, and k units of capital abroad. Subject to these capital holdings, profits in the product market are written as:

$$\pi^{B^*}(\alpha) \equiv \pi(\alpha, k, k) = \frac{A^2\alpha k}{(2b\alpha k + 1)} \tag{28}$$

Since firms have an equal amount of capital in each country (identical marginal cost curves), profit maximization will never include exports. Thus, trade costs do not enter into equilibrium profits when buying foreign capital.

3.1 Acquisition Stage

In the open economy model, firms must choose between four options in the acquisition stage. Respectively, the profits of selling, doing nothing, buying domestic capital, and buying foreign capital, can be written as:

$$\begin{aligned}\Pi^S(R_a) &= R_a \\ \Pi^N(\alpha) &= \pi^N(\alpha) \\ \Pi^B(\alpha, R_a) &= \pi^B(\alpha) - R_a \\ \Pi^{B^*}(\alpha, R_a) &= \pi^{B^*}(\alpha) - R_a - \delta\end{aligned}$$

Firms pay a fixed cost, R_a, for each lump of capital. This acquisition price will be endogenously determined, in equilibrium, using a market clearing condition for assets in each country. Given the assumption that countries are symmetric in every dimension, I will impose that the acquisition price is the same in each country. This is easily verified in equilibrium. For the moment, since firms are assumed to be small outside their own variety, R_a will be taken as given. Finally, note that in the last equation, firms acquiring abroad pay an additional fixed cost of serving the foreign market, δ. This is to embody the additional organizational, legal, or marketing costs associated with serving a foreign market.

A firm of productivity α chooses the acquisition option which maximizes profits in the acquisition market. Defining $V(\alpha, R_a)$ as acquisition market profits for a firm of productivity α, this decision is formally written as:

$$V(\alpha, R_a) = \max\left\{R_a, \pi^N(\alpha), \pi^B(\alpha) - R_a, \pi^{B^*}(\alpha) - R_a - \delta\right\} \tag{29}$$

The following normalization of (29) is again convenient:

$$\begin{aligned}\widehat{V}(\alpha, R_a) &\equiv V(\alpha, R_a) - \pi^N(\alpha) \\ &= \max\left\{R_a - \pi^N(\alpha), 0, \pi^B(\alpha) - \pi^N(\alpha) - R_a, \pi^{B^*}(\alpha) - \pi^N(\alpha) - R_a - \delta\right\}\end{aligned}$$

In $\widehat{V}(\alpha, R_a)$, the profits of each option are normalized relative to the outside option of doing nothing.

3.1.1 Domestic Acquisitions

Before characterizing $\widehat{V}(\alpha, R_a)$ as a function of productivity, it is useful to examine the effects of positive trade costs on the incentives to acquire, both domestically and abroad. Defining $\Delta\Pi(\alpha)$ as the product market profits of a domestic acquisition relative to doing nothing:

$$\begin{aligned}\Delta\Pi(\alpha) &= \pi^B(\alpha) - \pi^N(\alpha) \\ &= \left\{\begin{array}{lrl} \frac{A^2\alpha k}{2(2b\alpha k+1)(4b\alpha k+1)} & if\quad 0 & \leq \alpha < \frac{t}{4bk(A-t)} \\ \frac{A^2\alpha k}{(2b\alpha k+1)} - \frac{t(8bA\alpha k - 4b\alpha k t - t)}{8b(2b\alpha k+1)} - \frac{A^2\alpha k}{(4b\alpha k+2)} & \frac{t}{4bk(A-t)} & \leq \alpha < \frac{t}{2bk(A-t)} \\ \underbrace{\frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)}}_{\text{free trade incentives}} - \underbrace{\frac{tk\alpha(4A-t)}{8(b\alpha k+1)(2b\alpha k+1)}}_{\text{loss in market access}} & \frac{t}{2bk(A-t)} & \leq \alpha \end{array}\right\}\end{aligned} \tag{30}$$

In (30), the incentives to acquire a domestic firm embodied in $\Delta\Pi(\alpha)$ are split-up into three regions of productivity. For low values of α, firms cannot export before or after an acquisition, and the incentive to acquire a domestic firm is identical to the closed economy model. For firms in a middle range of productivity, the acquisition of additional capital provides the cost-improvement required to make exporting profitable. For high productivity firms, exporting is profitable before and after an acquisition. The incentives available to these firms are a combination of the incentives under free trade, and the negative effect of positive trade costs.

Despite the multiple regions of productivity, the following lemma proves that the incentives to acquire a domestic lump of capital are qualitatively identical to those in the closed economy.

Lemma 5 $\Delta\Pi(\alpha)$ *has the following properties:*

$$\begin{array}{lll}\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0 & if & \alpha < \widehat{\alpha} \\ \frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0 & if & \widehat{\alpha} < \alpha\end{array}$$

where,

$$\widehat{\alpha} = \left\{\begin{array}{lll}\frac{\sqrt{2}}{2bk} & if & t \leq \frac{2}{2+\sqrt{2}}A \\ \frac{\sqrt{2}}{4bk} & & t > \frac{2}{2+\sqrt{2}}A\end{array}\right\}.$$

Furthermore,

$$\Delta\Pi(0) = 0 \quad \lim_{\alpha\to\infty}\Delta\Pi(\alpha) = 0 \quad \frac{1}{2}\pi^B(\alpha) < \pi^N(\alpha)$$

Proof. See Appendix ■

Again, the incentive to acquire domestic capital is highest for mid-productivity firms. Low-productivity firms are constrained by their steep cost curve pre and post-acquisition. High productivity firms are constrained by bounds on variety specific revenues. Mid-productivity firms are relatively less-constrained by each, and have the largest incentive to acquire domestic capital.

3.1.2 Foreign Acquisitions and Trade Costs

Now turning attention to foreign acquisitions, the profits from acquiring foreign capital relative to doing nothing, $\Delta\Pi^*(\alpha)$, are written as:

$$\begin{aligned} \Delta\Pi^*(\alpha) &= \pi^{B^*}(\alpha) - \pi^N(\alpha) - \delta \qquad (31) \\ &= \left\{ \begin{array}{ll} \frac{A^2\alpha k}{2(2b\alpha k+1)} - \delta & \alpha \leq \frac{t}{2bk(A-t)} \\ \underbrace{\frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)} - \delta}_{\text{free trade incentives}} + \underbrace{\frac{t(4bA\alpha k - 2b\alpha kt - t)}{8b(b\alpha k+1)}}_{\text{gain in market access}} & \alpha > \frac{t}{2bk(A-t)} \end{array} \right\} \end{aligned}$$

In (31), $\Delta\Pi^*(\alpha)$ is split into two regions of productivity. The first part is for firms of low enough productivity such that exporting is not profitable given k units of capital at home. Since exporting is irrelevant for these firms, a foreign acquisition simply provides additional market access. In contrast, higher productivity firms export before a foreign acquisition, and thus acquiring a foreign firm not only affords *additional market access*, but also *diverts export production* to a newly purchased foreign affiliate. The former is embodied in the second term, which is a positive benefit of direct market access for potential exporters. The latter is embodied in the first term, which as will be explained shortly, is identical to the benefits of a domestic acquisition under free trade (except for the fixed cost of the foreign acquisition).

It is the production diversion, and the corresponding effects on production costs, which is crucial to the relationship between trade costs and foreign acquisition incentives. This can be seen by deriving the relevant properties of $\Delta\Pi^*(\alpha)$.

Lemma 6 $\Delta\Pi^*(\alpha)$ *has the following properties:*

$$\begin{aligned} \Delta\Pi^*(0) &= -\delta \\ \lim_{\alpha\to\infty} \Delta\Pi^*(\alpha) &= \frac{(2A-t)\,t}{4b} - \delta \\ \frac{1}{2}\pi^{B*}(\alpha) &\leq \pi^N(\alpha) \end{aligned}$$

Furthermore, if $t \leq \frac{2}{2+\sqrt{2}}A$, *then*

$$\begin{array}{lll} \frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} > 0 & \text{if} & \alpha < \frac{\sqrt{2}(2A+t+\sqrt{2}t)}{2bk(2A-2t-\sqrt{2}t)} \\ \frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} < 0 & & \alpha > \frac{\sqrt{2}(2A+t+\sqrt{2}t)}{2bk(2A-2t-\sqrt{2}t)} \end{array}$$

Conversely, if $t > \frac{2A}{2+\sqrt{2}}$, *then* $\frac{\partial \Delta \Pi^*(\alpha)}{\partial \alpha} > 0$ *for all* α.

Proof. See Appendix ■

Lemma 6 is an entirely novel result, and one which is central to the open economy model. It states that if trade costs are low ($t \leq \frac{2}{2+\sqrt{2}}A$), mid-productivity firms will have the highest incentive to acquire a foreign firm. In contrast, for high trade costs ($t > \frac{2}{2+\sqrt{2}}A$), high-productivity firms have the highest incentive to acquire a foreign firm.

The intuition for this result is best explained by how trade costs affect incentives for market access and variable factor efficiency. If trade costs are non-zero, purchasing a foreign firm always provides additional market access. Similar to Helpman, Melitz, and Yeaple (2004), the incentive to gain additional market access is increasing in productivity. High productivity firms are the largest exporters, and given that exporting costs are independent of productivity, these firms have the highest incentive to avoid these costs. Thus, absent any other incentives, high-productivity firms always have the highest incentive to acquire abroad.

However, in this particular model, there are additional incentives on the cost-side. Not only does purchasing a foreign firm provide additional market access, but it will divert export production that would otherwise be produced at home. Critically, given the fixed nature of capital at the time of production, higher output in one particular location increases the marginal cost of producing at that location.[15] By spreading production across capital at home and abroad, a firm can reduce production costs similar to a domestic acquisition. And, as has been discussed above, mid-productivity firms have the highest incentive to acquire to reduce production costs when faced with a bounded market for their variety.

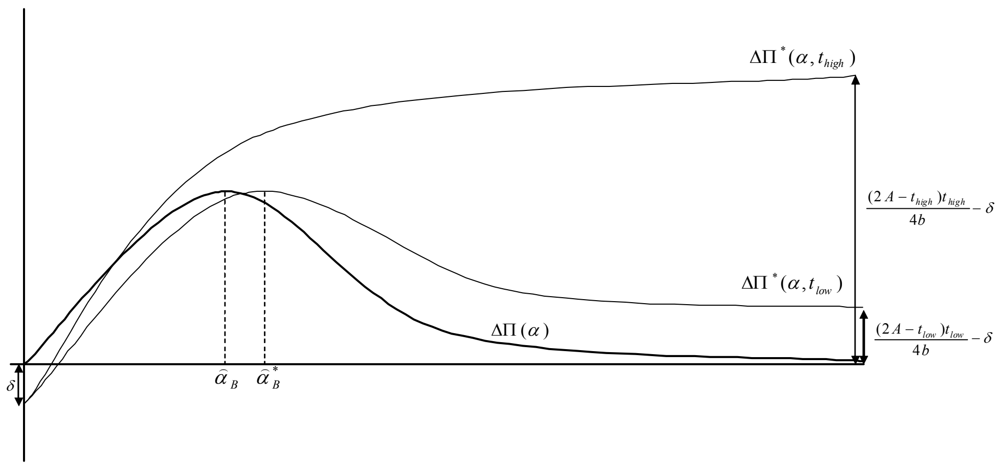
Thus, the central issue is whether market access (proximity) or production cost (concentration) considerations dominate. As characterized in Lemma 2, the relative size of trade costs provides a resolution to this issue. This is also illustrated in Figure 3. When trade costs are relatively high (t_{high} in Figure 3), exports are relatively low, and the incentives to gain additional market access dominate. Thus, high productivity firms have the highest incentive to acquire abroad. In contrast, when trade costs are low (t_{low} in Figure 3), foreign sales via exports are already significant and the incentives to gain additional market access are modest. Instead, the incentives to reduce production costs are relatively large, and mid-productivity firms have the highest incentive to acquire abroad

Essentially, one can think of foreign acquisitions as a skewed version of domestic acquisitions. If trade costs are zero, they function identically to domestic acquisitions, (except for the fixed cost δ). However, as trade costs increase, the incentives to acquire a foreign firm become skewed toward firms of higher productivity. Eventually, if trade costs are high enough, the incentive to acquire a foreign firm is an increasing function in productivity, and looks nothing like the incentive to acquire domestic firm.

While these properties are novel, the overall incentives must be compared with the aggregate measures (A and R_a) that are pinned down in general equilibrium. Again, A is fixed for clarity, where analysis of an endogenous value of A is relegated to a supplementary technical appendix. However, the acquisition price R_a is central to the policy results discussed in this paper, and will be characterized below. Crucially, depending on the size of the acquisition price, the non-monotone features of domestic and foreign acquisitions can lead to an array of possible equilibrium firm sortings. Rather than sift and discuss each possible outcome, I will focus on only one outcome to

[15] Indeed, this is the critical difference with other cost assumptions, such as total costs written as $C(q) = \frac{1}{\alpha k}q$. Here, diverting export production would yield no cost-side effects.

Figure 3: Acquisition Incentives - Open Economy



prove that an equilibrium exists, and to analyze the effects of policy parameters on the equilibrium sorting of firms and aggregate efficiency.

3.2 Equilibrium

With the incentives for domestic and foreign acquisitions in hand, I now characterize the equilibrium of the model subject to small trade and small foreign investment costs. Precisely, the following lemma characterizes the region over which I am solving the model.

Lemma 7 *There exists a* $\widehat{t}$ *and* $\widehat{\delta}$ *such that, over the space* $\left[0, \widehat{t}\right] \times \left[0, \widehat{\delta}\right]$, *any acquisition stage equilibrium must satisfy the following properties:*

1. *All active firms can export*

2. *Domestic and foreign acquisitions are not trivially unprofitable relative to the acquisition price.*

3. $\frac{(2A-t)t}{4b} - \delta < R_a$

Proof. See Appendix ■

With all details relegated to the appendix, Lemma 7 is proven using free trade as an analytical benchmark. To setup the equilibrium conditions of the model, I now discuss each condition outlined within Lemma 7.

Condition 1 simplifies the model substantially. Without Condition 1, export status conditional on acquisition status would have to be analyzed alongside acquisition choices, thus making the problem more complex. Also, restricting analysis to cases where all active firms can potentially export is similar Nocke and Yeaple (2007), in which all active firms either sell to, or operate in, the foreign market. Condition 2 states that at least some firms find a domestic acquisition or foreign acquisition profitable *relative to the acquisition price.* If this was not the case, then I could simply dispose of one type of acquisition depending on the value of trade costs. Finally, Condition 3 states that over $\left[0,\widehat{t}\right]\times\left[0,\widehat{\delta}\right]$, the highest productivity firms will not find any acquisition profitable. Simply put, I am proving that there exists a range of trade and investment costs in equilibrium such that high productivity firms do not invest, whether at home or abroad. Functionally, this guarantees the existence of two productivity cutoffs related to foreign acquisitions.

With Lemma 7 in-hand, I now turn to the characterization of firm-level behavior. Firms are indifferent between doing nothing and selling at α_S. This cutoff is defined by the following:

$$\pi^N\left(\alpha_S\right)=R_a \tag{32}$$

where,

$$For\ \alpha<\alpha_S\ ,\ S\succ N \tag{33}$$

Firms prefer selling if the acquisition price is greater than the return from staying in the market.

At $\underline{\alpha}_B$ and $\overline{\alpha}_B$, firms are indifferent between doing nothing and buying domestic capital. Precisely, these cutoffs are defined by the following equations:

$$\Delta\Pi\left(\underline{\alpha}_B\right)=R_a \tag{34}$$

$$\Delta\Pi\left(\overline{\alpha}_B\right)=R_a \tag{35}$$

where,

$$For\ \alpha\in\left(\underline{\alpha}_B,\overline{\alpha}_B\right),B\succ N \tag{36}$$

Here, similar to the closed economy model in section two, firms within a mid-range of productivity find a domestic acquisition profitable. The existence of $\underline{\alpha}_B$ and $\overline{\alpha}_B$ is guaranteed by Condition 2 in Lemma 7.

Similarly, at $\underline{\alpha}_B^*$ and $\overline{\alpha}_B^*$, firms are indifferent between foreign acquisitions and doing nothing. These two cutoffs are defined by:

$$\Delta\Pi^*\left(\underline{\alpha}_B^*\right)=R_a \tag{37}$$

$$\Delta\Pi^*\left(\overline{\alpha}_B^*\right)=R_a \tag{38}$$

where,

$$For\ \alpha\in\left(\underline{\alpha}_B^*,\overline{\alpha}_B^*\right),\ B^*\succ N \tag{39}$$

The existence of two productivity cutoffs, $\underline{\alpha}_B^*$ and $\overline{\alpha}_B^*$, is guaranteed by Conditions 2 and 3 in Lemma 7.

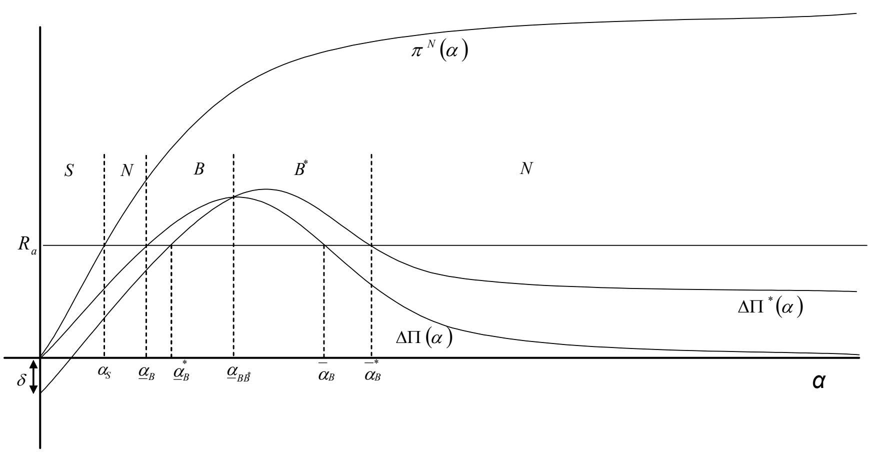
Finally, for $\frac{(2A-t)t}{4b}-\delta>0$, the indifference point between foreign acquisitions and domestic acquisitions, α_{BB^*}, is defined as follows:

$$\begin{aligned}\Delta\Pi\left(\alpha_{BB^*}\right)-\Delta\Pi^*\left(\alpha_{BB^*}\right)&=0\\ \alpha_{BB^*}&=\frac{(t^2+8\delta b)}{4bk((2A-t)t-4\delta b)}\end{aligned} \tag{40}$$

where,

$$\frac{\partial\left(\Delta\Pi\left(\alpha,A\right)-\Delta\Pi^*\left(\alpha,A\right)\right)}{\partial\alpha}=-\frac{tk\left(4A-t\right)}{4\left(2b\alpha k+1\right)^2}<0$$

Figure 4: Optimal Acquisition Choice - Open Economy



Thus,

$$For\ \alpha > \alpha_{BB^*}\ ,\ B^* \succ B \tag{41}$$

The most productive firms prefer foreign acquisitions to domestic acquisitions, where only the most efficient are able to recover the added fixed cost of foreign investment. For $\frac{(2A-t)t}{4b} - \delta < 0$, where the maximum market access benefit, $\frac{(2A-t)t}{4b}$, is dominated by the fixed cost, δ, domestic acquisitions are preferred to foreign acquisitions for all α.

Figure 4 illustrates one possible solution to the acquisition choice problem for a specific pair of t and δ in which both domestic and foreign acquisitions occur, and the three conditions in Lemma 3 are satisfied. In Figure 4, the least productive firms choose to sell and exit. Firms in a mid-range of productivity find some type of acquisition profitable, where the most-efficient firms within this group prefer foreign acquisitions. Of the remaining active firms, the least productive and most productive choose to do nothing in the acquisition market. For the highest productivity firms, the incentives to avoid trade costs are not sufficient to compensate for the acquisition price, which itself has been bid-up by mid-productivity firms.

Given Lemma 7, the following Proposition summarizes optimal acquisition choice as a function of small t and small δ.[16]

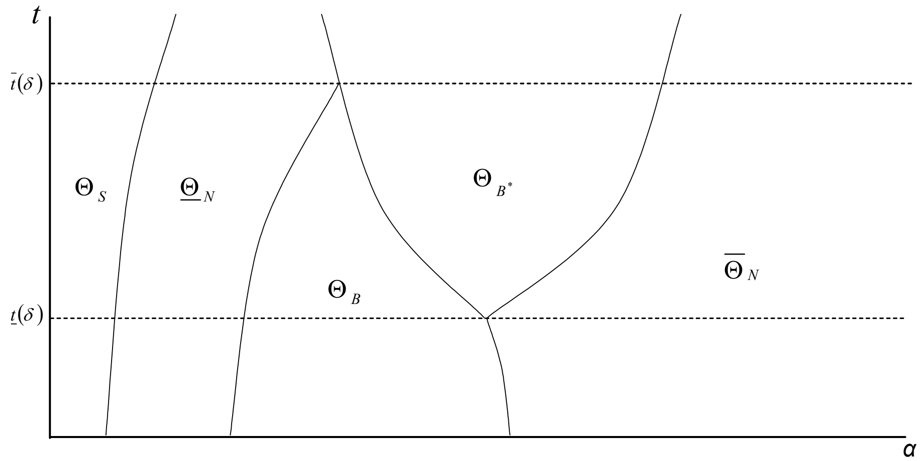
Proposition 2 *Given R_a, and $(t, \delta) \in [0, \widehat{t}] \times [0, \widehat{\delta}]$, there exist functions $\underline{t}(\delta)$ and $\overline{t}(\delta)$ such that the composition of the acquisition market is the following:*

$$\begin{array}{rrl} For & t \leq \underline{t}(\delta) & \textit{only domestic acquisitions occur} \\ & \underline{t}(\delta) < t < \overline{t}(\delta) & \textit{both domestic and foreign acquisitions occur} \\ & \overline{t}(\delta) \leq t & \textit{only foreign acquisitions occur} \end{array}$$

Furthermore, for $\overline{t}(\delta) \leq \widehat{t}$, optimal acquisition choice can be represented by Figure 5.

[16] The curvature in Figure 5 is approximate.

Figure 5: Optimal acquisition behavior as a function of trade costs



Proof. See Appendix ■

The role of trade costs in Figure 5 is fairly intuitive. In Figure 5, Θ_S represents the measure of firms that sell and exit the market. Given that all firms are exporters, higher trade costs diminish the value of staying in the market, leading to a larger measure of selling firms. Also in Figure 5, Θ_B and Θ_{B^*} represent the measure of firms that buy domestic and foreign capital, respectively. Intuitively, as trade costs increase, domestic acquisitions become less profitable and foreign acquisitions become more profitable. In Figure 5, this is illustrated in the contracting range of domestic acquisitions, Θ_B, and the expanding range of foreign acquisitions, Θ_{B^*}. Finally, there exist two regions, $\underline{\Theta}_N$ and $\overline{\Theta}_N$, such that firms do nothing.[17]

An important feature of Figure 5 is that the set of firms that acquire domestically, Θ_B, and the set of firms that acquire abroad, Θ_{B^*}, are non-nested. In other words, acquiring domestically is not the next best option every firm that acquires abroad, and vice versa. For some firms, the next best option to a domestic or foreign acquisition is no acquisition. The crucial implication of this feature for the forthcoming policy analyses is that there exist margins through which both domestic and foreign acquisition demand can influence *total* acquisition demand.

Lastly, Figure 5 describes a novel pattern of trade and investment. That is, exporters are not uniformly less productive than those firms investing abroad, where $\overline{\Theta}_N$ contains higher values of productivity than Θ_{B^*}. This suggests that in relatively integrated markets, such that the costs of international commerce are low, exporters should be the most productive. This could be an interesting prediction to test in future work using detailed census data. Indeed, this is similar to the discussion in Nocke and Yeaple (2007) in which the sorting of firms into modes of foreign investment depends crucially on the form of heterogeneity. The analogous result in this paper is that the basic characteristics of cost-structure and demand at the industry level motivate a similar degree of care in approaching empirical work. Indeed, one could use industry-specific regressions and non-parametric methods to evaluate the probability of exporting as function of productivity. As detailed by the above model, industries in which markets are relatively integrated may exhibit

[17] The formal representation of Θ_S, Θ_B, Θ_{B^*}, $\underline{\Theta}_N$, $\overline{\Theta}_N$ are contained in the appendix, as a function of t.

a non-monotone relationship between the probability of exporting and productivity.

General Equilibrium

To close the model, I now characterize the general equilibrium component of the acquisition market, R_a. The assumption of fixed R_a restricted the above discussion to a partial equilibrium analysis. Since interactions between buying and selling firms will ultimately dictate changes to industry efficiency, I now characterize how these elements determine the acquisition market clearing price, R_a, in general equilibrium.

Subject to firm-level acquisition decisions, the acquisition market must clear. Formally, this condition is written as:

$$\overbrace{\int_{\alpha\in\Theta_B} dG(\alpha) + \int_{\alpha\in\Theta_{B^*}} dG(\alpha)}^{Demand} = \overbrace{\int_{\alpha\in\Theta_S} dG(\alpha)}^{Supply} \tag{42}$$

It is straightforward to show that there exists a unique acquisition market clearing price, R_a. Domestic and foreign acquisition demand are both decreasing in the acquisition price, while the supply is increasing in the acquisition price. Thus, there exists an intersection of demand and supply at R_a such that there is as positive level of acquisition activity. Using the parameter space characterized in Lemma 7, and the acquisition behavior summarized in Proposition 2, this is proven in the following proposition.

Proposition 3 *Subject to the parameter space defined in Lemma 7 and optimal firm behavior in Proposition 2, there exists a unique R_a that clears the acquisition market.*

Proof. See Appendix ■

Proposition 3 proves that there exists a unique solution to the model presented in this section. Focusing on the acquisition market itself, higher trade costs shift up foreign acquisition demand and shift down domestic acquisition demand. In general, the acquisition market dictates how changes to the incentives affecting one type of firm (a selling firm, for example) spread throughout the economy affecting other types of firms. Given the restrictions in Lemma 7, falling trade costs directly impact all firms. However, the effects on buying firms will generally be larger than the effects on selling firms. I now precisely examine these effects, focusing on changes to firm-level behavior, and implications for aggregate efficiency.

4 Trade and Investment Liberalization

As discussed in the introduction, government policies regarding trade and investment remain quite heterogeneous. As such, understanding the effects of trade and investment policies on different types of investment, and any corresponding effects on aggregate measures of efficiency, is paramount. In this section I do exactly this, where I show that the degree to which liberalization improves aggregate productivity depends crucially on whether the liberalization is focused on trade or investment flows, and whether foreign acquisitions are an important component of the acquisition market.

Aggregate Productivity

To evaluate the effects of exogenous parameters, I first define the following measure of aggregate productivity, $\widetilde{\alpha}$:

$$\widetilde{\alpha} = s_{NA}\widetilde{\alpha}_{NA} + s_A\widetilde{\alpha}_A$$

Here, $\widetilde{\alpha}_{NA}$ represents the average productivity applied to units of capital which do not change ownership in the acquisition market, and $\widetilde{\alpha}_A$ reflects the average productivity applied to capital which changes ownership (acquired) in the acquisition market. Further, s_{NA} and s_A represent the respective shares of each group. Thus, $\widetilde{\alpha}$ will represent average productivity of firms operating in the product market, weighted by capital holdings. In terms of model parameters, $\widetilde{\alpha}$ is written as,

$$\begin{aligned}\widetilde{\alpha} \;=\; & \underbrace{\frac{1-G(\alpha_S)}{1-G(\alpha_S)+G(\overline{\alpha})-G(\underline{\alpha})}}_{s_{NA}} \cdot \underbrace{\frac{1}{1-G(\alpha_S)}\int_{\alpha_S}^{\infty}\alpha g(\alpha)d\alpha}_{\overline{\alpha}_{NA}} \\ & + \underbrace{\frac{G(\overline{\alpha})-G(\underline{\alpha})}{1-G(\alpha_S)+G(\overline{\alpha})-G(\underline{\alpha})}}_{s_A} \cdot \underbrace{\frac{1}{G(\overline{\alpha})-G(\underline{\alpha})}\int_{\underline{\alpha}}^{\overline{\alpha}}\alpha g(\alpha)d\alpha}_{\overline{\alpha}_A}\end{aligned}$$

where $\underline{\alpha}$ and $\overline{\alpha}$ represent the lower and upper bounds, respectively, of acquiring firms. These may be domestic acquiring firms if trade costs are low, foreign acquiring firms if trade costs are high, or some combination in a middle range of trade costs (within the bounds set in Lemma 7). Via the acquisition market clearing condition, $G(\alpha_S) = G(\overline{\alpha}) - G(\underline{\alpha})$, I can simplify $\widetilde{\alpha}$ as follows:

$$\widetilde{\alpha} = \int_{\alpha_S}^{\infty}\alpha g(\alpha)d\alpha + \int_{\underline{\alpha}}^{\overline{\alpha}}\alpha g(\alpha)d\alpha$$

Differentiating with respect to any exogenous variable x, and imposing the derivative of the acquisition market clearing condition yields the following:

$$\frac{\partial\widetilde{\alpha}}{\partial x} = \frac{\partial\overline{\alpha}}{\partial x}g(\overline{\alpha})\underbrace{(\overline{\alpha}-\alpha_S)}_{+} - \frac{\partial\underline{\alpha}}{\partial x}g(\underline{\alpha})\underbrace{(\underline{\alpha}-\alpha_S)}_{+} \tag{43}$$

Using (43), I will parsimoniously evaluate the effects of t and δ on aggregate productivity. The form of $\frac{\partial\widetilde{\alpha}}{\partial x}$ is fairly intuitive, since $\overline{\alpha}$ rising and/or $\underline{\alpha}$ falling represents an expansion of acquisition demand. This pushes up the acquisition price, and leads to a greater share of inefficient firms that sell and exit. Generally, when exogenous factors increase acquisition demand, aggregate productivity rises.

Trade Liberalization

To begin this section, I examine the effects of trade costs on acquisition decisions, and the corresponding effects on aggregate productivity. Specifically, I examine the effects of trade costs when the combination of t and δ is prohibitive to foreign acquisitions occurring in equilibrium. In Figure 5, this is the region where $t < \underline{t}(\delta)$. These effects are summarized in the following Proposition.

Proposition 4 *For $t < \underline{t}(\delta)$, where no foreign acquisitions occur, higher trade costs diminish* *acquisition activity. In terms of productivity cutoffs:*

$$\frac{\partial \alpha_S}{\partial t} < 0 \quad \frac{\partial \underline{\alpha}_B}{\partial t} > 0 \quad \frac{\partial \overline{\alpha}_B}{\partial t} < 0$$

Further, applying (43), $\frac{\partial \widetilde{\alpha}}{\partial t} < 0$.

Proof. See Appendix. ■

The effects of trade costs are driven by the interaction of the marginal acquiring and marginal selling firms. At a constant acquisition price, higher trade costs diminish domestic acquisition demand. Since higher trade costs reduce the effective size of the world market, fewer firms have sufficient incentives to increase production via domestic acquisitions. On the supply side, higher trade costs reduce the value of staying in the market, which increases the supply of selling firms. In equilibrium, the effects on the demand side dominate. Since selling-firms are very small exporters, the marginal effect of trade costs on these firms is minimal. Thus, higher trade costs shrink equilibrium acquisition behavior.[18]

The effect of higher trade costs leads to lower aggregate industry productivity via reallocation in the acquisition market. Since Proposition 2 dictates that acquisitions transfer capital from the least efficient firms to more efficient firms, and acquisition activity shrinks with higher trade costs, average industry productivity falls, and thus $\frac{\partial \widetilde{\alpha}}{\partial t} < 0$.

The response of aggregate productivity to trade costs is similar to that in Melitz and Ottaviano (2008), although their results are via a different exit mechanism. In their work, firms exit if they cannot profitably produce in the product market, where smaller trade costs make this outcome more likely. In my model, all firms have the opportunity to profitably sell to the product market, since the marginal cost of the first unit of production is zero. However, firms of low productivity find it in their best interest to sell their lump of capital to firms who are better able to utilize it.

The results of Proposition 4 are qualitatively identical to the results in Breinlich (2008). He also tests this prediction on a firm-level sample of US and Canadian firms, where the response of the acquisition market to trade costs is confirmed in the data. That is, trade liberalization expands acquisition activity. However, one important feature within the US-Canada case study is that domestic acquisitions comprise roughly 95% of all acquisitions. Thus, Breinlich's empirical work is well motivated by a model with only domestic acquisitions.

As many industries experience waves of both domestic and foreign acquisitions, I now derive the equilibrium effects of trade costs when both domestic and foreign acquisitions occur simultaneously. In Figure 5, this occurs when $\underline{t}(\delta) < t < \overline{t}(\delta)$. The effects of trade costs within this region are summarized in Proposition 5.

Proposition 5 *For* $\underline{t}(\delta) < t < \overline{t}(\delta)$, *where both domestic and foreign acquisitions occur, the effect of trade costs on acquisition activity is ambiguous. In terms of productivity cutoffs:*

$$\frac{\partial \alpha_S}{\partial t} \lesseqgtr 0 \quad \frac{\partial \underline{\alpha}_B}{\partial t} > 0 \quad \frac{\partial \alpha_{BB*}}{\partial t} < 0 \quad \frac{\partial \overline{\alpha}^*_B}{\partial t} > 0$$

Applying (43), $\frac{\partial \widetilde{\alpha}}{\partial t} \gtrless 0$. *However, if* $\frac{g(\overline{\alpha}^*_B)}{g(\underline{\alpha}_B)}$ *is sufficiently large,* $\frac{\partial \widetilde{\alpha}}{\partial t} > 0$.

[18] This result would remain if the marginal selling firm did not export.

Proof. See Appendix. ■

Since the highest productivity firms *do not* acquire, foreign acquisition activity may expand to higher productivity firms. This is the key source of the ambiguity in Proposition 5. Both domestic and foreign acquisitions have an effect on total acquisition demand. Since foreign acquisitions expand and domestic acquisitions contract in response to higher trade costs, the effect of trade costs on the acquisition market is theoretically ambiguous.[19]

The refinement of Proposition 5 is more striking. Within the acquisition market clearing condition, the effects on $\overline{\alpha}_B^*$ will be dominant over $\underline{\alpha}_B$ so long as $g(\overline{\alpha}_B^*)$ is not too small relative to $g(\underline{\alpha}_B)$ Precisely, if $g(\overline{\alpha}_B^*)$ is relatively large, then $\frac{\partial \widetilde{\alpha}}{\partial t} > 0$, and trade liberalization will reduce aggregate productivity.

To complete the analysis, I examine the region of trade costs such that $t > \overline{t}(\delta)$, where only foreign acquisitions occur. The effects within this region are summarized in the following Proposition:

Proposition 6 *For $t > \overline{t}(\delta)$, where only foreign acquisitions occur, higher trade costs increase acquisition activity. In terms of productivity cutoffs:*

$$\frac{\partial \alpha_S}{\partial t} > 0 \quad \frac{\partial \underline{\alpha}_B^*}{\partial t} \lesseqgtr 0 \quad \frac{\partial \overline{\alpha}_B^*}{\partial t} > 0$$

Applying (43), $\frac{\partial \widetilde{\alpha}}{\partial t} > 0$

Proof. See Appendix. ■

In Proposition 6, the qualitative movement of productivity cutoffs is nearly opposite of the movement in Proposition 3. Since foreign acquisitions expand with higher trade costs, the demand for domestic assets increases. Thus, more low-productivity firms exit the market via a sell-off. Overall, when acquisition demand is comprised entirely of foreign firms, trade liberalization reduces aggregate productivity.

In Propositions 5 and 6, the effect of trade costs on aggregate productivity is different than in Melitz and Ottaviano (2008). Again, this highlights the assumption of firm-exit via a sell-off. In their work, the marginal exiting firm is determined by the profitability of production. Independent of the decisions facing other (more productive) firms, higher trade costs decrease market toughness, leading to fewer exiting firms that find production unprofitable. In my model, the measure of exiting firms is determined by the relative value of the acquisition price, where the relative value of the acquisition price is positively affected by the willingness of other firms to pay for an acquisition. In the case outlined in Proposition 6, foreign firms are the only acquiring firms, and higher trade costs increase their willingness to pay for an acquisition. Thus, a greater share of (inefficient) firms find it in their best interest to sell and exit. Overall, productivity improves.

[19] In contrast, a CES demand system subject to the same cost structure would not deliver such a prediction. Since revenues are effectively unbounded, the incentives to acquire another firm are increasing in productivity, and the most efficient firms will choose foreign acquisitions. In equilibrium, total acquisition demand must decrease with higher trade costs, as domestic acquisitions are the only type of acquisition that affect total acquisition demand on the margin. Thus, higher trade costs, which promote foreign acquisitions relative to domestic, unambiguously reduce aggregate productivity.

Investment Liberalization

Continuing, I now discuss the effects of foreign investment liberalization on acquisition decisions, and the corresponding effects on aggregate productivity. The proxy for foreign investment liberalization will be δ, with larger values of δ representing more restrictive investment policies. Naturally, foreign investment liberalization will only affect equilibria if foreign investment occurs ($\underline{t}(\delta) < t$). In both cases, the effects of δ are qualitatively identical. With higher foreign investment costs, foreign acquisition demand for domestic assets decreases. This depresses the price for assets, which makes domestic acquisitions more profitable, and allows for more inefficient firms to remain in the market. Overall, aggregate productivity falls.[20]

4.1 Discussion

The results of this section can be summarized as a single theme: Whether trade or investment, if the manipulation of a policy results in an overall reduction in demand for industry specific assets, the effects on aggregate productivity will be negative. With respect to trade liberalization, aggregate productivity may fall if foreign firms are a relatively important source of demand for domestic assets.

Practically, these issues are important, as a contentious point of political debate ranges from the treatment of foreign imports to the treatment of foreign firms buying domestic assets. Poorly motivated policy may have unintended consequences for the utilization of assets and growth, and thus precise empirical work is necessary to evaluate the effects of policies on measure of efficiency. Unfortunately, episodes of liberalization rarely involve only one type of liberalization. For example, consider the case of India in the early 1990's, during which ownership restrictions were relaxed and import tariffs were liberalized according to multilateral commitments (Chibber and Majumdar, 1999; Panagariya, 2004). On one hand, reducing foreign investment restrictions would lead to an increase in demand by foreign firms for Indian assets. This would tend to increase aggregate productivity. On the other hand, the liberalization of tariffs would temper the incentive to locate directly in the Indian market, and would put downward pressure on acquisition demand. In this case, the role of the global acquisition market would be ambiguous as trade and investment liberalization have conflicting influences on aggregate productivity.

Overall, significant empirical work is necessary to ascertain the role of the asset (acquisition) market in changes to aggregate productivity. For example, consider the well-known paper by Trefler (2004), in which he examines the effects of the Canada-US free trade agreement on plant-level and industry-level productivity. With regard to the latter, Trefler estimates that the total impact of the FTA was to raise industry level productivity by 5-8%. However, Trefler (2004) differentiates the source of gains only by the degree to which tariffs are liberalized, and not issues related to within-industry reallocation via acquisitions. Breinlich (2008) fills-in this gap showing that an important channel of reallocation resulting from the Canada-US free trade agreement was in fact via domestic mergers. On a basic level, these results are jointly supportive of the prediction in Proposition 4. However, as foreign acquisitions rarely occurred, the precise role of foreign acquisitions in this particular case study remains elusive.

A number of authors have examined the impact of cross-border transactions on firm-performance, generally finding that inward FDI improves performance vis-a-vis domestic owned firms. For ex-

[20] A rigorous derivation of this result is omitted to save space, and is available upon request.

ample, see Arndt and Mattes (2008) and Arnold and Sarzynska-Javorcik (2009). In particular, the latter has a keen focus on the effect of inward acquisitions vis-a-vis domestic entrants and other types of foreign ownership, finding a positive effect of inward acquisitions on productivity, employment, and investment. However, the propensity-score matching approach used in Arnold and Sarzynska-Javorcik (2009) abstracts from the effects of tariff and investment liberalization. Thus, it still remains unclear whether the traditional, productivity enhancing effects of trade liberalization are tempered to any degree by a reduction in demand for domestic assets by foreign firms.

5 Conclusion

This paper presented an industry-equilibrium model of acquisition behavior in which firms trade capital after productivity has been realized. The incentives for acquisition, driven by firm heterogeneity in productivity, result from cost-lowering acquisitions and a linear demand framework. The main results of the model show that in a closed economy, mid-productivity firms are the most likely to acquire another firm. In an open economy model, while foreign acquiring firms are more productive than domestic acquiring firms, both groups of firms reside in a mid-range of productivity if trade costs are relatively small.

In evaluating the changing costs of trade, I show that trade liberalization improves domestic productivity only under certain conditions. Precisely, trade liberalization necessarily improves aggregate productivity only if foreign acquisitions do not occur. Generally, if foreign owners are an important source of acquisition demand, trade liberalization will hurt aggregate productivity by reducing foreign demand for domestic assets.

Overall, this paper adds a number of empirical questions onto the ever growing list of issues to evaluate with firm-level data. The basic prediction that mid-productivity firms are more likely to acquire has already been evaluated in a companion paper, Spearot (2009), using the *Compustat* database. Of note, Spearot (2009), shows that mid-productivity firms are most likely to acquire in industries in which firms are likely to have some price-setting power (as defined by Rauch, 1999, classifications). In industries in which firms are likely to act as price-takers, high productivity firms are the most likely to acquire, just as the theory predicts. However, the predictions of the open economy model, in particular the relationship between trade liberalization and aggregate productivity via reallocation in the acquisition market, have yet to be tested. This is an issue I plan to take up in future empirical work.

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A Closed Economy Proofs

A.1 Demand Generalization

Suppose that we have a monopolist that maximizes the following

$$\Pi = P(q) \cdot q - \frac{q^2}{2\alpha k}$$

Taking a derivative with respect to q, optimal output is determined by:

$$P'(q) \cdot q + P(q) = \frac{q}{\alpha k} \tag{44}$$

By the envelope theorem, the marginal value of additional capital can be written as:

$$\begin{aligned} \frac{\partial \Pi}{\partial k} &= \underbrace{\left(P'(q) \cdot q + P(q) - \frac{q}{\alpha k}\right)}_{=0} \frac{\partial q}{\partial k} + \frac{q^2}{\alpha k^2} \\ &= \frac{q^2}{\alpha k^2} > 0 \end{aligned} \tag{45}$$

Note that the marginal value of additional capital (net of capital costs) is always positive. Using (44), equation (45) can be written as:

$$\frac{\partial \Pi}{\partial k} = \alpha \left(P'(q) \cdot q + P(q)\right)^2 = \alpha \left(MR(q)\right)^2$$

Differentiating with respect to α, we get:

$$\frac{\partial^2 \Pi}{\partial k \partial \alpha} = \left(MR(q)\right)^2 + 2\alpha \left(MR(q)\right) \frac{\partial MR(q)}{\partial q} \frac{\partial q}{\partial \alpha}$$

As the final intermediate step, note that using (44) $\frac{\partial q}{\partial \alpha}$ is written as,

$$\frac{\partial q}{\partial \alpha} = \frac{kMR(q)}{1 - \alpha k \frac{\partial MR(q)}{\partial q}}$$

and thus $\frac{\partial^2 \Pi}{\partial k \partial \alpha}$ is simplified as:

$$\frac{\partial^2 \Pi}{\partial k \partial \alpha} = \underbrace{\left(MR(q)\right)^2}_{>0} \left(\frac{\overbrace{1 + \alpha k \frac{\partial MR(q)}{\partial q}}^{?}}{\underbrace{1 - \alpha k \frac{\partial MR(q)}{\partial q}}_{>0}} \right) \tag{46}$$

Thus, $\frac{\partial^2 \Pi}{\partial k \partial \alpha} > 0$ if $\alpha k < -\frac{1}{\frac{\partial MR(q)}{\partial q}}$, and $\frac{\partial^2 \Pi}{\partial k \partial \alpha} > 0$ if $\alpha k > -\frac{1}{\frac{\partial MR(q)}{\partial q}}$.

A.2 Proof of Lemma 1

To prove Lemma 1 for small R_a, I need to establish that $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$. Once this ranking is established, Lemma 1 is immediate via the preference conditions in (16) and (19). To show $\alpha_S < \underline{\alpha}_B$, first note that from (15) and (17) it must be the case that:

$$\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) = \pi^N(\alpha_S)$$

Rearranging,

$$\frac{1}{2}\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) = \pi^N(\alpha_S) - \frac{1}{2}\pi^B(\underline{\alpha}_B)$$

Since $\frac{1}{2}\pi^B(\underline{\alpha}_B) < \pi^N(\underline{\alpha}_B)$, the RHS must also be negative in equilibrium. This is only possible if $\alpha_S < \underline{\alpha}_B$. By definition, $\underline{\alpha}_B < \overline{\alpha}_B$. Using this result and $\alpha_S < \underline{\alpha}_B$, this completes the proof that $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$.

A.3 Proof of Lemma 2

Holding A fixed, differentiating the acquisition profit functions (15), (17), and (18) with respect to R_a, and using the properties of $\frac{\pi^B(\alpha)}{\partial\alpha} - \frac{\pi^N(\alpha)}{\partial\alpha}$, I get:

$$\begin{aligned}
\overset{>0}{\frac{\pi^N(\alpha_S(R_a(A), A))}{\partial\alpha}}\frac{\partial\alpha_S(R_a(A), A)}{\partial R_a(A)} &= 1 \\
\overbrace{\left(\frac{\pi^B(\underline{\alpha}_B(R_a(A), A))}{\partial\alpha} - \frac{\pi^N(\underline{\alpha}_B(R_a(A), A))}{\partial\alpha}\right)}^{>0}\frac{\partial\underline{\alpha}_B(R_a(A), A)}{\partial R_a(A)} &= 1 \\
\overbrace{\left(\frac{\partial\pi^B(\overline{\alpha}_B(R_a(A), A))}{\partial\alpha} - \frac{\pi^N(\overline{\alpha}_B(R_a(A), A))}{\partial\alpha}\right)}^{<0}\frac{\partial\overline{\alpha}_B(R_a(A), A)}{\partial R_a(A)} &= 1
\end{aligned}$$

These derivatives clearly yield $\frac{\partial\alpha_S(R_a(A),A)}{\partial R_a(A)} > 0$, $\frac{\partial\underline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} > 0$, and $\frac{\partial\overline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} < 0$. Differentiating (20) and (21) with respect to $R_a(A)$, and imposing $\frac{\partial\alpha_S(R_a(A),A)}{\partial R_a(A)} > 0$, $\frac{\partial\underline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} > 0$, and $\frac{\partial\overline{\alpha}_B(R_a(A),A)}{\partial R_a(A)} < 0$,

$$\begin{aligned}
\frac{\partial K_D(R_a(A))}{\partial R_a} &= M_E k g(\overline{\alpha}_B(R_a(A), A))\frac{\partial\overline{\alpha}_B(R_a(A), A)}{\partial R_a(A)} \\
&\quad - M_E k g(\underline{\alpha}_B(R_a(A), A))\frac{\partial\underline{\alpha}_B(R_a(A), A)}{\partial R_a(A)} \\
&< 0
\end{aligned} \tag{47}$$

$$\frac{\partial K_S(R_a(A))}{\partial R_a(A)} = M_E k\left(g(\alpha_S(R_a(A), A))\frac{\partial\alpha_S(R_a(A), A)}{\partial R_a(A)}\right) > 0 \tag{48}$$

Naturally, $K_D(R_a(A))$ is decreasing and $K_S(R_a(A))$ is increasing in the acquisition price. Thus, if $K_D(R_a)$ is larger (smaller) than $K_S(R_a)$ at low (high) values of R_a, there exists a unique R_a

that clears the acquisition market. When $R_a \to 0$, using (10), (11) and (12):

$$\alpha_S\left(R_a\left(A\right), A\right) \to 0,\ \underline{\alpha}_B\left(R_a\left(A\right), A\right) \to 0,\ \overline{\alpha}_B\left(R_a\left(A\right), A\right) \to \infty$$

This yields,

$$K_D\left(R_a\left(A\right)\right) \to M_E k,\ K_S\left(R_a\left(A\right)\right) \to 0 \tag{49}$$

Similarly, as $R_a \to \frac{\left(3-2\sqrt{2}\right)A^2}{4b}$:

$$\alpha_S\left(R_a\left(A\right), A\right) > 0,\ \underline{\alpha}_B\left(R_a\left(A\right), A\right) \to \frac{\sqrt{2}}{4bk},\ \overline{\alpha}_B\left(R_a\left(A\right), A\right) \to \frac{\sqrt{2}}{4bk}$$

which yields:

$$\begin{aligned} K_D\left(R_a\left(A\right)\right) &\to 0 \\ K_S\left(R_a\left(A\right)\right) &> 0 \end{aligned} \tag{50}$$

Thus, using (49) and (50), there exists a unique value $\widehat{R}_a\left(A\right)$ that clears the acquisition market.

B Open Economy Proofs

B.1 Proof of Lemma 5

To begin the derivation of Lemma 5, I will focus on $\alpha \leq \frac{t}{4bk(A-t)}$. Differentiating $\frac{A^2\alpha k}{2(4\alpha kb+1)(2\alpha kb+1)}$ with respect to α yields:

$$\frac{\partial \Delta\Pi\left(\alpha\right)}{\partial \alpha} = \frac{A\alpha k\left(1-8b^2\alpha^2k^2\right)}{2(4\alpha kb+1)^2(2\alpha kb+1)^2}$$

Clearly, the positive root of $\Delta\Pi\left(\alpha\right)$ is $\alpha = \frac{\sqrt{2}}{4bk}$. However, this maximum is irrelevant if $\frac{t}{4bk(A-t)} < \frac{\sqrt{2}}{4bk}$. This condition simplifies to $\frac{\sqrt{2}}{4bk}\left(\frac{2A-(2+\sqrt{2})t}{A-t}\right) > 0$. This is satisfied if $t < \frac{2}{2+\sqrt{2}}A$. Thus, $\frac{\partial \Delta\Pi(\alpha)}{\partial \alpha} > 0$ for $\alpha \leq \frac{t}{4bk(A-t)}$

Now turning to $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{2bk(A-t)}$, over this range, $\Delta\Pi\left(\alpha\right)$ has two roots (in α). They are written as:

$$\begin{aligned} \alpha_1 &= \frac{\sqrt{2}\left(-2A+\left(2+\sqrt{2}\right)t\right)}{4bk\left(-A+\left(A-t\right)\left(\sqrt{2}+1\right)\right)} \\ \alpha_2 &= -\frac{\sqrt{2}\left(2\left(A-t\right)+\sqrt{2}t\right)}{4bk\left(A+\left(A-t\right)\left(\sqrt{2}-1\right)\right)} < 0 \end{aligned}$$

Clearly, α_2 is not relevant. The root α_1 is relevant if the following condition holds.

$$\alpha_1 - \frac{t}{4bk(A-t)} = \frac{\sqrt{2}\,(2A-t)\overbrace{\left(\left(2+\sqrt{2}\right)t-2A\right)}^{\text{(-) if } t<\frac{2}{2+\sqrt{2}}A}}{8bk\,(A-t)\underbrace{\left(-A+(A-t)\left(\sqrt{2}+1\right)\right)}_{\text{(+) if } t<\frac{2}{2+\sqrt{2}}A \text{ (see below)}}} < 0$$

To see that $-A+(A-t)\left(\sqrt{2}+1\right) > 0$ if $t < \frac{2}{2+\sqrt{2}}A$, rearrange the first expression to read $t < \left(1-\frac{1}{1+\sqrt{2}}\right)A$. Simplifying, we get $t < \frac{2}{2+\sqrt{2}}A$. Thus, since the numerator and denominator always have an opposite sign, it must always be the case that $\alpha_1 < \frac{t}{4bk(A-t)}$, and thus not relevant in the mid range of productivity.

Both $\alpha_1 < \frac{t}{4bk(A-t)}$ and $\alpha_2 < 0$ imply that if $t < \frac{2}{2+\sqrt{2}}A$, the roots of $\Delta\Pi\left(\alpha\right)$ do not occur over the range $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{2bk(A-t)}$. Thus, the sign of $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha}$ is constant over this range. To see when $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$, note that:

$$\frac{\partial\Delta\Pi\left(\frac{t}{4bk(A-t)}\right)}{\partial\alpha} = \frac{k(A-t)^2\overbrace{\left(2A^2-4At+t^2\right)}^{\text{(+) if } t<\frac{2}{2+\sqrt{2}}A}}{2(2A-t)^2}$$

Clearly, the sign of $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha}$ over the mid range of productivity is dependent on the size of trade costs relative to the market, and is positive if $t < \frac{2}{2+\sqrt{2}}A$.

Finally, for $\frac{t}{2bk(A-t)} < \alpha$, I can write:

$$\frac{\partial\Delta\Pi\left(\alpha\right)}{\partial\alpha} = \frac{k\left(2A-t\right)\left(4-8b^2\alpha^2k^2\right)}{2(4\alpha kb+2)^2(2\alpha kb+2)^2}$$

The positive solution to $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} = 0$ is $\frac{\sqrt{2}}{2bk}$. It is also clear that for $\alpha < \frac{\sqrt{2}}{2bk}$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} > 0$, and for $\alpha > \frac{\sqrt{2}}{2bk}$, $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < 0$.

To finish proving the properties in Lemma 5, I must show that $\frac{1}{2}\pi^B\left(\alpha\right) < \pi^N\left(\alpha\right) < \pi^B\left(\alpha\right)$ for $\alpha > 0$. The condition $\pi^N\left(\alpha\right) < \pi^B\left(\alpha\right)$ is trivial through lower costs. The condition $\frac{1}{2}\pi^B\left(\alpha\right) < \pi^N\left(\alpha\right)$ is true if $\Delta\Pi\left(\alpha\right) < \pi^N\left(\alpha\right)$. Expanding $\Delta\Pi\left(\alpha\right)$, I get $\pi^B\left(\alpha\right) - \pi^N\left(\alpha\right) < \pi^N\left(\alpha\right)$, which simplifies to $\frac{1}{2}\pi^B\left(\alpha\right) < \pi^N\left(\alpha\right)$. Thus, I will now show that $\Delta\Pi\left(\alpha\right) < \pi^N\left(\alpha\right)$.

To do this, I will first show that $\Delta\Pi\left(0\right) = \pi^N\left(0\right)$, and then $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} < \frac{\partial\pi^N(\alpha)}{\partial\alpha}$ for all finite α. Showing $\Delta\Pi\left(0\right) = \pi^N\left(0\right)$ is straightforward, given the multiplicative nature of α. For $\alpha \leq \frac{t}{4bk(A-t)}$, I can write $\frac{\partial\Delta\Pi(\alpha)}{\partial\alpha} - \pi^N\left(\alpha\right)$ as:

$$\frac{\partial\Delta\Pi\left(\alpha\right)}{\partial\alpha} - \pi^N\left(\alpha\right) = -\frac{2A^2\alpha^2k^2b}{(4\alpha kb+1)(2\alpha kb+1)} < 0$$

For $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{2bk(A-t)}$, I can write $\frac{\partial \Delta\Pi(\alpha)}{\partial\alpha} - \frac{\partial \pi^N(\alpha)}{\partial\alpha}$ as:

$$\frac{\partial \Delta\Pi\left(\alpha\right)}{\partial\alpha} - \pi^N\left(\alpha\right) = -\frac{kt\left(2A-t\right)}{(4\alpha kb+2)^2} < 0$$

Finally, for $\frac{t}{2bk(A-t)} < \alpha$, I can write $\frac{\partial \Delta\Pi(\alpha)}{\partial\alpha} - \frac{\partial \pi^N(\alpha)}{\partial\alpha}$ as:

$$\frac{\partial \Delta\Pi\left(\alpha\right)}{\partial\alpha} - \pi^N\left(\alpha\right) = -\frac{4bk^2\alpha\left(2A-t\right)^2\left(3b\alpha k+2\right)}{(4\alpha kb+2)^2(2\alpha kb+2)^2} < 0$$

B.2 Proof of Lemma 6

Clearly, $\Delta\Pi^*(0) = -\delta$. In addition, it is straightforward to show that $\lim_{\alpha\to\infty}\Delta\Pi^*(\alpha) = \frac{(2A-t)t}{4b} - \delta$. To derive the slope properties of $\Delta\Pi^*(\alpha)$, note that for $\alpha \leq \frac{t}{2bk(A-t)}$, $\frac{A^2\alpha k}{2(2b\alpha k+1)}$ is equal to the no acquisition profit function in the closed economy, $\pi^N(\alpha)$. Thus, for $\alpha \leq \frac{t}{2bk(A-t)}$, $\frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} > 0$. For $\alpha > \frac{t}{2bk(A-t)}$, with some work, one can show that the only positive root of $\Delta\Pi^*(\alpha)$ is $\widehat{\alpha} = \frac{\sqrt{2}(2A+t+\sqrt{2}t)}{2bk(2A-2t-\sqrt{2}t)}$. To start, differentiate $\Delta\Pi^*(\alpha)$ with respect to α, set equal to zero, and solve for α:

$$\widehat{\alpha}_1 = \frac{\overbrace{\left(\left(2\sqrt{2}-2\right)A+t\right)}^{+}}{2bk\underbrace{\left(2A-2t-\sqrt{2}t\right)}_{+\text{ if } t<\frac{2}{2+\sqrt{2}}A}} \qquad \widehat{\alpha}_2 = -\frac{\overbrace{\left(2A-\left(\sqrt{2}-1\right)t\right)}^{+}}{2bk\underbrace{\left(2\left(A-t\right)+\sqrt{2}t\right)}_{+}} < 0$$

Clearly, $\widehat{\alpha}_2$ is irrelevant. Regarding $\widehat{\alpha}_1$, it is only relevant if $t < \frac{2}{2+\sqrt{2}}A$. To see this, note that $\widehat{\alpha}_1 > \frac{t}{2bk(A-t)}$ if the following holds:

$$\widehat{\alpha}_1 - \frac{t}{2bk(A-t)} = \frac{(\sqrt{2}-1)(2A-t)A}{2bk\left(A-t\right)\left(\left(2-\sqrt{2}\right)A-t\right)} > 0$$

This clearly holds if $t < \left(2-\sqrt{2}\right)A$. Note that $t < \left(2-\sqrt{2}\right)A$ can be written as $t < \left(2-\sqrt{2}\right)\frac{2+\sqrt{2}}{2+\sqrt{2}}A = \frac{4-2+2\sqrt{2}-2\sqrt{2}}{2+\sqrt{2}}A = \frac{2}{2+\sqrt{2}}A$.

Now, all that is left is identifying the sign of $\frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha}$ on either side of $\widehat{\alpha}$. At $\alpha = \frac{t}{2bk(A-t)}$, $\frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} = \frac{k(A-t)^2}{2} > 0$. The second derivative at $\widehat{\alpha}_1$ gives us guidance to the shape at the peak. Precisely:

$$\frac{\partial^2\Delta\Pi^*\left(\widehat{\alpha}_1\right)}{\partial\alpha^2} = -\frac{\overbrace{(17\sqrt{2}-24)}^{+}bk^2(2A-2t-\sqrt{2}t)^4}{4\left(2A-t\right)A}$$

Clearly, $\frac{\partial^2\Delta\Pi^*(\widehat{\alpha}_1)}{\partial\alpha^2} < 0$ only if $t \neq \frac{2}{2+\sqrt{2}}A$. Thus, if $t \neq \frac{2}{2+\sqrt{2}}A$, $\widehat{\alpha}_1$ is a maximum, and $\frac{\partial\Delta\Pi^*(\alpha)}{\partial\alpha} < 0$ for finite $\alpha > \widehat{\alpha}_1$. As $t \to \frac{2}{2+\sqrt{2}}A$, $\widehat{\alpha}_1 \to \infty$, where $\frac{\partial^2\Delta\Pi^*(\widehat{\alpha}_1)}{\partial\alpha^2} \to 0$.

To show that $\frac{1}{2}\pi^{B^*}(\alpha) \leq \pi^N(\alpha) < \pi^{B^*}(\alpha)$, note that for low productivity $\left(\alpha \leq \frac{t}{2bk(A-t)}\right)$, $\frac{1}{2}\pi^{B^*}(\alpha) = \pi^N(\alpha)$. For $\alpha > \frac{t}{2bk(A-t)}$, firms that do nothing can now export and $\pi^N(\alpha)$ is larger relative to $\pi^{B^*}(\alpha)$. Thus, $\frac{1}{2}\pi^{B^*}(\alpha) \leq \pi^N(\alpha)$.

B.3 Proof of Lemma 7

The existence of R_a is straightforward. Acquisition demand is decreasing (from positive demand) starting from $R_a = 0$. Eventually, acquisition demand is 0 with R_a high enough. This is since the incentives derived in $\Delta\Pi(\alpha)$ and $\Delta\Pi^*(\alpha)$ are bounded above. The opposite is the case with acquisition supply. For $R_a = 0$, firms have no incentive to sell. For R_a high enough, all firms sell. Thus, using the intermediate value theorem, there must exist a $R_a > 0$ such that the acquisition market clears.

Continuing, it will be convenient to use the case of free trade as an analytical benchmark. Setting $t = 0$, we find that acquisition incentives may be written as:

$$\Delta\Pi(\alpha|t=0) = \frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)} \quad \Delta\Pi^*(\alpha|t=0) = \frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)} - \delta$$

Clearly, these incentives are identical outside of the fixed cost of foreign investment, δ. The intuition is straightforward. With $t = 0$, we have an integrated world market that, holding A fixed, is twice the size of each market. Both domestic and foreign acquisitions provide two lumps of capital, and by the equalization of marginal costs across production locations (lumps of capital), both domestic and foreign acquisitions will produce the same amount for the integrated world market.

Taking into account the fixed costs of foreign investment, we see that foreign acquisitions will not occur in free trade. Further, note that $\frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)}$ is qualitatively identical to closed economy acquisition incentives. The only difference is that the slope of aggregate world demand curve in the open economy is $1/2$ as steep as in the closed economy. As the proofs in the closed economy model are valid for any b, there exists an equilibrium value of R_a is such that $R_a < \max_{\alpha}\left\{\frac{A^2\alpha k}{2(b\alpha k+1)(2b\alpha k+1)}\right\} = \frac{(3-2\sqrt{2})}{2b}A^2$.

Moving forward, writing the equilibrium value of R_a as $R_a(t,\delta)$, I now formally define $\left[0,\widehat{t}\right] \times \left[0,\widehat{\delta}\right]$. First, I will define the space $\left[0,\widehat{t}\right]$ for any δ. Then, over the space $\left[0,\widehat{t}\right]$, I will define $\left[0,\widehat{\delta}\right]$.

Step 1: Fix δ. Note that $\frac{(2A-t)t}{4b} - \delta < R_a(t,\delta)$ is satisfied if $\frac{(2A-t)t}{4b} < R_a(t,\delta)$. Note that $\lim_{t\longrightarrow 0}\frac{(2A-t)t}{4b} = 0$. Since $R_a(t,\delta) > 0$, there exists a $t_1(\delta) > 0$ such that $\frac{(2A-t)t}{4b} < R_a(t,\delta)$ for $t \in [0, t_1(\delta)]$. This also satisfies the exporter condition for all active firms, where by manipulating the exporting cutoffs and profit functions, I can show that all active firms are exporters if $\frac{At}{4b} < R_a$. For $A > t$, which is the condition for non-prohibitive trade, it is the case that $\frac{At}{4b} < \frac{(2A-t)t}{4b}$.

Next, I must ensure that each acquisition option is profitable relative to the acquisition price for at least some range of firms. For domestic acquisitions, this is satisfied if $\max_{\alpha}\Delta\Pi(\alpha) = \frac{(3-2\sqrt{2})}{8b}(2A-t)^2 > R_a(t,\delta)$. Since $\frac{\partial\Delta\Pi(\alpha)}{\partial t} \leq 0$, it is possible that for some t, $\frac{(3-2\sqrt{2})}{8b}(2A-t)^2 < R_a(t,\delta)$. However, the free trade equilibrium developed above dictates that $\frac{(3-2\sqrt{2})}{2b}A^2 > R_a(0,\delta)$. This is convenient, since $\lim_{t\longrightarrow 0}\frac{(3-2\sqrt{2})}{8b}(2A-t)^2 = \frac{(3-2\sqrt{2})}{2b}A^2$. Thus, there must exist a $t_2(\delta) > 0$ such that $\frac{(3-2\sqrt{2})}{8b}(2A-t)^2 > R_a(t,\delta)$ holds for $t \in [0, t_2(\delta)]$. Summarizing so far, the first two conditions are jointly satisfied for $t \in [0, \widetilde{t}(\delta)]$, where $\widetilde{t}(\delta) = \min\{t_1(\delta), t_2(\delta)\} > 0$.

The above paragraph pins down an upper bound $\widetilde{t}(\delta) > 0$ for each value of δ. Thus, the minimum upper bound of t can be characterized as follows:

$$\widehat{t} = \min_{\delta}\left\{\widetilde{t}(\delta)\right\} > 0$$

Before moving to step two, first note that $\max_{\alpha} \Delta\Pi^* (\alpha|t=0) = \frac{(3-2\sqrt{2})}{2b} A^2 - \delta$. Since $\frac{\partial \Delta\Pi^*(\alpha)}{\partial t} > 0$ for firms that can export, this implies that $\max_{\alpha} \Delta\Pi^* (\alpha) \geq \max_{\alpha} \Delta\Pi^* (\alpha|t=0)$. This is will prove to be a useful relationship to use below.

Step 2: Fix $t \in [0, \widehat{t}]$. Over this region of trade costs, $\frac{(3-2\sqrt{2})}{8b} (2A - t)^2 > R_a(t, \delta)$. Since $\frac{(3-2\sqrt{2})}{2b} A^2 > \frac{(3-2\sqrt{2})}{8b} (2A-t)^2$, this implies that $\frac{(3-2\sqrt{2})}{2b} A^2 > R_a(t, \delta)$ over this same region. Thus, there must exist a $\widetilde{\delta}(t) = \frac{(3-2\sqrt{2})}{8b} (2A-t)^2 - R_a(t, \delta) > 0$ such that for $\delta \in [0, \widetilde{\delta}(t)]$, $\frac{(3-2\sqrt{2})}{2b} A^2 - \delta > R_a(t, \delta)$, and foreign acquisitions are not trivially unprofitable relative to the acquisition price. Thus, the minimum upper bound of δ can be characterized as follows:

$$\widehat{\delta} = \min_{t \in [0, \widehat{t}]} \left\{\widetilde{\delta}(t)\right\} > 0$$

Thus, in equilibrium, there exists a subspace $\left[0, \widehat{t}\right] \times \left[0, \widehat{\delta}\right]$ such that all active firms are exporters, neither acquisition choice is trivially unprofitable, and the limit of $\Delta\Pi^*(\alpha)$ is lower than the equilibrium acquisition price.

B.4 Proof of Proposition 2

To prove Proposition 1, I must first establish that $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$. To show $\alpha_S < \underline{\alpha}_B$, first note that from (32) and (34) it must be the case that:

$$\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) = \pi^N(\alpha_S)$$

Rearranging,

$$\frac{1}{2}\pi^B(\underline{\alpha}_B) - \pi^N(\underline{\alpha}_B) = \pi^N(\alpha_S) - \frac{1}{2}\pi^B(\underline{\alpha}_B)$$

Since $\frac{1}{2}\pi^B(\underline{\alpha}_B) < \pi^N(\underline{\alpha}_B)$, the RHS must also be negative in equilibrium. This is only possible if $\alpha_S < \underline{\alpha}_B$. By definition, $\underline{\alpha}_B < \overline{\alpha}_B$. Using this result and $\alpha_S < \underline{\alpha}_B$, it is clear that $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$.

To show that $\alpha_S < \underline{\alpha}^*_B < \overline{\alpha}^*_B$, the analysis is similar. Equilibrium conditions dictate that:

$$\pi^{B^*}(\underline{\alpha}^*_B) - \pi^N(\underline{\alpha}^*_B) - \delta = \pi^N(\alpha_S)$$

Subtracting $\frac{1}{2}\pi^{B^*}(\underline{\alpha}^*_B)$, we get:

$$\frac{1}{2}\pi^{B^*}(\underline{\alpha}^*_B) - \pi^N(\underline{\alpha}^*_B) - \delta = \pi^N(\alpha_S) - \frac{1}{2}\pi^{B^*}(\underline{\alpha}^*_B)$$

Since all firms are exporters, using the result in Lemma 6, it must be the case that $\frac{1}{2}\pi^{B^*}(\underline{\alpha}^*_B) -$

$\pi^N(\underline{\alpha}_B^*) < 0$. Thus, the RHS, $\pi^N(\alpha_S) - \frac{1}{2}\pi^{B^*}(\underline{\alpha}_B^*)$ must also be negative. This can only be the case if $\underline{\alpha}_B^* > \alpha_S$. Finally, $\overline{\alpha}_B^* > \underline{\alpha}_B^*$ follows from the shape of $\Delta\Pi^*(\alpha)$. Thus, $\alpha_S < \underline{\alpha}_B^* < \overline{\alpha}_B^*$.

Using $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$ and $\alpha_S < \underline{\alpha}_B^* < \overline{\alpha}_B^*$, for $\alpha < \alpha_S$, firms prefer selling. Formally, defining Θ_s as the measure of firms that sell, $\Theta_s = [0, \alpha_S)$.

The relationship $\alpha_S < \underline{\alpha}_B^* < \overline{\alpha}_B^*$ summarizes the decision between doing nothing and buying a foreign firm. The relationship $\alpha_S < \underline{\alpha}_B < \overline{\alpha}_B$ summarizes the decision between doing nothing and buying a domestic firm. The productivity cutoff α_{BB^*}, defined by (40), and the corresponding preference condition in (41) summarize the decision between buying a foreign firm and buying a domestic firm. I now jointly analyze these three decisions, completing the proof of Proposition 2.

A few straightforward properties that will be used in the proof are the following:

$$\begin{aligned}
\frac{\partial \alpha_{BB^*}}{\partial t} &= \frac{-\left(\frac{\partial \Delta\Pi^*(\alpha_{BB^*})}{\partial t} - \frac{\partial \Delta\Pi(\alpha_{BB^*})}{\partial t}\right)}{\frac{\partial \Delta\Pi^*(\alpha_{BB^*})}{\partial \alpha} - \frac{\partial \Delta\Pi(\alpha_{BB^*})}{\partial \alpha}} < 0 \\
\frac{\partial \overline{\alpha}_B^*}{\partial t} &= \frac{-\left(\frac{\partial \Delta\Pi^*(\overline{\alpha}_B^*)}{\partial t}\right)}{\frac{\partial \Delta\Pi^*(\overline{\alpha}_B^*)}{\partial \alpha}} > 0\ ,\ \frac{\partial \underline{\alpha}_B^*}{\partial t} = \frac{-\left(\frac{\partial \Delta\Pi^*(\underline{\alpha}_B^*)}{\partial t}\right)}{\frac{\partial \Delta\Pi^*(\underline{\alpha}_B^*)}{\partial \alpha}} < 0 \\
\frac{\partial \overline{\alpha}_B}{\partial t} &= \frac{-\left(\frac{\partial \Delta\Pi(\overline{\alpha}_B)}{\partial t}\right)}{\frac{\partial \Delta\Pi(\overline{\alpha}_B)}{\partial \alpha}} < 0\ ,\ \frac{\partial \underline{\alpha}_B}{\partial t} = \frac{-\left(\frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial t}\right)}{\frac{\partial \Delta\Pi^*(\underline{\alpha}_B)}{\partial \alpha}} > 0
\end{aligned} \tag{51}$$

The strategy to prove Proposition 2 is to start at free trade and show how the composition of the acquisition market changes with gradually increasing trade costs. To begin, fix $\delta < \widehat{\delta}$ and $t = 0$. According to (40), α_{BB^*} is not defined for $t = 0$ and $\delta > 0$, and thus $B \succ B^*$. Since $B \succ B^*$ for all α, then $\Delta\Pi^*(\alpha) < \Delta\Pi(\alpha)$ for all α. More generally, α_{BB^*} is not defined for t such that $(2A - t)t/4b - \delta < 0$. Since $(2A - t)t/4b$ is increasing in t for $t < A$, there exists a range of very low t such that $\Delta\Pi^*(\alpha) < \Delta\Pi(\alpha)$ for all α, and thus $B \succ B^*$ for all α.

Within this range of very low t, it is straightforward to show that $\underline{\alpha}_B < \underline{\alpha}_B^* < \overline{\alpha}_B^* < \overline{\alpha}_B$. To see this, first suppose to the contrary that $\overline{\alpha}_B^* > \overline{\alpha}_B$. Since by definition, $\frac{\partial \Delta\Pi(\overline{\alpha}_B)}{\partial \alpha} < 0$ for $\alpha \geq \overline{\alpha}_B$, this would imply that $\Delta\Pi(\overline{\alpha}_B^*) < R_a$. Given the equilibrium condition $\Delta\Pi^*(\overline{\alpha}_B^*) = R_a$, this is contradiction of $\Delta\Pi^*(\alpha) < \Delta\Pi(\alpha)$ for all α. A similar argument applies to $\underline{\alpha}_B < \underline{\alpha}_B^*$. Thus, for t such that $(2A - t)t/4b - \delta < 0$, it must be that $\underline{\alpha}_B < \underline{\alpha}_B^* < \overline{\alpha}_B^* < \overline{\alpha}_B$.

Now consider the range of t such that $(2A - t)t/4b - \delta > 0$. At t close to $\widetilde{t}$ such that $(2A - \widetilde{t})\widetilde{t}/4b - \delta = 0$, α_{BB^*} is very high (approaches ∞ from below with lower t) and decreases with higher t. Note that since $\frac{\partial \overline{\alpha}_B^*}{\partial t} > 0$, $\frac{\partial \overline{\alpha}_B}{\partial t} < 0$, and $\overline{\alpha}_B > \overline{\alpha}_B^*$, there exists a unique value of t such that $\overline{\alpha}_B = \overline{\alpha}_B^*$. Supposing that this t exists and is labeled $\underline{t}'(\delta)$, at this t it must be the case that $\Delta\Pi(\overline{\alpha}_B^*) = \Delta\Pi^*(\overline{\alpha}_B^*)$. This implies that at $\underline{t}'(\delta)$, $\alpha_{BB^*} = \overline{\alpha}_B = \overline{\alpha}_B^*$.

Thus, for $t < \underline{t}'(\delta)$, $\underline{\alpha}_B < \underline{\alpha}_B^* < \overline{\alpha}_B^* < \overline{\alpha}_B < \alpha_{BB^*}$. Using (39) and (36), foreign acquisitions are not the preferred acquisition choice for any value of α. If foreign acquisitions are profitable for some region of productivity, domestic acquisitions are more profitable over this same region. If $\widehat{t} < \underline{t}'(\delta)$, the above ranking still holds. Defining $\underline{t}(\delta) = \min\left\{\widehat{t}, \underline{t}'(\delta)\right\}$, we have the following:

$$For\ t < \underline{t}(\delta),\ \left\{ \begin{array}{llllll} \Theta_S = & (0, \alpha_s) & \underline{\Theta}_N = & (\alpha_s, \underline{\alpha}_B) & \Theta_B = & (\underline{\alpha}_B, \overline{\alpha}_B) \\ & & \overline{\Theta}_N = & (\overline{\alpha}_B, \infty) & \Theta_{B^*} = & \varnothing \end{array} \right\}$$

As an aside, note that $\frac{\partial \underline{t}'(\delta)}{\partial \delta} > 0$. This is a straightforward calculation obtained by differentiating $\Delta\Pi\left(\overline{\alpha}_B, \underline{t}'(\delta)\right) = \Delta\Pi^*\left(\overline{\alpha}_B, \underline{t}'(\delta)\right)$ with respect to t, and substituting in the derivative of $\Delta\Pi\left(\overline{\alpha}_B(\underline{t}'(\delta)), \underline{t}'(\delta)\right) = R_a$ with respect to t. This yields $\left(\underbrace{\frac{\partial\Delta\Pi^*(\overline{\alpha}_B)}{\partial\alpha}}_{-}\underbrace{\frac{\partial\overline{\alpha}_B}{\partial t}}_{-} + \underbrace{\frac{\partial\Delta\Pi^*(\overline{\alpha}_B)}{\partial t}}_{+}\right)\underbrace{\frac{\partial\underline{t}'(\delta)}{\partial\delta}}_{>0} = -\underbrace{\frac{\partial\Delta\Pi^*(\overline{\alpha}_B)}{\partial\delta}}_{+}$. As trade costs increase, a given firm of productivity $\overline{\alpha}_B$ remains indifferent between domestic and foreign acquisitions if there is a suitable increase in foreign investment costs. Also, $\underline{t}'(0) = 0$. Intuitively, if the cost of foreign investment is zero, a firm is indifferent between a foreign and domestic acquisition only if the relative incentive to acquire abroad is also zero. Functionally, this implies that $\underline{t}'(\delta) < \widehat{t}$ for δ sufficiently low.

Moving forward, since $\frac{\partial\overline{\alpha}_B^*}{\partial t} > 0$ and $\frac{\partial\overline{\alpha}_B}{\partial t} < 0$, for $t \underset{(<)}{>} \underline{t}'(\delta)$, $\overline{\alpha}_B \underset{(>)}{<} \overline{\alpha}_B^*$. In addition, the following shows that for $t \underset{(<)}{>} \underline{t}'(\delta)$, $\alpha_{BB^*} \underset{(>)}{<} \overline{\alpha}_B$. Evaluating $\frac{\partial\alpha_{BB^*}}{\partial t} - \frac{\partial\overline{\alpha}_B}{\partial t}$ at $\alpha_{BB^*} = \overline{\alpha}_B$ yields $\frac{\partial\alpha_{BB^*}}{\partial t} - \frac{\partial\overline{\alpha}_B}{\partial t} < 0$. This can be seen by writing:

$$\frac{\partial\alpha_{BB^*}}{\partial t} - \frac{\partial\overline{\alpha}_B}{\partial t} = \frac{-\left(\frac{\partial\Delta\Pi^*(\alpha_{BB^*})}{\partial t} - \frac{\partial\Delta\Pi(\alpha_{BB^*})}{\partial t}\right)}{\frac{\partial\Delta\Pi^*(\alpha_{BB^*})}{\partial\alpha} - \frac{\partial\Delta\Pi(\alpha_{BB^*})}{\partial\alpha}} - \frac{-\left(\frac{\partial\Delta\Pi(\overline{\alpha}_B)}{\partial t}\right)}{\frac{\partial\Delta\Pi(\overline{\alpha}_B)}{\partial\alpha}}$$

Imposing $\alpha_{BB^*} = \overline{\alpha}_B$, I can simply the above expression as:

$$\frac{\partial\alpha_{BB^*}}{\partial t} - \frac{\partial\overline{\alpha}_B}{\partial t} = \frac{-\overbrace{\frac{\partial\Delta\Pi^*\left(\overline{\alpha}_B\right)}{\partial t}}^{+}\overbrace{\frac{\partial\Delta\Pi\left(\overline{\alpha}_B\right)}{\partial\alpha}}^{-} + \overbrace{\frac{\partial\Delta\Pi\left(\overline{\alpha}_B\right)}{\partial t}}^{-}\overbrace{\frac{\partial\Delta\Pi^*\left(\overline{\alpha}_B\right)}{\partial\alpha}}^{-}}{\underbrace{\left(\frac{\partial\Delta\Pi^*(\overline{\alpha}_B)}{\partial\alpha} - \frac{\partial\Delta\Pi(\overline{\alpha}_B)}{\partial\alpha}\right)}_{+}\underbrace{\frac{\partial\Delta\Pi(\overline{\alpha}_B)}{\partial\alpha}}_{-}} < 0$$

Thus, since $\alpha_{BB^*} > \overline{\alpha}_B$ for $t < \underline{t}'(\delta)$ and $\alpha_{BB^*} = \overline{\alpha}_B$ at $t = \underline{t}'(\delta)$, α_{BB^*} must be less than $\overline{\alpha}_B$ for $t > \underline{t}'(\delta)$.

Further, note that for t slightly above $\underline{t}'(\delta)$, where α_{BB^*} is sufficiently close to $\overline{\alpha}_B$ and $\overline{\alpha}_B^*$, $\underline{\alpha}_B$ and $\underline{\alpha}_B^*$ are both be less than α_{BB^*}. This implies that $\underline{\alpha}_B < \underline{\alpha}_B^*$. To see this, suppose to the contrary that $\underline{\alpha}_B > \underline{\alpha}_B^*$. Since by definition, $\frac{\partial\Delta\Pi(\underline{\alpha}_B)}{\partial\alpha} > 0$ for $\alpha < \overline{\alpha}_B$, this would imply that $\Delta\Pi\left(\underline{\alpha}_B^*\right) < R_a$. Given the equilibrium condition $\Delta\Pi^*\left(\overline{\alpha}_B^*\right) = R_a$, this is contradiction of $\Delta\Pi^*\left(\alpha\right) < \Delta\Pi\left(\alpha\right)$ for $\alpha < \alpha_{BB^*}$. In summary, for t greater than $\underline{t}'(\delta)$, $\alpha_{BB^*} < \overline{\alpha}_B < \overline{\alpha}_B^*$, and for t slightly greater than $\underline{t}'(\delta)$, $\underline{\alpha}_B < \underline{\alpha}_B^* < \alpha_{BB^*}$.

Continuing, increase t and note that $\frac{\partial\underline{\alpha}_B^*}{\partial t} < 0$ and $\frac{\partial\underline{\alpha}_B}{\partial t} > 0$. Given that $\underline{\alpha}_B < \underline{\alpha}_B^*$ for t close to $\underline{t}'(\delta)$, there exists a unique value of t, labeled $\overline{t}'(\delta)$, such that $\underline{\alpha}_B = \underline{\alpha}_B^*$. At this t it must be the case that $\Delta\Pi\left(\underline{\alpha}_B\right) = \Delta\Pi^*\left(\underline{\alpha}_B\right)$. This implies that at $\overline{t}'(\delta)$, $\alpha_{BB^*} = \underline{\alpha}_B = \underline{\alpha}_B$. Thus, for $\underline{t}(\delta) < t < \overline{t}'(\delta)$, $\underline{\alpha}_B < \underline{\alpha}_B^* < \alpha_{BB^*} < \overline{\alpha}_B < \overline{\alpha}_B^*$. If $\underline{t}(\delta) < \widehat{t} < \overline{t}'(\delta)$, the same ranking holds. Thus, defining $\overline{t}(\delta) = \min\left\{\widehat{t}, \overline{t}'(\delta)\right\}$, for $\underline{t}(\delta) < t < \overline{t}(\delta)$:

$$\textit{For } \underline{t}(\delta) < t < \overline{t}(\delta), \quad \left\{\begin{array}{llllll} \Theta_S = & (0, \alpha_s) & \underline{\Theta}_N = & (\alpha_s, \underline{\alpha}_B) & \Theta_B = & (\underline{\alpha}_B, \alpha_{BB^*}) \\ & & \overline{\Theta}_N = & (\overline{\alpha}_B^*, \infty) & \Theta_{B^*} = & (\alpha_{BB^*}, \overline{\alpha}_B^*) \end{array}\right\}$$

By similar derivations to above, note that $\frac{\partial\overline{t}'(\delta)}{\partial\delta} > 0$, $\overline{t}'(0) = 0$. Thus, for low enough δ, the value

$\overline{t}(\delta)$ will exist.

Moving forward, using $\frac{\partial \underline{\alpha}_B^*}{\partial t} < 0$ and $\frac{\partial \underline{\alpha}_B}{\partial t} > 0$, $t \underset{(<)}{>} \overline{t'}(\delta)$, $\underline{\alpha}_B^* \underset{(>)}{<} \underline{\alpha}_B$. Further, the following shows that for $t \underset{(<)}{>} \overline{t'}(\delta)$, $\alpha_{BB^*} \underset{(>)}{<} \underline{\alpha}_B^*$. Evaluating $\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \underline{\alpha}_B^*}{\partial t}$ at $\alpha_{BB^*} = \underline{\alpha}_B^* (= \underline{\alpha}_B)$ yields $\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \underline{\alpha}_B^*}{\partial t} < 0$. This can be seen by writing:

$$\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \underline{\alpha}_B^*}{\partial t} = \frac{-\left(\frac{\partial \Delta\Pi^*(\alpha_{BB^*})}{\partial t} - \frac{\partial \Delta\Pi(\alpha_{BB^*})}{\partial t}\right)}{\frac{\partial \Delta\Pi^*(\alpha_{BB^*})}{\partial \alpha} - \frac{\partial \Delta\Pi(\alpha_{BB^*})}{\partial \alpha}} - \frac{-\left(\frac{\partial \Delta\Pi^*\left(\underline{\alpha}_B^*\right)}{\partial t}\right)}{\frac{\partial \Delta\Pi^*\left(\underline{\alpha}_B^*\right)}{\partial \alpha}}$$

Imposing $\alpha_{BB^*} = \underline{\alpha}_B^* (= \underline{\alpha}_B)$, I can simply the above expression as:

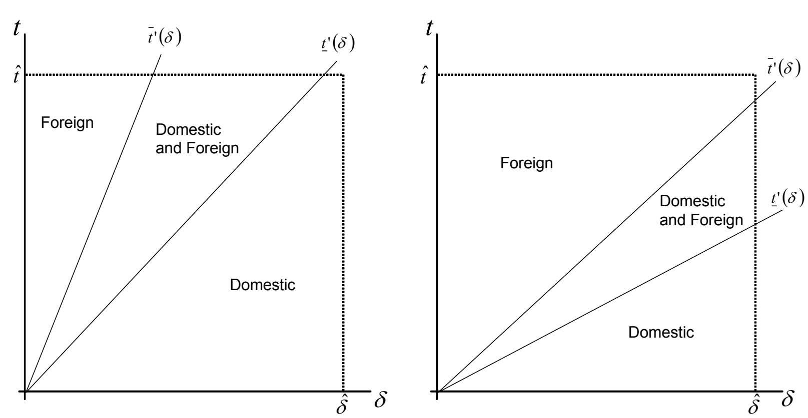
$$\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \underline{\alpha}_B^*}{\partial t} = \frac{-\overbrace{\frac{\partial \Delta\Pi^*\left(\underline{\alpha}_B^*\right)}{\partial t}}^{+}\overbrace{\frac{\partial \Delta\Pi\left(\underline{\alpha}_B^*\right)}{\partial \alpha}}^{+} + \overbrace{\frac{\partial \Delta\Pi\left(\underline{\alpha}_B^*\right)}{\partial t}}^{-}\overbrace{\frac{\partial \Delta\Pi^*\left(\alpha_{BB^*}\right)}{\partial \alpha}}^{+}}{\underbrace{\left(\frac{\partial \Delta\Pi^*\left(\underline{\alpha}_B^*\right)}{\partial \alpha} - \frac{\partial \Delta\Pi\left(\underline{\alpha}_B^*\right)}{\partial \alpha}\right)}_{+}\underbrace{\frac{\partial \Delta\Pi\left(\underline{\alpha}_B^*\right)}{\partial \alpha}}_{+}} < 0$$

Since $\alpha_{BB^*} > \underline{\alpha}_B^*$ for $t < \overline{t'}(\delta)$, $\alpha_{BB^*} < \underline{\alpha}_B^*$ for $t > \overline{t'}(\delta)$. In summary, I have shown that $\alpha_{BB^*} < \underline{\alpha}_B^* < \underline{\alpha}_B < \overline{\alpha}_B < \overline{\alpha}_B^*$. Thus, for $t > \overline{t}(\delta)$, I can state the following:

$$For\ \overline{t}(\delta) < t,\ \left\{ \begin{array}{llll} \Theta_S = & (0, \alpha_s) & \underline{\Theta}_N = (\alpha_s, \underline{\alpha}_B^*) & \Theta_B = \{\ \varnothing\ \} \\ & & \overline{\Theta}_N = (\overline{\alpha}_B^*, \infty) & \Theta_{B^*} = (\underline{\alpha}_B^*, \overline{\alpha}_B^*) \end{array} \right\}$$

Domestic acquisitions never occur over this region.

To summarize the results of Proposition 2, the following figure (with assumed linear curvature) illustrates the composition of the acquisition market for $\left[0, \widehat{t}\right] \times \left[0, \widehat{\delta}\right]$ under two different relative positions of $\underline{t}'(\delta)$ and $\overline{t'}(\delta)$. Independent of the relative position of $\widehat{t}$, $\widehat{\delta}$, $\underline{t}'(\delta)$ and $\overline{t'}(\delta)$, the two-panels in the above figure show that for a given value of δ, the higher are trade costs, the more likely a firm will acquire abroad.



B.5 Proof of Proposition 3

It is straightforward to show how the productivity cutoffs are affected by R_a. Precisely, $\frac{\partial \alpha_S}{\partial R_a} > 0$, $\frac{\partial \underline{\alpha}_B}{\partial R_a} > 0$ $\frac{\partial \overline{\alpha}_B}{\partial R_a} < 0$, $\frac{\partial \underline{\alpha}_B^*}{\partial R_a} < 0$, $\frac{\partial \overline{\alpha}_B^*}{\partial R_a} > 0$, and $\frac{\partial \alpha_{BB^*}}{\partial R_a} = 0$. In the below analysis, define $K_D(R_a)$ and $K_D^*(R_a)$ as domestic and foreign acquisition demand, respectively. Define $K_S(R_a)$ as acquisition supply.

If $(2A-t)t/4b - \delta < 0$, then α_{BB^*} does not exist. This implies that domestic acquisitions are always preferred to foreign acquisitions. Thus, $K_D^*(R_a) = 0$. In this case, the proof a unique market clearing price R_a^* is identical to the closed economy.

Now, suppose that $(2A-t)t/4b - \delta > 0$. Foreign acquisitions are now profitable at some point along the range of α. Begin at $R_a = 0$. Here, $K_D^*(R_a) = (G(\infty) - G(\alpha_{BB^*}))\, kM_E = (1 - G(\alpha_{BB^*}))\, kM_E$, $K_D(R_a) = (G(\alpha_{BB^*}) - G(0)) = G(\alpha_{BB^*})kM_E$ and $K_S(R_a) = 0$. Acquisition demand is greater than acquisition supply and cannot be an equilibrium.

Increasing R_a, it is straightforward to show that $\frac{\partial K_D^*(R_a)}{\partial R_a} \leq 0$, $\frac{\partial K_D(R_a)}{\partial R_a} \leq 0$, $\frac{\partial K_S(R_a)}{\partial R_a} \geq 0$. When $R_a = \frac{A^2}{4b} + \frac{(A-t)^2}{4b}$, $K_D(R_a) = 0$, $K_D^*(R_a) = 0$, $K_S(R_a) = 1$. This also cannot be an equilibrium since supply is greater than demand. However, by the intermediate value theorem, there is a unique intersection of $K_D(R_a) + K_D^*(R_a)$ and $K_S(R_a)$ at $R_a^* > 0$. This completes the proof.

B.6 Proof of Proposition 4

Differentiating the equilibrium conditions in (32), (34), (35), and (42) for $t < \underline{t}(\delta)$, we get:

$$\begin{bmatrix} -\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & \frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial \alpha} & 0 \\ -\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & 0 & \frac{\partial \Delta\Pi(\overline{\alpha}_B)}{\partial \alpha} \\ g(\alpha_S) & g(\underline{\alpha}_B) & -g(\overline{\alpha}_B) \end{bmatrix} \begin{bmatrix} \frac{\partial \alpha_S}{\partial t} \\ \frac{\partial \underline{\alpha}_B}{\partial t} \\ \frac{\partial \overline{\alpha}_B}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial t} \\ \frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta\Pi(\overline{\alpha}_B)}{\partial t} \\ 0 \end{bmatrix}$$

Using the equilibrium conditions in (32), (34) and (35), note that:

$$\frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial t} = \frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta\Pi(\overline{\alpha}_B)}{\partial t} = \frac{At}{2bk(2A-t)} > 0$$

we can solve the system of equations and get (unless otherwise labeled, individual terms are positive):

$$\begin{aligned} \frac{\partial \alpha_S}{\partial t} &= \frac{\bar{1}}{D}\left(-\frac{\partial \bar{\Delta\Pi}(\overline{\alpha}_B)}{\partial \alpha} g(\underline{\alpha}_B) + \frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial \alpha} g(\overline{\alpha}_B)\right)\left(\frac{At}{2bk(2A-t)}\right) < 0 \\ \frac{\partial \underline{\alpha}_B}{\partial t} &= \frac{\bar{1}}{D}\left(\frac{\partial \bar{\Delta\Pi}(\overline{\alpha}_B)}{\partial \alpha} g(\alpha_S)\right)\left(\frac{At}{2bk(2A-t)}\right) > 0 \\ \frac{\partial \overline{\alpha}_B}{\partial t} &= \frac{\bar{1}}{D}\left(\frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial \alpha} g(\alpha_S)\right)\left(\frac{At}{2bk(2A-t)}\right) < 0 \end{aligned}$$

where:

$$D = \frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial \alpha}\frac{\partial \overline{\Delta\Pi}\left(\overline{\alpha}_B\right)}{\partial \alpha} g(\alpha_S) - \frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial \alpha}\frac{\partial \pi^N\left(\alpha_S\right)}{\partial \alpha} g(\overline{\alpha}_B) + \frac{\partial \overline{\Delta\Pi}\left(\overline{\alpha}_B\right)}{\partial \alpha}\frac{\partial \pi^N\left(\alpha_S\right)}{\partial \alpha} g(\underline{\alpha}_B) < 0$$

B.7 Proof of Proposition 5

Differentiating the equilibrium conditions in (32), (34), (38), and (42), for $\underline{t}(\delta) < t < \overline{t}(\delta)$, we get:

$$\begin{bmatrix} -\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & \frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial \alpha} & 0 \\ -\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & 0 & \frac{\partial \Delta\Pi^*(\overline{\alpha}_B^*)}{\partial \alpha} \\ g(\alpha_S) & g(\underline{\alpha}_B) & -g(\overline{\alpha}_B^*) \end{bmatrix} \begin{bmatrix} \frac{\partial \alpha_S}{\partial t} \\ \frac{\partial \underline{\alpha}_B}{\partial t} \\ \frac{\partial \overline{\alpha}_B^*}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta\Pi(\underline{\alpha}_B)}{\partial t} \\ \frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta\Pi^*(\overline{\alpha}_B^*)}{\partial t} \\ 0 \end{bmatrix}$$

In addition to the work in the previous proposition, the following result follows directly from differentiating profit functions:

$$\frac{\partial \Delta\Pi^*\left(\overline{\alpha}_B^*\right)}{\partial t} - \frac{\partial \overline{\pi}^N\left(\alpha_S\right)}{\partial t} > 0$$

Continuing, I can write:

$$\begin{aligned} \frac{\partial \alpha_S}{\partial t} &= -\frac{1}{D}\overbrace{\frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial \alpha} g(\overline{\alpha}_B^*)\left(\frac{\partial \Delta\Pi^*\left(\overline{\alpha}_B^*\right)}{\partial t} - \frac{\partial \overline{\pi}^N\left(\alpha_S\right)}{\partial t}\right)}^{>0} \\ &\quad -\frac{1}{D}\frac{\partial \overline{\Delta\Pi}^*\left(\overline{\alpha}_B^*\right)}{\partial \alpha} g(\underline{\alpha}_B)\overbrace{\left(\frac{\partial \pi^N\left(\alpha_S\right)}{\partial t} - \frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial t}\right)}^{>0} \lesseqgtr 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \underline{\alpha}_B}{\partial t} &= -\frac{1}{D}\overbrace{\frac{\partial \pi^N\left(\alpha_S\right)}{\partial \alpha} g(\overline{\alpha}_B^*)\left(\frac{\partial \Delta\Pi^*\left(\overline{\alpha}_B^*\right)}{\partial t} - \frac{\partial \overline{\pi}^N\left(\alpha_S\right)}{\partial t}\right)}^{>0} \\ &\quad -\frac{1}{D}\overbrace{\left(\frac{\partial \pi^N\left(\alpha_S\right)}{\partial \alpha} g(\overline{\alpha}_B^*) - \frac{\partial \Delta\Pi^*\left(\overline{\alpha}_B^*\right)}{\partial \alpha} g(\alpha_S)\right)}^{>0}\overbrace{\left(\frac{\partial \pi^N\left(\alpha_S\right)}{\partial t} - \frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial t}\right)}^{>0} > 0 \\ \frac{\partial \overline{\alpha}_B^*}{\partial t} &= -\frac{1}{D}\frac{\partial \pi^N\left(\alpha_S\right)}{\partial \alpha} g(\underline{\alpha}_B)\overbrace{\left(\frac{\partial \pi^N\left(\alpha_S\right)}{\partial t} - \frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial t}\right)}^{>0} \\ &\quad -\frac{1}{D}\overbrace{\left(\frac{\partial \Delta\Pi\left(\underline{\alpha}_B\right)}{\partial \alpha} g(\alpha_S) + \frac{\partial \pi^N\left(\alpha_S\right)}{\partial \alpha} g(\underline{\alpha}_B)\right)\left(\frac{\partial \Delta\Pi^*\left(\overline{\alpha}_B^*\right)}{\partial t} - \frac{\partial \overline{\pi}^N\left(\alpha_S\right)}{\partial t}\right)}^{>0} > 0 \end{aligned}$$

where:

$$D=\frac{\partial\Delta\Pi\left(\underline{\alpha}_B\right)}{\partial\alpha}\frac{\overset{-}{\partial\Delta\Pi^*}\left(\overline{\alpha}_B^*\right)}{\partial\alpha}g(\alpha_S)-\frac{\partial\Delta\Pi\left(\underline{\alpha}_B\right)}{\partial\alpha}\frac{\partial\pi^N\left(\alpha_S\right)}{\partial\alpha}g(\overline{\alpha}_B^*)+\frac{\overset{-}{\partial\Delta\Pi^*}\left(\overline{\alpha}_B^*\right)}{\partial\alpha}\frac{\partial\pi^N\left(\alpha_S\right)}{\partial\alpha}g(\underline{\alpha}_B)<0$$

and,

$$\frac{\partial\pi^N\left(\alpha_S\right)}{\partial t}-\frac{\partial\Delta\Pi\left(\underline{\alpha}_B\right)}{\partial t}=\frac{At}{2bk\left(2A-t\right)}$$

Finally, differentiating (40),

$$\frac{\partial\alpha_{BB^*}}{\partial t}=-\frac{\overbrace{\left(\frac{\partial\Delta\Pi^*\left(\alpha_{BB^*}\right)}{\partial t}-\frac{\partial\Delta\Pi\left(\alpha_{BB^*}\right)}{\partial t}\right)}^{>0}}{\underbrace{\left(\frac{\partial\Delta\Pi^*\left(\alpha_{BB^*}\right)}{\partial\alpha}-\frac{\partial\Delta\Pi\left(\alpha_{BB^*}\right)}{\partial\alpha}\right)}_{>0}}<0$$

Evaluating the effects of productivity, $\frac{\partial\widetilde{\alpha}}{\partial t}>0$ if:

$$\frac{g(\overline{\alpha}_B^*)}{g(\underline{\alpha}_B)}>\frac{\left(\underline{\alpha}_B-\alpha_S\right)}{\left(\overline{\alpha}_B^*-\alpha_S\right)}\frac{\frac{\partial\underline{\alpha}}{\partial t}}{\frac{\partial\overline{\alpha}_B^*}{\partial t}}$$

As $\frac{\partial\underline{\alpha}}{\partial t}$ and $\frac{\partial\overline{\alpha}_B^*}{\partial t}$ are both finite in $g(\overline{\alpha}_B^*)$ and $g(\underline{\alpha}_B)$, and Proposition 3 proves that $\underline{\alpha}_B>\alpha_S$ and $\overline{\alpha}_B^*>\alpha_S$, there exists a $\frac{g(\overline{\alpha}_B^*)}{g(\underline{\alpha}_B)}$ large enough such that trade liberalization improves aggregate productivity.

B.8 Proof of Proposition 6

Differentiating the equilibrium conditions in (32), (37), (38), and (42), for $\overline{t}(\delta)<t$, we get:

$$\begin{bmatrix}-\frac{\partial\pi^N(\alpha_S)}{\partial\alpha} & \frac{\partial\Delta\Pi^*\left(\underline{\alpha}_B^*\right)}{\partial\alpha} & 0\\ -\frac{\partial\pi^N(\alpha_S)}{\partial\alpha} & 0 & \frac{\partial\Delta\Pi^*(\overline{\alpha}_B^*)}{\partial\alpha}\\ g(\alpha_S) & g(\underline{\alpha}_B^*) & -g(\overline{\alpha}_B^*)\end{bmatrix}\begin{bmatrix}\frac{\partial\alpha_S}{\partial t}\\ \frac{\partial\underline{\alpha}_B^*}{\partial t}\\ \frac{\partial\overline{\alpha}_B^*}{\partial t}\end{bmatrix}=\begin{bmatrix}\frac{\partial\pi^N(\alpha_S)}{\partial t}-\frac{\partial\Delta\Pi^*\left(\underline{\alpha}_B\right)}{\partial t}\\ \frac{\partial\pi^N(\alpha_S)}{\partial t}-\frac{\partial\Delta\Pi^*(\overline{\alpha}_B^*)}{\partial t}\\ 0\end{bmatrix}$$

Continuing, we can write:

$$\begin{aligned}\frac{\partial\alpha_S}{\partial t} &= -\frac{1}{D}\overbrace{\left(-\frac{\overset{-}{\partial\Delta\Pi^*}\left(\overline{\alpha}_B^*\right)}{\partial\alpha}\right)g(\underline{\alpha}_B^*)\left(\frac{\partial\Delta\Pi^*\left(\underline{\alpha}_B^*\right))}{\partial t}-\frac{\overset{-}{\partial\pi^N}\left(\alpha_S\right)}{\partial t}\right)}^{>0}\\ &\quad -\frac{1}{D}\overbrace{\frac{\partial\Delta\Pi\left(\underline{\alpha}_B\right)}{\partial\alpha}g(\overline{\alpha}_B^*)\left(\frac{\partial\Delta\Pi^*\left(\overline{\alpha}_B^*\right)}{\partial t}-\frac{\overset{-}{\partial\pi^N}\left(\alpha_S\right)}{\partial t}\right)}^{>0}>0\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \underline{\alpha}_B^*}{\partial t} &= \frac{1}{D} \overbrace{\left(- \frac{\partial \Delta \overset{-}{\Pi^*} \left(\overline{\alpha}_B^* \right)}{\partial \alpha} g(\alpha_S) \right) \left(\frac{\partial \Delta \Pi^* \left(\underline{\alpha}_B^* \right))}{\partial t} - \frac{\partial \overset{-}{\pi^N} \left(\alpha_S \right)}{\partial t} \right)}^{>0} \\
&\quad - \frac{1}{D} \overbrace{\frac{\partial \pi^N \left(\alpha_S \right)}{\partial \alpha} g(\overline{\alpha}_B^*) \left(\frac{\partial \Delta \Pi^* \left(\overline{\alpha}_B^* \right)}{\partial t} - \frac{\partial \Delta \Pi^* \left(\underline{\alpha}_B^* \right))}{\partial t} \right)}^{>0} \lesseqgtr 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \overline{\alpha}_B^*}{\partial t} &= - \frac{1}{D} \overbrace{\frac{\partial \Delta \Pi \left(\underline{\alpha}_B \right)}{\partial \alpha} g(\alpha_S) \left(\frac{\partial \Delta \Pi^* \left(\overline{\alpha}_B^* \right)}{\partial t} - \frac{\partial \overset{-}{\pi^N} \left(\alpha_S \right)}{\partial t} \right)}^{>0} \\
&\quad - \frac{1}{D} \overbrace{\frac{\partial \pi^N \left(\alpha_S \right)}{\partial \alpha} g(\underline{\alpha}_B) \left(\frac{\partial \Delta \Pi^* \left(\overline{\alpha}_B^* \right)}{\partial t} - \frac{\partial \Delta \Pi^* \left(\underline{\alpha}_B^* \right))}{\partial t} \right)}^{>0} > 0
\end{aligned}
$$

where:

$$
D = \frac{\partial \Delta \Pi^* \left(\underline{\alpha}_B^* \right)}{\partial \alpha} \frac{\partial \Delta \overset{-}{\Pi^*} \left(\overline{\alpha}_B^* \right)}{\partial \alpha} g(\alpha_S) - \frac{\partial \Delta \Pi^* \left(\underline{\alpha}_B^* \right)}{\partial \alpha} \frac{\partial \pi^N \left(\alpha_S \right)}{\partial \alpha} g(\overline{\alpha}_B^*) + \frac{\partial \Delta \overset{-}{\Pi^*} \left(\overline{\alpha}_B^* \right)}{\partial \alpha} \frac{\partial \pi^N \left(\alpha_S \right)}{\partial \alpha} g(\underline{\alpha}_B) < 0
$$

Evaluating the effects of trade costs on aggregate productivity:

$$
\frac{\partial \widetilde{\alpha}}{\partial t} = \frac{\partial \overline{\alpha}_B^*}{\partial t} g(\overline{\alpha}_B^*)(\overline{\alpha}_B^* - \alpha_S) - \frac{\partial \underline{\alpha}_B^*}{\partial t} g(\underline{\alpha}_B^*)(\underline{\alpha}_B^* - \alpha_S)
$$

Differentiating the acquisition market clearing condition, we get:

$$
g(\underline{\alpha}_B^*) \frac{\partial \underline{\alpha}_B^*}{\partial t} = g(\overline{\alpha}_B^*) \frac{\partial \overline{\alpha}_B^*}{\partial t} - g(\alpha_S) \frac{\partial \alpha_S}{\partial t}
$$

Substituting for $g(\underline{\alpha}_B^*) \frac{\partial \underline{\alpha}_B^*}{\partial t}$, $\frac{\partial \widetilde{\alpha}}{\partial t}$ is simplified as:

$$
\frac{\partial \widetilde{\alpha}}{\partial t} = \frac{\partial \overline{\alpha}_B^*}{\partial t} g(\overline{\alpha}_B^*)(\overline{\alpha}_B^* - \underline{\alpha}_B^*) + g(\alpha_S) \frac{\partial \alpha_S}{\partial t} (\underline{\alpha}_B^* - \alpha_S) > 0
$$